Contents

Chapter 1

Electromagnetic Waves

Sound is a compression wave — to travel, it needs a medium to compress: air, water, etc. (Regardless of what you have seen in movies, sound does not travel through a vacuum!)

Light is an electromagnetic wave — it causes fluctuations in the electric and magnetic fields that are everywhere. It can cross a vacuum, as it does to reach us from the sun.

Electromagnetic waves travel at about 300 million meters per second in a vacuum. The waves travel slower through different types of matter. For example, an electromagnetic wave travels at 225 million meters per second in water.

Electromagnetic waves come in different frequencies. For example, the light coming out of a red laser pointer is usually about 4.75×10^{14} Hz. The wifi data sent by your computer is carried on an electromagnetic wave too. It is usually close to 2.4×10^6 Hz or 5×10^6 Hz.

Because we know how fast the waves are moving, we sometimes talk about their wavelengths instead of their frequencies. Since we know its speed and its oscillation rate, we can calculate its wavelength. Since we want the distance per oscillation, we can divide the speed by the frequency. The light coming out of a laser pointer is $300 \times 10^6 / 4.75 \times 10^{14} = 630 \times 10^{-9}$ m, or 630 nm.

Exercise 1 Wavelengths

A green laser pointer emits light at 5.66×10^{14} Hz. What is its wavelength in a vacuum?

Working Space

Answer on Page 53

This can be simplified as:

$$\lambda = \frac{c}{f}$$

where $c = 3 \times 10^8$ m/s (the speed of light in a vacuum)

We have given names to different ranges of the electromagnetic spectrum in Figure 1.1

Name	Hertz	Meters
Gamma rays	$10^{20} - 10^{24}$	$10^{-14} - 10^{-11}$
X-rays	$10^{16} - 10^{20}$	$10^{-11} - 10^{-8}$
Ultraviolet	$10^{15} - 10^{16}$	$10^{-8} - 10^{-7}$
Blue	$\sim 6 \times 10^{14}$	$\sim 5 \times 10^{-7}$
Red	$\sim 4 \times 10^{14}$	$\sim 7 \times 10^{-7}$
Infrared	$10^{12} - 10^{14}$	$10^{-6} - 10^{-3}$
Microwaves	$10^{9} - 10^{12}$	$10^{-3} - 10^{-1}$
Radio waves	$10^{3} - 10^{9}$	$10^{-1} - 10^{5}$

Figure 1.1: A table of different ranges of the electromagnetic spectrum.

(You may have heard of "cosmic rays" and wonder why they are not listed in this table. Cosmic rays are actually the nuclei of atoms that have been stripped of their electron cloud. These particles come flying out of the sun at very high speeds. They were originally thought to be electromagnetic waves, and were mistakenly named "rays".)

In general, the lower frequency the wave is, the better it passes through a mass. A radio wave, for example, can pass through the walls of your house, but visible light cannot. The people who designed the microwave oven chose the frequency of 2.45 GHz because the energy from those waves tended to get absorbed in the first few inches of food that it passed through.

1.1 The greenhouse effect

Humans have dug up a bunch of long carbon-based molecules (like oil and coal) and burned them, releasing large amounts of CO_2 into the atmosphere. It may not be obvious why that has made the planet warmer, but the answer is electromagnetic waves.

A warm object gives off infrared electromagnetic waves. That's why, for example, motion detectors in security systems are actually infrared detectors: even in a dark room, your body gives off a lot of infrared radiation.

You may have heard of "heat-seeking missiles." These are more accurately called "Infrared homing missiles" because they follow objects giving off infrared radiation – hot things like jet engines.

The sun beams a lot of energy to our planet in the form of electromagnetic radiation: visible light, infrared, ultraviolet. (How much? At the top of the atmosphere directly facing the sun, we get 1,360 watts of radiation per square meter. That is a lot of power!)

Some of that radiation just reflects back into space. 23% is reflected by the clouds and the atmosphere, while 7% makes it all the way to the surface of the planet and is reflected back into space.

The other 71% is absorbed. 48% is absorbed by the surface and 23% is absorbed by the atmosphere. All of that energy warms the planet and the atmosphere so that it gives off infrared radiation. The planet lives in equilibrium; the infrared radiation leaving our atmosphere is exactly the same amount of energy as that 71% of the radiation that it absorbs.

(If the planet absorbs more energy than it releases, the planet gets hotter. Hotter things release more infrared. When the planet is in equilibrium again, it stops getting hotter.)

So, what is the problem with CO_2 and other large molecules in the atmosphere? They absorb the infrared radiation instead of letting it escape into space. This means the planet must be hotter to maintain equilibrium.

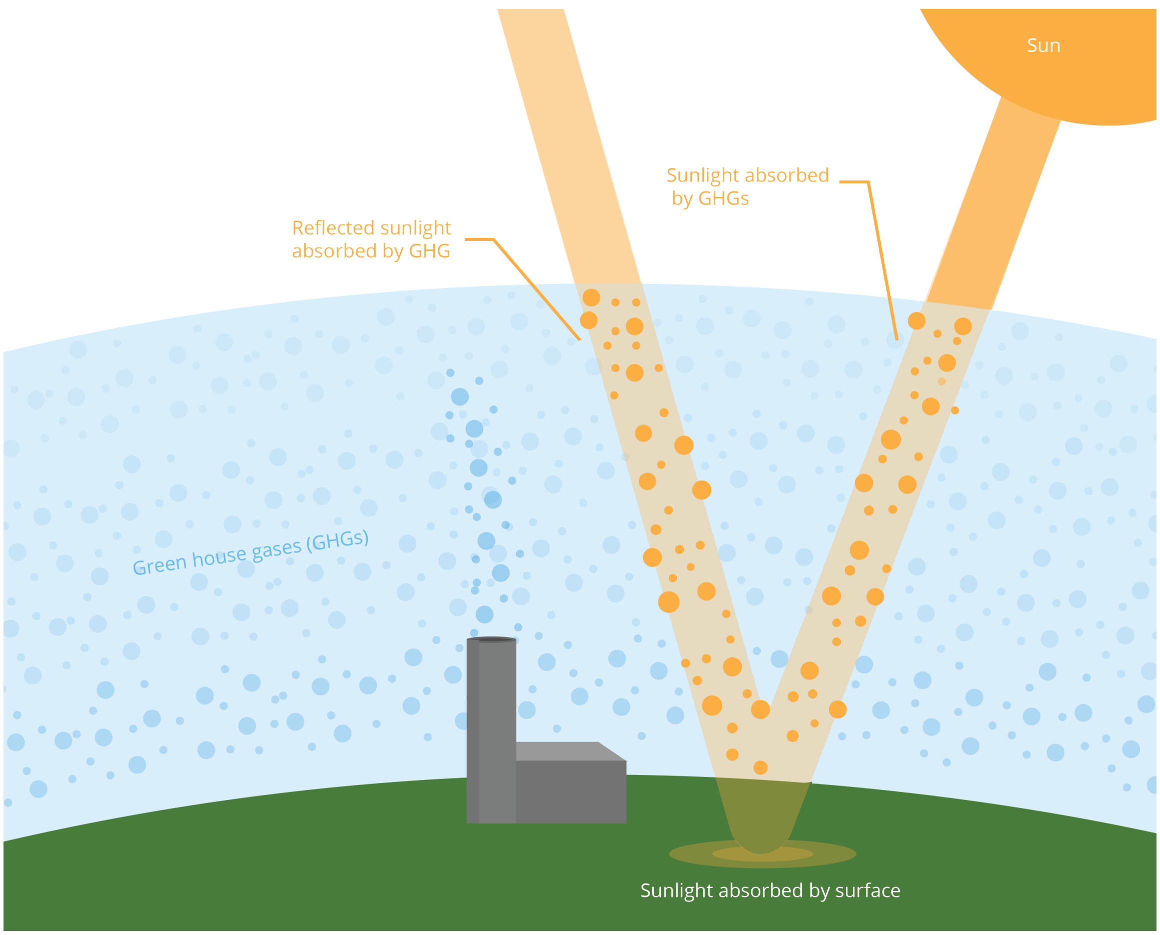


Figure 1.2: Greenhouse gas effects from the solar rays.

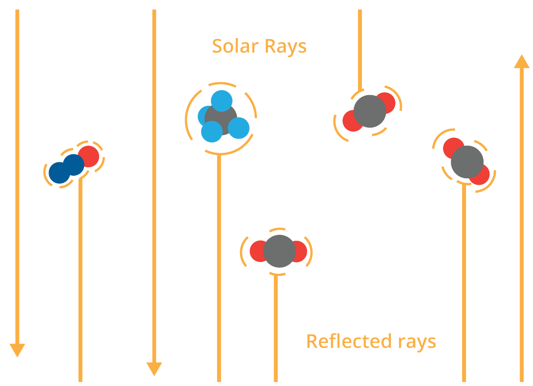


Figure 1.3: A zoomed in version of greenhouse gases.

The planet is getting hotter, and it is creating a multitude of problems:

- Weather patterns are changing, which leads to extreme floods and droughts.
- Ice and snow in places like Greenland are melting and flowing into the oceans. This is raising sea levels.
- Biomes with biodiversity are resilient. Rapidly changing climate is destroying biodiversity everywhere, which is making these ecosystems very fragile.
- In many places, permafrost, which has trapped large amounts of methane in the ground for millenia, is melting.

That last item is particularly scary, because methane is a large gas molecule — it absorbs even more infrared radiation than CO_2. As it escapes the permafrost, the problem will get worse.

Scientists are working on four kinds of solutions:

- **Stop increasing the amount of greenhouse gases in our atmosphere.** It is hoped that non-carbon based energy systems like solar, wind, hydroelectric, and nuclear could let us stop burning carbon. Given the methane already being released, it maybe too late for this solution to work on its own.
- **Take some of greenhouse gases out of our atmosphere and sequester them somewhere.** The trunk of a tree is largely made up of carbon molecules. When you grow a tree where there had not been one before, you are sequestering carbon inside the tree. There are also scientists that are trying to develop a process that pulls greenhouse gases out of the air and turn them into solids.
- **Decrease the amount of solar radiation that is absorbed by our planet and its atmosphere.** Clouds reflect a lot of radiation back into space. Could we increase the cloudiness of our atmosphere? Or maybe launch mirrors into orbit around our

planet?

- **Adapt to the changing climate.** These scientists are assuming that global warming will continue, and are working to minimize future human suffering. How will we relocate a billion people as the oceans claim their homes? When massive heat waves occur, how will we keep people from dying? As biodiversity decreases, how can we make sure that species that are important to human existence survive?

What are the greenhouse gases and how much does each contribute to keeping the heat from exiting to space? These numbers are still being debated, but this will give you a feel:

Water vapor	H_2O	36 - 72 %
Carbon dioxide	CO_2	9 - 26 %
Methane	CH_4	4 - 9 %
Ozone	O_3	3 - 7 %

Notice that while we talk a lot about carbon dioxide, the most important greenhouse gas is actually water. Why don't we talk about it? Given the enormous surfaces of the oceans, it is difficult to imagine any way to permanently decrease the amount of water in the air. Additionally, a great deal of water in the air is in the form of clouds, which help reflect radiation before it is absorbed.

Chapter 2

Reflection

What happens when light hits a mass?

In a previous chapter, we talked about light as a wave, and we mentioned that each color in the rainbow is a different wavelength. You can also think of light as particles of energy called *photons*. Every photon comes with an amount of energy that determines what color it is. When we are talking about light interacting with objects, your intuition will be right more often if you think of light as a beam of photons.

When a photon comes from the sun and hits an object, one of several things can happen:

- The energy of the photon is absorbed by the object. It makes the object a little warmer. If a large proportion of photons hitting the mass are absorbed like this, we say the object is "black".
- The photon bounces off the object. If the surface is very smooth, the photons bounce in a predictable manner, and we call this *reflection* and we say the object is "shiny".
- If the surface is rough and the photons are not absorbed, the photons are scattered in random directions. We call this *diffusion*. If most of the photons hitting an object are bounced in random directions, we say that the object is "white".
- The photon passes through the mass. If the mass has smooth surfaces and a consistent composition, the photons will pass throught the mass in a predictable manner. We say that the mass is "transparent".
- If the photons pass through, but in an unpredictable, scattering manner, we say the mass is "translucent".

No object absorbs every photon, but chemists are always coming up with "blacker" materials. Vantablack, for example, is a super-black paint that absorbs 99.965% of all photons in the visible spectrum.

No object reflects every photon, but a mirror is pretty close. Let's talk about reflections in a mirror.

2.1 Reflection

When a beam of light hits a mirror, it bounces off the mirror at the same angle it approached from. That is, if it approaches nearly perpendicular to the mirror, it departs nearly perpendicular to the mirror. If it hits the mirror at a glancing angle, it departs at an angle close to the mirror's surface. A good video to see this in action can be found here: https://www.youtube.com/shorts/qA_VQZfTiUA

2.1.1 Law of Reflection

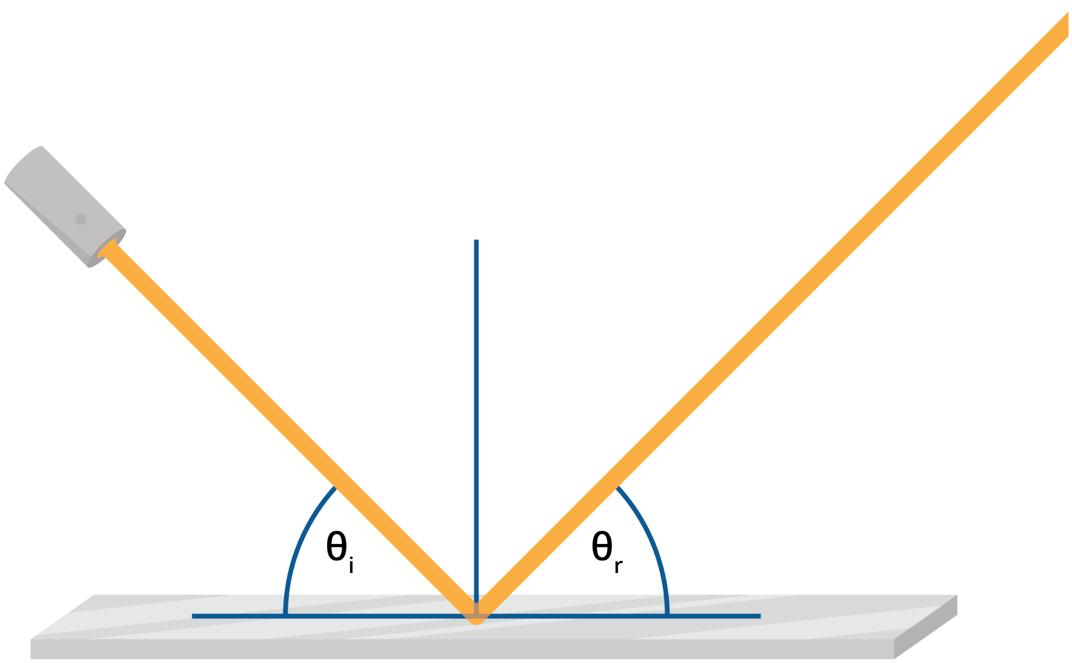


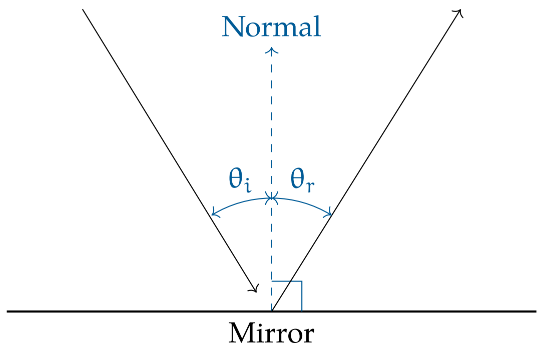
Figure 2.1: The angle of *incidence* is equal to the angle of *reflection*.

Law of Reflection

The angle of incidence, denoted as θ_i, is equal to the angle of reflection, denoted as θ_r. This law can be mathematically expressed as:

$$\theta_i = \theta_r$$

where θ_i is the angle between the incident light ray and the normal to the surface, and θ_r is the angle between the reflected light ray and the normal.



Exercise 2 Law of Reflection

Working Space

You are standing 4 meters from a mirror hung on a wall. The bottom of the mirror is the same height as your chin, so you can't see your whole body. You stick a piece of masking tape to your body.

You walk forward until you are only 3 meters from the mirror, then put a piece of masking tape on your body at the new cut-off point. Is the new masking tape higher or lower on your body?

Answer on Page 53

Exercise 3 Photons and Color

Working Space

There are red photons.

Are there black photons?

Are there white photons?

Are there yellow photons?

Answer on Page 53

2.2 Ellipses and Curved Mirrors

Flat mirrors are common and useful, but things get more interesting once you bend the mirrors. In this section, we are going to talk about a few different kinds of curved mirrors.

2.2.1 The Reflective Properties of Circles and Spheres

For example, if you were inside a circular room (a cylinder, actually), you could imagine standing in the center and pointing a flash light in any horizontal direction. The beam of light would bounce right back to you. See Figure 2.2

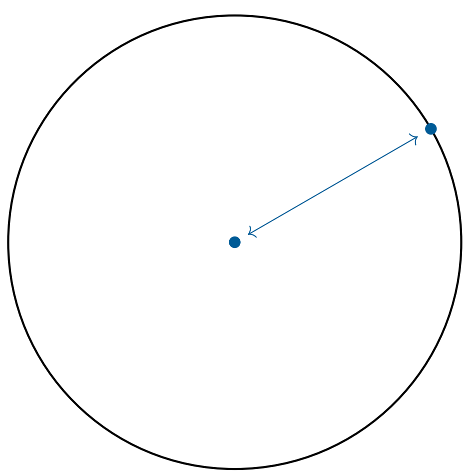


Figure 2.2: A circle would reflect back onto itself.

How do you know this? Because the tangent line is always perpendicular to the radius to the point of tangency:

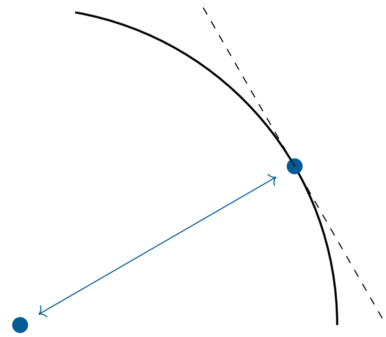


Figure 2.3: The reflection on a circle uses a radius which hits a tangent line as a perpendicular bisector.

You could create a spherical room with mirror walls. You would create a platform in the center where you could stand, and if you pointed your flashlight in any direction, its beam of light would shine back at you.

2.2.2 Ellipses and Ellipsoids

Intuitively, you know what an ellipse is: an oval. However, the ellipse is actually an oval with some special properties. This is a good time to talk about those properties.

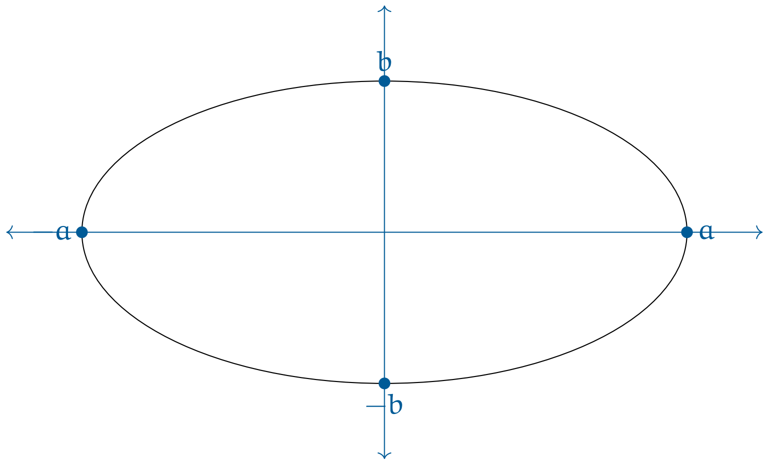
Mathematicians talk about a *standard* ellipse. A standard ellipse is centered on the origin $(0, 0)$ and its long axis is parallel with the x-axis or the y-axis.

Equation for a Standard Ellipse

To be precise, a standard ellipse is the set of points (x, y) that are solutions to the equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Note that $(a, 0), (-a, 0), (0, b), (0, -b)$ are all part of the set. The complete set looks like this:



The area contained inside this ellipse is given by

$$A = \pi a b$$

We can now talk about two special points: the *foci*. Each focal point is on the long axis of the ellipse. Let's assume for a moment that $a > b$. (Everything works the same if $b > a$, but it gets confusing if we try to deal with both cases simultaneously.)

If p is a point on the ellipse, the distance from p to focal point 1 plus the distance from p to focal point 2 is always $2a$.

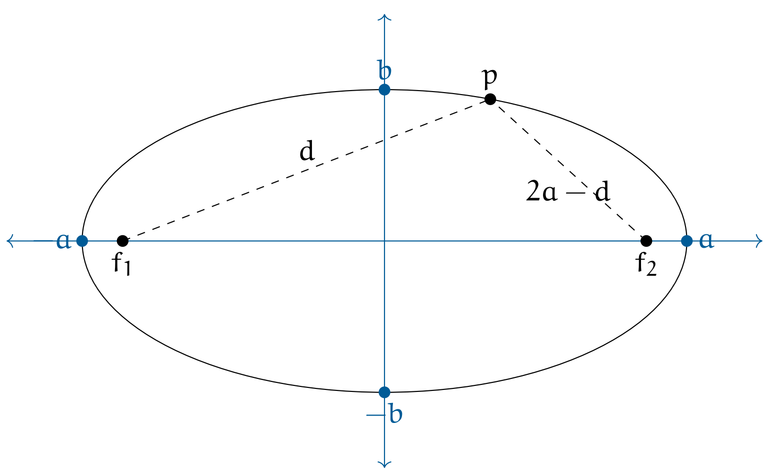


Figure 2.4: The both foci and point p forms a triangle with legs $2a - d$ or d.

How do we find the foci? We know they are on the long axis and that they are symmetrical across the short axis. All we need to know is how far are they from the short axis.

Distance from Center to the Foci

If you have an ellipse with a long axis that extends a from the center, and a short axis that extends b from the center, the foci lie on the long axis and are c from the center. Where

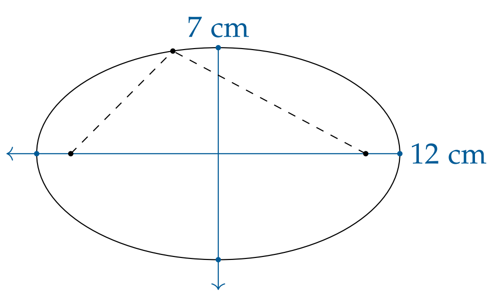
$$c = \sqrt{a^2 - b^2}$$

Exercise 4 Foci of an ellipse

Working Space

You need to draw an ellipse that is 12 cm long and 7 cm wide. You have a string, two pushpins, a ruler, and a pencil. Using the ruler, you draw two perpendicular axes.

You will stick one pin at each focal point. Each end of the string will be tied to a push pin. Using the pencil to keep the string taut, you will draw an ellipse.



How far from the short axis are the pushpins placed?

How long is the string between them?

Answer on Page 54

The Reflective Property of Ellipses

Here is something else that is wonderful about an ellipse: Pick any point p on the ellipse. Draw a line from p to each focal point. Draw the line tangent to the ellipse at p. You will see that the angle between the tangent and the line to focal point 1 is equal to the angle between the tangent and the line to focal point 2. See Figure 2.5

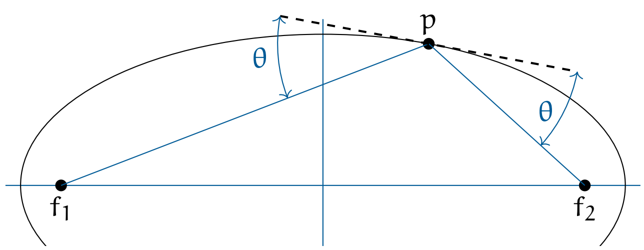


Figure 2.5: Reflective Property of Ellipses.

This is known as "The Reflective Property of Ellipses". Imagine you and your friend Fred are at an ellipse-shaped skating rink, and the edge of the rink is mirrored. You sit at one focal point and your friend sits at the other. If you point a flashlight at the mirror (in any direction!), the beam will bounce off the wall and head directly for Fred.

If Fred ducks out of the way, the beam will bounce again and head back to you.

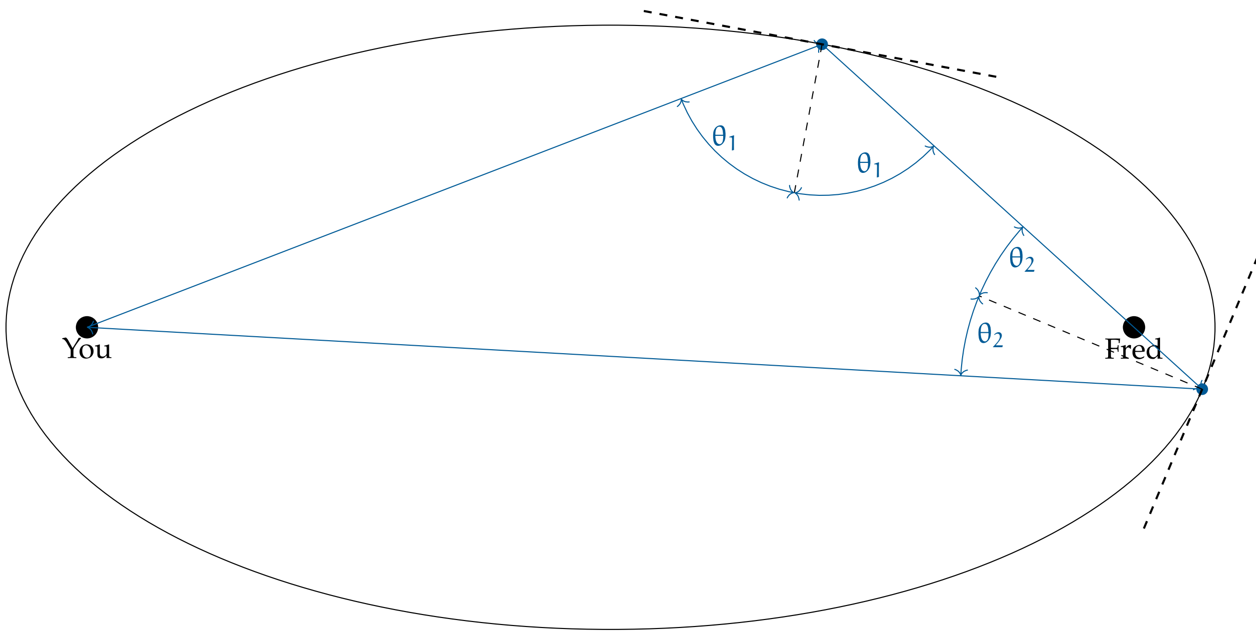


Figure 2.6: Your laser intersects Fred.

This will work for sound as well. If you whisper while on the focal point, Fred (at the other focal point) will hear you surprisingly well, because all the soundwaves that hit the wall will bounce (just like the light) straight at Fred.

2.2.3 Elliptical Orbits

One more fun fact about ellipses: We often imagine the planets traveling in circular orbits with the sun at the center — they actually travel in elliptical orbits, with the sun as one of the focal points.

The Earth is closest to the sun around January 3rd: 147 million km.

The Earth is farthest from the sun around July 3rd: 152 million km.

(Note that these dates are not the same as the solstices: The southern hemisphere is tilted the most toward the sun around December 21 and tilted most away around June 21.)

2.2.4 Ellipsoids

Just as we can pull the ideas of a circle into three dimensions to make a sphere, we can extend the ideas of the ellipse into three dimensions to talk about ellipsoids. Ellipsoids are like blimps.

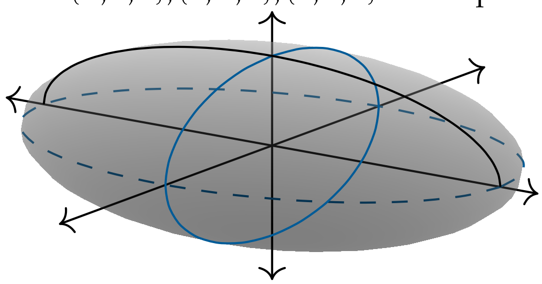
The standard ellipsoids are centered at the origin and aligned with the three axes.

Equation for a Standard Ellipsoid

To be precise, a standard ellipse is the set of points (x, y, z) that satify the equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

Note that $(a, 0, 0), (0, b, 0), (0, 0, c)$ are all part of the set. The complete set looks like



this:

The volume bounded by this ellipsoid is

$$V = \frac{4}{3}\pi abc$$

Of course, a, b, and c can be any positive number, but in the real world, we find ourselves working regularly with ellipsoids where two of the numbers are the same.

Oblate Spheroid

If two axes have the same length and one is shorter, you get something that looks like a sphere compressed in one direction — like a pumpkin. These are called *oblate spheroids*.

The Earth is actually an oblate spheroid; the axis that goes through the north and south pole is shorter than the axes that pass through the equator. How much shorter? Just a little, relatively speaking. The equator is 6,378 km from the center of the Earth; the north pole is 21 km closer.

Prolate Spheroid

If two axes have the same length and one is longer, you get something that looks like a sphere stretched in one direction — like a rugby ball. It is called a *prolate spheroid*.

Like an ellipse, prolate spheroids have two focal points.

Focal Points of a Prolate Spheroid

If the long axis has a radial length of a and the two shorter axes have radial length b, then the focal points are on the long axis. The distance from the center to the focal point is given by

$$c = \sqrt{a^2 - b^2}$$

For any point p on the prolate spheroid, the sum of the distances from p to the focus points will always be $2a$.

It has the reflective property: A photon shot in any any direction from one focal point will bounce off the wall and head directly at the other.

Exercise 5 Volume of Ruby Ball

Working Space

Some jokesters once thought it would be fun to make something that looked like a rugby ball, but made out of lead.

A rugby ball is about 30 cm long and has a circumference of 60 cm at its midpoint. A cubic centimeter of lead has a mass of 11.34 grams.

How much would a solid (not hollow) lead ruby ball weigh?

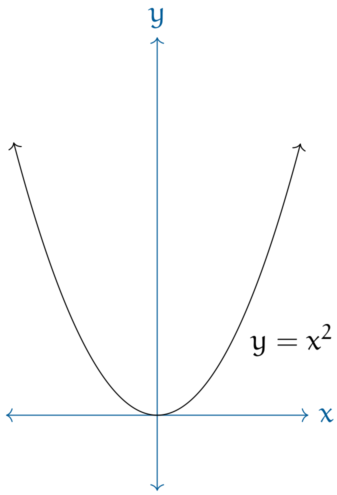
Answer on Page 54

2.2.5 Parabolas and Parabolic Reflectors

You are familiar with quadratic functions:

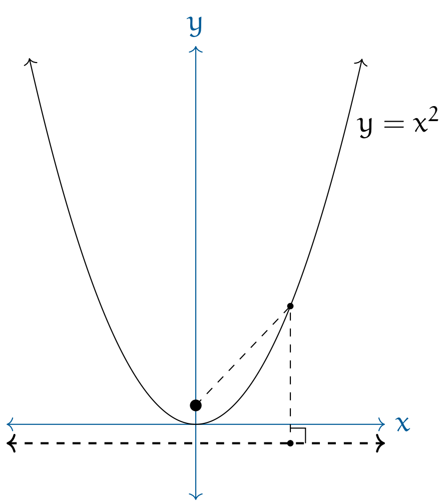
$$y = ax^2 + bx + c$$

If a is not zero, the graph of a quadratic is a curved line called a *parabola*. The first parabola that most mathematicians think of is the graph of $y = x^2$:



Every parabola has a *focus* and a *directix*. The focus is a point on the parabola's axis of symmetry. The directrix is a line perpendicular to the axis of symmetry. Every point on the parabola is equal distance from the focus and the directrix.

For the graph of $y = x^2$, the focus is the point $(0, \frac{1}{4})$. The directrix is the line $y = -\frac{1}{4}$:



For example, the point $(1, 1)$ is on this parabola. It is $5/4$ from the directorix. How far is it from the focus? 1 horizontally and $3/4$ vertically.

$$\sqrt{1^2 + \left(\frac{3}{4}\right)^2} = \frac{5}{4}$$

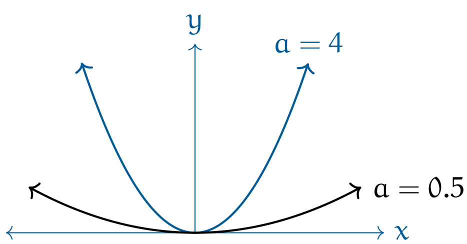
Thus, we have confirmed that $(1, 1)$ is equal distances from the focus and the directrix.

When we think about a parabola and its properties, we usually rotate and translate it to be symmetric around the y-axis, flip it so that it is low in the middle and rising on both sides, and push it up or down until the low point is is on the x-axis.

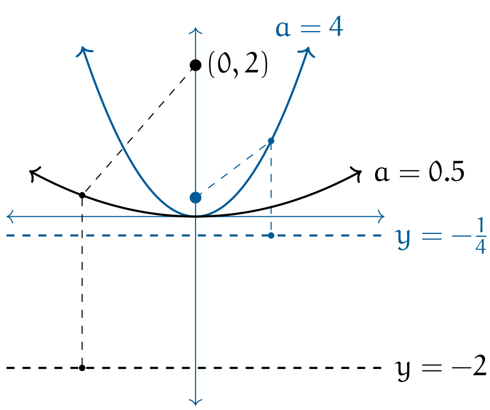
Then, they can all be written:

$$y = \frac{a}{4}x^2$$

where $a > 0$. If a is small, the parabola opens wider.

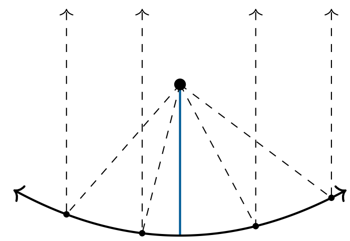


The focus is at $(0, \frac{1}{a})$ and the directorix is the line $y = -\frac{1}{a}$.



Reflective Property of a Parabola

Assume you have a parabola-shaped mirror. A beam of light shot from the focus will bounce off the mirror in the direction of the axis of symmetry:



This is why your flashlight has a parabolic mirror. The lightbulb is at the focus, so any photons that hit the mirror are redirected straight forward.

(Note that in the real world, we use parabolic dishes: a parabola rotated around its axis of symmetry.)

The reflection works exactly the same in reverse. There are solar cookers that are big parabolic mirrors. They let you put a pot on the focus point. You move the dish until its axis of symmetry is pointed at the sun.

You will also see a lot of antennas have parabolic dishes. Note that photons that come in parallel to the axis of symmetry are redirected to a single point: where the receiver is.

Figure 2.7: A real life parabolic dish as a satellite.

Sometimes in a science museum, you will see two parabolic dishes far apart and pointed at each other. One person speaks with their mouth at the focus of one. The other person listens with their ear at the focus of the other. Even though you are very far apart, it sounds like they are really, really close.

Figure 2.8: Two parabolic dishes send waves between eachother.

This is because the speaker's parabolic wall focuses the sound energy in a nice beam the size of the wall pointed straight at the listener's parabolic wall. The listener's wall focuses the energy of that beam at the listener's ear.

Chapter 3

Refraction

The refraction of light is a phenomenon where light changes its direction when it passes from one medium to another. The change in direction is due to a change in the speed of light as it moves from one medium to another.

This phenomenon is explained by Snell's law, which states:

$$n_1 \cdot \sin(\theta_1) = n_2 \cdot \sin(\theta_2) \tag{3.1}$$

where:

- n_1 and n_2 are the indices of refraction for the first and second media, respectively. The index of refraction is the ratio of, c the speed of light in a vacuum to, v the speed of light in the medium, $n = \frac{c}{v}$. It is a dimensionless quantity. $n \geq 1$ for all materials, such that $n_{\text{vacuum}} = 1$ and $n_{\text{air}} \approx 1$. See a list of all mediums here: https://en.wikipedia.org/wiki/List_of_refractive_indices
- θ_1 and θ_2 are the angles of incidence and refraction, respectively. These angles are measured from the normal (perpendicular line) to the surface at the point where light hits the boundary.

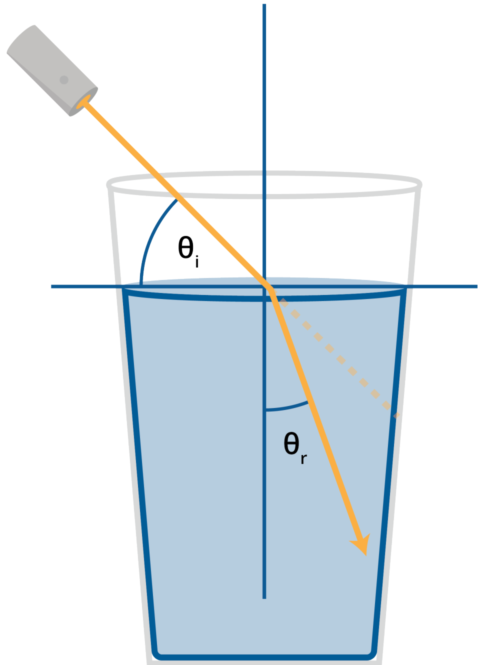


Figure 3.1: Refraction occurs when light changes the medium it is in.

The angle of incidence (θ_1) is the angle between the incident ray and the normal to the interface at the point of incidence. Similarly, the angle of refraction (θ_2) is the angle between the refracted ray and the normal.

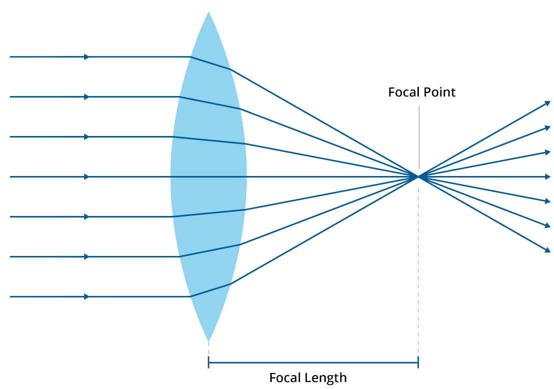
When light travels from a medium with a lower refractive index to a medium with a higher refractive index, it bends towards the normal. Conversely, when light travels from a medium with a higher refractive index to one with a lower refractive index, it bends away from the normal.

Chapter 4

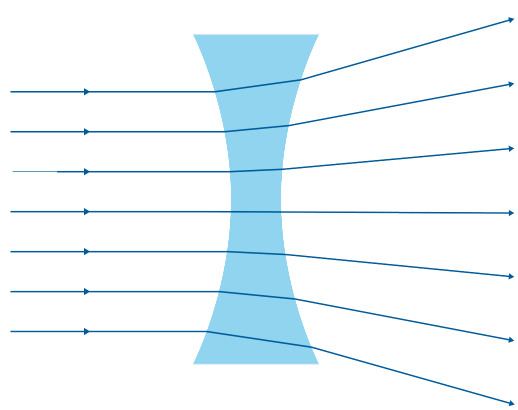
Lenses

Lenses are optical devices with perfect or approximate axial symmetry that transmit and refract light, converging or diverging the beam. There are two main types of lenses, distinguished by their shape and the way they refract light:

- **Converging (or Convex) Lenses:** These are thicker at the center than at the edges. When parallel light rays enter a convex lens, they converge to a point called the focal point. Examples of converging lenses include magnifying glasses and camera lenses.



- **Diverging (or Concave) Lenses:** These are thinner at the center than at the edges. When parallel light rays enter a concave lens, they diverge or spread out. These lenses are often used in glasses to correct nearsightedness.



4.1 Focal Length

The focal length of a lens is the distance between the center of the lens and the focal point. It is determined by the lens shape and the refractive index of the lens material. For a converging lens, the focal length is positive; for a diverging lens, the focal length is negative.

4.2 Refractive Index

The refractive index of a material is a measure of how much the speed of light is reduced inside the material. The refractive index n of a material is given by the ratio of the speed of light in a vacuum c to the speed of light v in the material:

$$n = \frac{c}{v}$$

The refractive index affects how much a light ray changes direction, or refracts, when entering the material at an angle. A higher refractive index indicates that light travels slower in that medium, and the light ray will bend more towards the normal.

Lenses work by refracting light at their two surfaces. By choosing the right lens shape and material, lenses can be designed to bring light to a focus, spread it out, or perform more complex transformations.

Chapter 5

Oscillations

We just finished up a chapter about circular motion, where objects move around in a circle at either constant (uniform/harmonic) or changing (non-uniform) speeds.

In this chapter, we will study oscillatory motion, which is a type of motion where an object moves back and forth around an equilibrium point. Oscillations are common in many physical systems, such as springs, pendulums, and even electrical circuits. Oscillations are important not only because they appear everywhere in physical systems, but also because they provide a connection between simple motion and more complex behaviors. Small oscillations often behave in a very predictable way and can be described using *simple harmonic motion* (SHM), which is governed by restoring forces proportional to displacement.

We will also give an introduction to *differential equations* used to model oscillatory systems, often found in engineering and physics.

5.1 Sine and Cosine Graphs [Review]

Let's recall the two pillars of trigonometry and, consequently, simple harmonic motion: Sine and Cosine. Recall that these two functions are periodic, meaning they repeat their values in regular intervals or periods. The sine and cosine functions have a period of 2π radians or 360°, repeating their values every 2π radians.

As discussed in the circular motion chapter, the sine function represents the y-coordinate of a point on the unit circle as it moves around the circle, while the cosine function represents the x-coordinate. This is the fundemental behavior of the unit circle, where a point moves around the circle at a constant angular speed ω. The position of the point at any time t can be described using the following parametric equations:

$$x(t) = \cos(\omega t), \qquad y(t) = \sin(\omega t)$$

The general forms of the sine and cosine functions can be expressed as:

$$x(t) = A\cos(\omega t + \phi), \qquad y(t) = A\sin(\omega t + \phi)$$

where:

- A is the amplitude, representing the maximum displacement from the equilibrium position.
- ω is the angular speed, which determines how quickly the oscillations occur.
- ϕ is the phase shift, which determines the initial position of the oscillation at $t = 0$.

Recall that the derivatives of these functions are also periodic, a key property that is useful for analyzing oscillatory motion:

$$\frac{d}{dt}\cos(\omega t) = -\omega \sin(\omega t)$$

$$\frac{d}{dt}\sin(\omega t) = \omega \cos(\omega t)$$

$$\frac{d^2}{dt^2}\cos(\omega t) = -\omega^2 \cos(\omega t)$$

In the motion we will talk about,

- $x(t)$ will be our position function
- $v(t)$ describes our velocity function, which can be the first derivative of our position
- $a(t)$ is our acceleration function, derived from $v(t)$

We notice that the second derivative produces a negative multiple of the original function. This can be simplified in terms of acceleration and position functions: $a(t) = -\omega^2 x(t)$.

5.2 Simple Harmonic Motion

Simple Harmonic Motion will reference many definitions we have seen before and will continue to see in this chapter. Here are some key terms to know:

Cycle A single complete execution of a periodically repeated motion. Another way to think about it is a particle, block, or object making a *round-trip*.

Period The time it takes to complete one cycle of motion. An identity that comes up often is $T = \frac{2\pi}{\omega}$.

Frequency The number of cycles completed per unit time. It is the reciprocal of the period, represented as **Hz**.[1]

[1] One **Hz** is equal to one cycle per second. For example, a frequency of 5 Hz means that 5 cycles are completed in one second. On a monitor, a refresh rate of 60 Hz means the screen updates 60 times per second. Higher refresh rates can lead to smoother motion perception.

Equilibrium The position at which the "default" position of an oscillating object is located. In terms of mechanics, this is the position where all the forces acting on the object are balanced or zero.

Restoring Force The force that acts to bring an oscillating object back to its equilibrium position. It is typically proportional to the displacement from equilibrium and acts in the opposite direction.

Angular Speed The rate at which an object moves around a circle, measured in radians per second.

So far, we have discussed oscillatory motion in general terms. Now, let's focus on a specific type of oscillatory motion known as Simple Harmonic Motion (SHM). SHM is characterized by the following properties:

- The motion occurs around an equilibrium position.
- The *restoring force* is directly proportional to the displacement from the equilibrium position and acts in the opposite direction.
- The motion is sinusoidal in nature, meaning it can be described using sine or cosine functions.
- The period and frequency of the motion are constant, regardless of the amplitude.
- The acceleration always points toward equilibrium and increases in size as the displacement grows.

So the motion continously repeats itself in a regular pattern. It speeds up near equilibrium, and slows down to a stop at maximum displacement. Examples of these systems are

- A mass attached to a spring, either vertically or horizontally
- A simple pendulum
- Vibrations of a tuning fork or guitar string

We saw in our pendulums chapter how the system has both centripetal acceleration and acceleration due to gravity, causing oscillations back and forth with changing speed. Like all mechanical systems, an SHM system has both kinetic and potential energy. For general simple harmonic motion systems, we can describe its energy as follows:

- The total mechanical energy E of the system is the sum of its kinetic energy K and potential energy U:
$$E = K + U$$
- The kinetic energy K, system dependent, is highest when the object passes through the equilibrium position.

- The potential energy U depends on the specific system, such as spring potential energy or gravitational potential energy, but is highest at the maximum displacement from equilibrium.

5.3 Springs

Let's talk about springs! Springs are mechanical devices, usually made out of some rigid metal material, that store and release energy through deformation. When a spring is stretched or compressed from its equilibrium position, it exerts a restoring force that tries to bring it back to its original shape. The most common type of spring is a coil spring, which consists of a linear wire would into a helical shape.

5.3.1 Hooke's Law

In 1676, physicist Robert Hooke formulated a principle that describes the behavior of springs, now known as Hooke's Law. Hooke's Law states that the force exerted by an ideal spring is directly proportional to the displacement from its equilibrium position.

Mathematically, Hooke's Law can be expressed as:

$$F_s = -kx$$

where:

- F_s is the restoring force exerted by the spring (in newtons, N)
- k is the spring constant (in newtons per meter, N/m), which measures the stiffness of the spring. It is spring-specific
- x is the displacement from the equilibrium position (in meters, m)
- The negative sign represents the fact that the force attempts to bring the spring back to its equilibrium position.

Since we know that force is related to mass and acceleration through Newton's Second Law, we can combine Hooke's Law with Newton's Second Law to analyze the motion of a mass attached to a spring. See Figure 5.1 for a diagram of a mass-spring system.

$$F_s = ma = -kx \tag{5.1}$$

$$ma = -kx \tag{5.2}$$

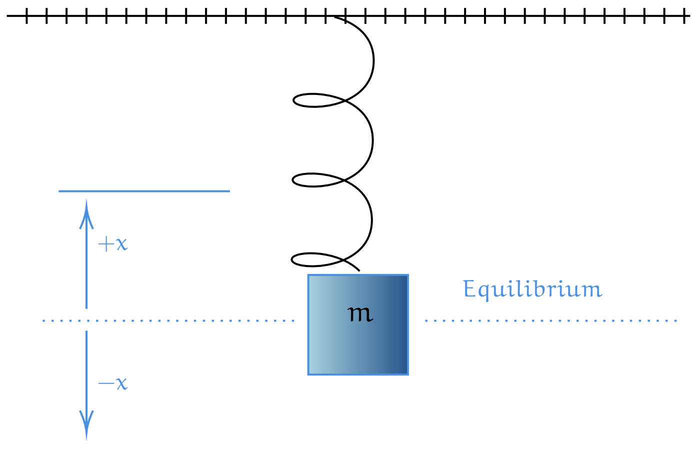


Figure 5.1: A diagram showing a block of mass m displaced $+x$ and $-x$ from equilibrium.

But since above we found a relation between acceleration and position, we can write:

$$m\frac{d^2x}{dt^2} = -kx \tag{5.3}$$

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \tag{5.4}$$

We can see that this equation describes simple harmonic motion, where the acceleration is proportional to the negative of the displacement. In this form, we can identify the angular speed ω of the oscillation as:

$$\omega = \sqrt{\frac{k}{m}}$$

as the term in front of the x in equation (5.4) is ω^2. Thus, we can further define the period as

$$T = 2\pi\sqrt{\frac{m}{k}}$$

and the frequency as

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

In a later section of this chapter, we will solve this differential equation to find the position function $x(t)$ of general mass-spring systems.

Exercise 6 Mass-Spring Frequency Change

A block is attached to a spring and set into oscillatory motion, and its frequency is measured. If this block were removed and replace with a block of 1/2 the mass, how would the frequency of oscillations compare to the original frequency?

1. The frequency would be halved.
2. The frequency would remain the same.
3. The frequency would increase by a factor of $\sqrt{2}$.
4. The frequency would decrease by a factor of $\sqrt{2}$.

Working Space

Answer on Page 54

5.3.2 Spring Potential Energy

Let's recall that Work from a physics standpoint is defined as the transfer of energy through a certain distance via a force. When we alter the length of a spring through either compression or extenson, we are performing work on the spring. This work is stored as potential energy within the spring, which can be released when the spring returns to its equilibrium position.

Let's find an equation for this change in potential energy, which gets stored up in the spring. Recall the two mathematical definitions for work:

- Work as the product of force and distance: $W = F \cdot d$
- Work as the integral of force over a distance: $W = \int F \, dx$

We need to use the integral definition, since, by our definition of Hooke's Law, the force exerted by a spring varies with displacement, so every tiny bit of distance, or dx is different. For any spring starting at equilibrium (0), and moving to some final position x_f, we can write:

$$\begin{aligned} W &= \int_{x_i}^{x_f} F_x \, dx \\ W_s &= \int_{x_i}^{x_f} F_s \, dx \\ &= \int_0^{x_f} -kx \, dx \\ &= \left. \frac{-kx^2}{2} \right|_0^{x_f} \\ &= \frac{-k(x_f)^2}{2} \end{aligned}$$

This work done on the spring is stored as potential energy, so we can say that the change in potential energy of the spring is equal to the negative of the work done by the spring force:

$$\Delta U_s = -W_s = \frac{1}{2} k x_f^2 \tag{5.5}$$

We can see this graphically in Figure 5.2, where the area under the force vs. displacement graph represents the work done on the spring, which is equal to the change in potential energy.

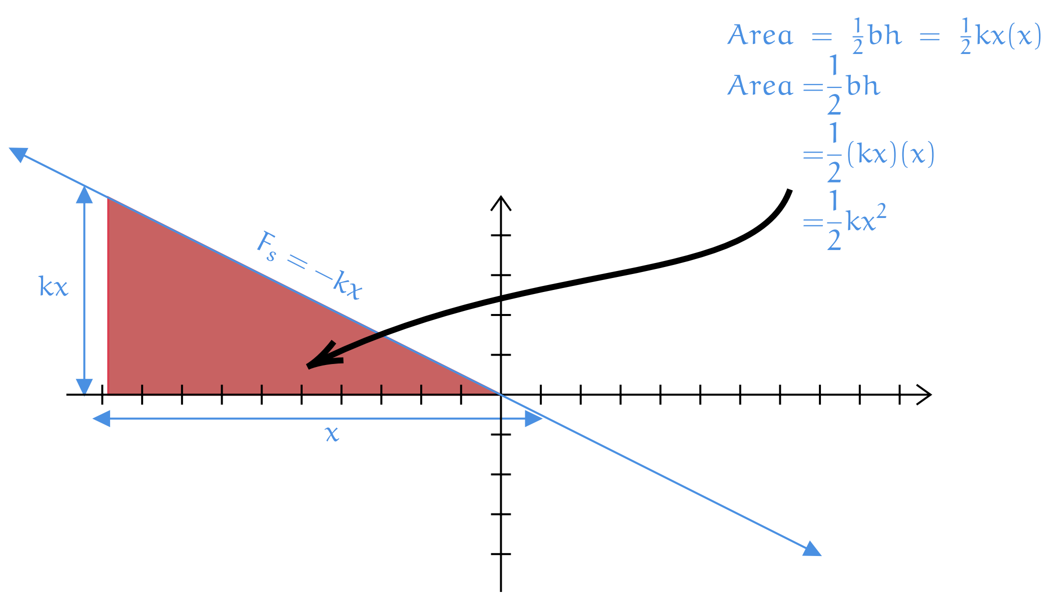


Figure 5.2: A graph of force versus displacement for a spring, equal to the work done and change in potential energy.

Notice that Equation (5.5)is similar in structure to the kinetic energy equation,$K = \frac{1}{2}mv^2$. This is not a coincidence, as both equations describe forms of energy in mechanical systems, and are derived from linear systems. The big idea is: When the effort (force) needed increases in direct proportion to what you're trying to change, the energy required ends up depending on the square of that change. For a spring

- the farther you stretch or compress it, the more force it exerts to return to equilibrium (Hooke's Law)
- the more energy is stored in the spring (spring potential energy)

For kinetic energy

- the faster you try to move an object, the more force is needed to accelerate it (Newton's Second Law)
- the more energy the object has due to its motion (kinetic energy)

Exercise 7 Finding the equation for a mass-spring system

This question is from an AP-style physics review. A block of mass 4 kg on a frictionless, horizontal table is attached to a spring with spring constant $k = 400\,\text{N/m}$ and undergoes simple harmonic oscillations about its equilibrium position and its amplitude $A = 6$ cm. If the block is at $x = 6$ cm at time $t = 0$ and has a velocity of 0 cm/s at that time, which of the following equations (with x in cm and t in seconds) describes the block's position as a function of time?

(A) $x(t) = 6\cos(10t)$

(B) $x(t) = 6\sin(10t)$

(C) $x(t) = 6\sin(10t + \frac{1}{2}\pi)$

(D) $x(t) = 6\sin(10\pi t + \frac{1}{2}\pi)$

(E) $x(t) = 6\sin(10\pi t - \frac{1}{2}\pi)$

Working Space

Answer on Page 55

Exercise 8

Working Space

This question is from an AP-style physics review.

Which of the following characteristics are true of simple harmonic motion? Select all that apply.

I The acceleration is constant.

II The restoring force is proportional to the displacement from equilibrium.

III The frequency is independent of the amplitude.

(a) II only

(b) I and II only

(c) I and III only

(d) II and III only

Answer on Page 56

Exercise 9 Energy Transfer in a Mass-Spring System

Working Space

A pinball machine uses a spring to launch a ball of mass 0.2 kg. The spring has a spring constant of 500 N/m and is compressed by 0.24 m before release. Assuming energy is conserved and no frictional losses occur, what is the speed of the ball as it leaves the spring?

Answer on Page 56

5.4 Mass-Spring Systems and Linear Differential Equations

We have already seen that the motion of a mass attached to a spring can be model e d using different ial equations. Here, we will examine this connection more clo sel y.

5.4.1 Undamped Simple Harmonic Motion

Starting from Newton's Second Law and Hooke's Law, we derived the differential e quation for simple harmonic motion:

$$m\frac{d^2x}{dt^2} + kx = 0. \tag{5.6}$$

The term$m\,\frac{d^2x}{dt^2}$comes directly from *Newton's Second Law*, representing the net force required to accelerate the mass. The termkxari ses from *Hooke's Law*, which states that the spring exerts a restoring fo rce proportional to displacement and directed toward equilibrium. Equation (5.6) is an example of a **second-order linear differential equation with constant coefficients without damping**. It describes an ideal mass-spring s ystem with no exter nal influences—one that oscillates forever, undamped and unfor ced. The$ma(t)$term is *Newton's Second Law*part of the equation, while the $kx(t)$ term comes from *Hooke's Law*. This is a second-order linear diff erential equation with constant coefficie nts. What if the spring system is be ing driven by a third force, like friction or damping? We can add an additional force term$F(t)$to th e equation:$F_f(t) = -cv = -cx'$for forcing function.

5.4.2 Introducing Damping

Real systems are often influenced by additional forces. One common example is a **damping force**, such as friction or air resistance, which opposes the mo tion of the mass. A typical damping f orce is proportional to the velocity and ca n be wri tten as

$$F_d(t) = -c\,\frac{dx}{dt},$$

where$c$is the damping coefficient. Including this force in Newton's Second Law m odifies our differe ntial equation. Instead of the undamped equation (5.6), we obtain:

$$ma(t) = -c\,v(t) - k\,x(t) \tag{5.7}$$

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = 0 \tag{5.8}$$

This is still a second-order linear differential equation with constant coeffici e nts, but its solutions behave very differently. Depending on the value of c, th e system may oscillate with decreasing amplitude, fail to oscillate at all, or re t urn to equilibrium as quickly as possible. Finding the roots of the characteristic equation associated with Equation (5.8) allows us to classify the system's behavior into three categories. Recall that the quadratic formula gives us the roots using our coefficients:

$$r = \frac{-c \pm \sqrt{c^2 - 4mk}}{2m}$$

Without going into too much linear algebra, the function of a mass spring is giv e n

Characteristic Roots	Homogeneous Solution	Damping Type
$r_1, r_2 \in \mathbb{R}$, both negative	$y_h = C_1 e^{r_1 t} + C_2 e^{r_2 t}$	Overdamped
r real, repeated	$y_h = (C_1 + C_2 t)e^{rt}$	Critically damped
$r = \alpha \pm i\beta$, $\alpha < 0$	$y_h = e^{\alpha t}(C_1 \cos(\beta t) + C_2 \sin(\beta t))$	Underdamped
$r = \pm i\beta$	$y_h = C_1 \cos(\beta t) + C_2 \sin(\beta t)$	Simple Harmonic Motion

Table 5.1: Classification of solutions to the damped harmonic oscillator based on characteristic roots.

by$y(t) = y_h(t) + y_p(t)$, wherebyy_h is the homogeneous solution (the p art we solved above by setting the differential equation to 0), and y_p is the p articular solution, which depends on any external forcing functions. In this section, we focused on the homogeneous solution. If the equation (5.8) had a forcing function$F(t)$on the right side, we would need to find a particular solutiony_p to account for that.

Exercise 10 Linear Systems 1

Working Space

A 2 kg mass is attached to a spring with $k = 200\,\text{N/m}$ with no damping. Which differential equation describes the motion?

1. $x'' + 100x = 0$
2. $x'' + 10x = 0$
3. $x'' + 200x = 0$
4. $2x'' + 100x = 0$

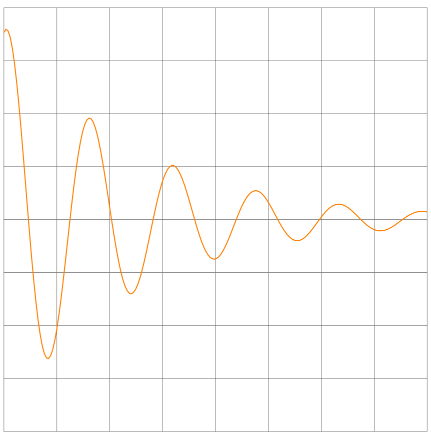
Answer on Page 56

Exercise 11 Linear Systems 2

Work

Answer the following questions about the graph below, which shows the position of a mass on a spring as a function of time. The equation of motion for this system is given by:

$$x(t) = 0.15e^{-0.40t}\cos(4t + 0.30)$$



- Is this system undergoing simple harmonic motion, or is it damped? If it is damped, is it underdamped, overdamped, or critically damped?
- What would need to be changed in the equation of motion to make this system undergo simple harmonic motion instead?

Answer on Page 56

Exercise 12 Linear Systems 3

Working Space

Given that a unit (1 kg) mass obeys:

$x'' + 16x = 0, \quad x(0) = 0.20, \quad v(0) = x'(0) = 0$

1. Solve for $x(t)$
2. Find the acceleration, $a(t)$, and the velocity, $v(t)$, as functions of time.

Answer on Page 57

5.5 Pendulums

You may be surprised to know that pendulums act the exact same way as roller coa s ters from the circ ular chapter, in terms of forces. But, similar to springs, th e oscillate back and forth unless acted upon by a damping force. A pendulum has a n equilibrium position$\theta = 0$such that it is parallel to its gravitational com p onent. A pendulum also has a maximum angle$\theta_{max}$from its equilibrium p os i tion. It will swing through a set arc in a repetitive motion, which is a type of m otion we call simple harmonic motion, similar to springs (which we will cover al s o cover in the oscillations chap ter). The tension force is always directed towards the piv ot. This is diagramed below in Figure 5.3

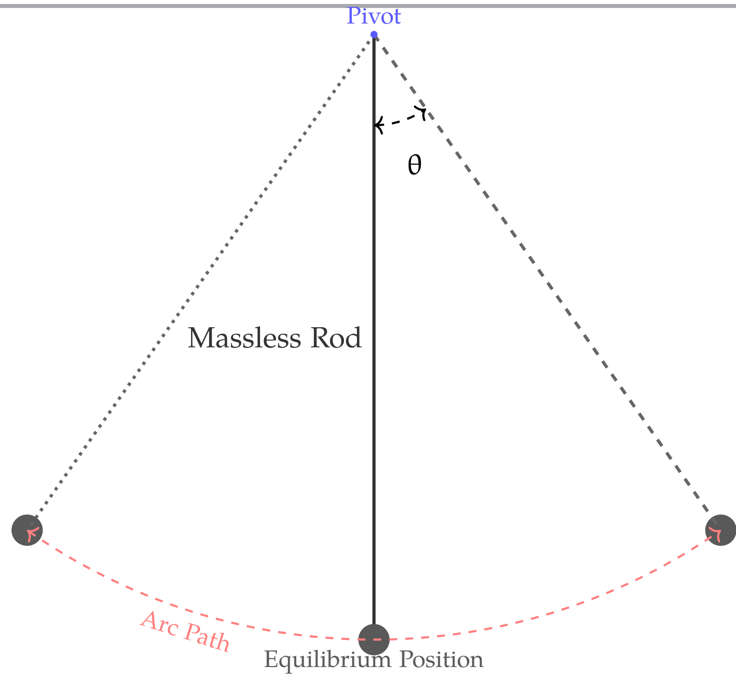


Figure 5.3: The anatomy of a pendulum.

When displaced by an angle$\theta$from the vertical equilibrium position ($\theta = 0$), gravity prov ides a *restoring force*that acts along the arc of motion . This restoring force is given by$F = -mg \sin\theta$, where the negative sign in dicates that the force always points back toward the equ i librium position. The te nsion in the string keeps the bob moving along its curved path, while gravi t y con tinually tries to return it to rest. The net force shifts such that the acceler a tion arrow i s *not*always pointing to the center. The net force is a combi n ation of the tension (centripet a l) force and the gravitational (tangential), so r ather it points at different points along the arc o f the circle, as it as alway s changing due to the height of the pendulum. This is demonstrated in Fi gure 5.4. Note the following differences in energy and forces at the e quilibrium po sitio n versus the maximum angle positions, summarized in Table 5.2. Note here that the magnitude of the restoring force$-mg\sin(\theta)$, which is no t

At $\theta = 0$	**At** $\theta = \theta_{max}$
$F_{restoring} = 0$	$F_{restoring} = \max$
a_t (tangential acceleration) $= 0$	$a_t = \max$
$PE = 0$	$PE = mgh$
$KE = \max$	$KE = 0$
$V = \max$	$v = 0$

Table 5.2: Comparison of pendulum quantities at equilibrium and maximum displacement

*directly*pr oportional to the angular displacementθ. However, for s mall angles (typically less than about 1 radian or 180°), we can use the small-angle approximation$\sin(\theta) \approx \theta$(in r adians), and state the restoring force as$F \approx -mg\theta$. This allows us to treat the pendulum' s m otion as simple harmonic motion for small oscillations, where the restoring force is approximatel y proportional to the angular displacement.

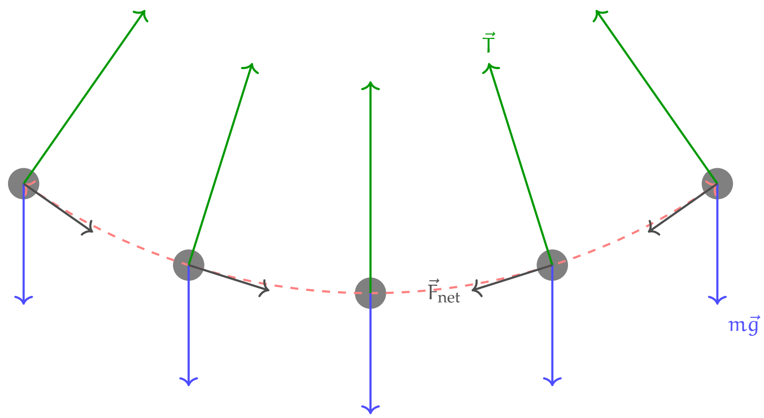


Figure 5.4: The net force at different locations of the arc.

Exercise 13 Pendulum Period Dependence

A simple pendulum consists of a mass m attached to a massless rod L, swinging under the influence of gravity. Derive the equation for the period of a simple pendulum. Which of the following changes would affect the period T of the pendulum's oscillation?

1. Increasing the mass m of the pendulum bob.
2. Increasing the length L of the string.
3. Increasing the amplitude (maximum angle) of the swing.
4. Decreasing the acceleration due to gravity g.

Working Space

Answer on Page 57

This exercise showed you that the period of a simple pendulum depends on the len g th of the pendulum and the acceleration due to gravity, but not on the mass of t h e bob or the amplitude of the swing (for small angles). One derivation from th e e xercise above was the Equation **??**, which can be rearrang ed a s$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta$. Solving this differenti al e qu ation[2]will yi e ld the angu lar position function$\theta(t) = \theta_{max}\sin(\omega t + \phi_0)$o r equivallently,$\theta(t) = \theta_{max}\sin(\sqrt{\frac{g}{L}}t + \phi_0)$of t he pendulum over time, which describes its os ci llatory motion. Note that this dif ferential equation does not the depend on mass of the bob nor t he a mplitude of th e swingθ_{max}, a core factor of pendulum harmonic motion.

Exercise 14 Pendulum on the Moon

A simple pendulum has a period of 2 seconds when measured on Earth. If the same pendulum were taken to the Moon, where the acceleration due to gravity is approximately $\frac{1}{6}$, what would be its new period?

a Approximately 0.8 seconds

b Approximately 2 seconds

c Approximately 4.9 seconds

d Approximately 6.8 seconds

Working Space

Answer on Page 58

5.6 Alternating Current

We have talked about Alternating Current (AC) in previous chapters, but now we will explore its connection to oscillations. AC is an electric current that periodically reverses direction, in contrast to Direct Current (DC), which flows in a single direction and provides constant voltage. The voltage and current in an AC circuit vary sinusoidally with time, making them ideal for modeling using oscillatory functions. FIXME expand this

[2]We will go more in depth about this in a future chapter.

Chapter 6

Orbits

Gravity is the force that attracts two bodies toward each other. It is responsible for the behavior of orbital motion. Gravity is described by Newton's Law of Universal Gravitation, which states that every point mass attracts every other point mass with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. The formula for gravitational force is:

$$F_g = \frac{Gm_1m_2}{r^2}$$

Where F_g is the gravitational force between two objects, G is the gravitational constant, m_1 and m_2 are the masses of the objects, and r is the distance between the centers of the two objects. G is a constant with a value of approximately 6.674×10^{-11} N m^2/kg^2.

If we think about acceleration due to gravity where one mass is significantly larger than the other, we can rewrite the equation as:

$$A_g = \frac{Gm_1m_2}{r^2m_2} = \frac{Gm_1}{r^2}$$

Where m_1 is the mass of the more massive object and m_2 is the mass of the significantly less massive object. This cancelling of the m_2 mass is why the acceleration due to gravity is independent of the mass of the object in free fall.

A satellite stays in orbit around the planet because the pull of the planet's gravity causes it to accelerate toward the center of the planet.

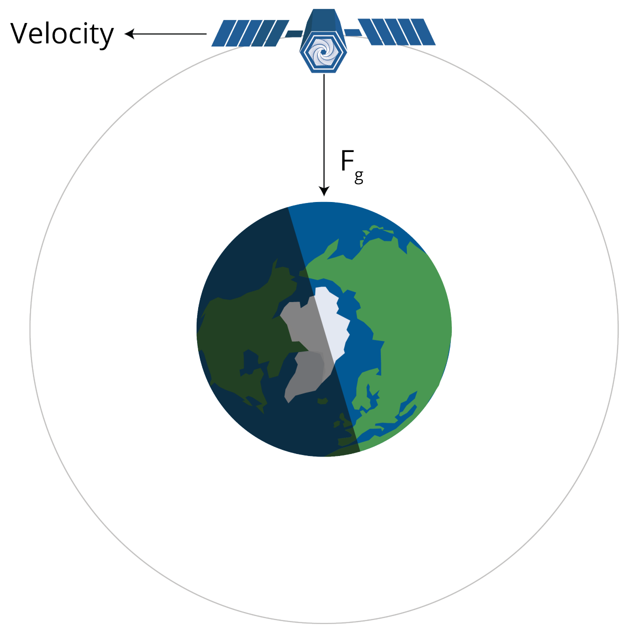


Figure 6.1: A satellite's centripetal force is gravity.

The satellite must be moving at a very particular speed to keep a constant distance from the planet — to travel in a circular orbit. If it is moving too slowly, it will get closer to the planet. If it is going too fast, it will get farther from the planet.

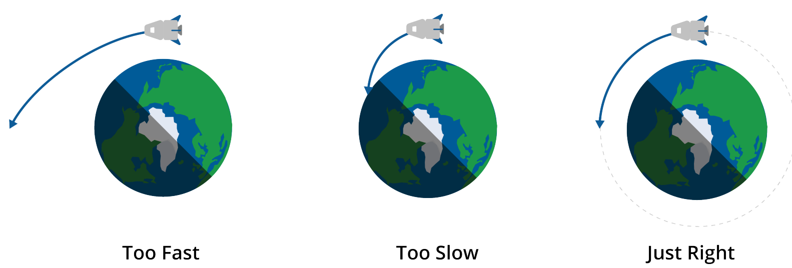


Figure 6.2: A diagram showing the required speeds for entering orbit.

The radius of a satellite in a low orbit is typically about 2 million meters above the ground. At that distance, the acceleration due to gravity is more like $6.8m/s^2$, instead of the $9.8m/s^2$ that we experience on the surface of the planet.

How fast does the satellite need to be moving in a circle with a radius of 8.37 million meters to have an acceleration of $6.8m/s^2$? Real fast.

Recall that the acceleration vector is

$$a = \frac{v^2}{r}$$

Thus the velocity v needs to be:

$$v = \sqrt{ar} = \sqrt{6.8(8.37x10^6)} = 7,544 \text{ m/s}$$

(That's 16,875 miles per hour.)

When a satellite falls out of orbit, it enters the atmosphere at that 7,544 m/s. The air rushing by generates so much friction that the satellite gets very, very hot, and usually disintegrates.

6.1 Astronauts are not weightless

Some people see astronauts floating inside an orbiting spacecraft and think there is no gravity: that the astronauts are so far away that the gravity of the planet doesn't affect them. This is incorrect. The gravity might be slightly less (maybe 6 newtons per kg instead of 9.8 newtons per kg), but the weightlessness they experience is because they and the spacecraft are in free fall. They are moving so fast (in a direction perpendicular to gravity) that they don't collide with the planet.

Exercise 15 Mars Orbit

The radius of Mars is 3.39 million meters. The atmosphere goes up another 11 km. Let's say you want to put a satellite in a circular orbit around Mars with a radius of 3.4 million meters.

The acceleration due to gravity on the surface of Mars is $3.721 \mathrm{m/s}^2$. We can safely assume that it is approximately the same 11 km above the surface.

How fast does the satellite need to be traveling in its orbit? How long will each orbit take?

Working Space

Answer on Page 59

6.2 Geosynchronous Orbits

The planet Earth rotates once a day. Satellites in low orbits circle the Earth many times a day. Satellites in very high orbits circle less than once per day. There is a radius at which a satellite orbits exactly once per day. Satellites at this radius are known as "geosynchronous" or "geostationary", because they are always directly over a place on the planet.

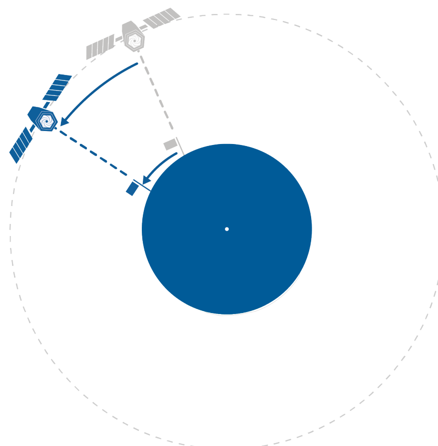


Figure 6.3: A satellite in geosynchronous orbit.

The radius of a circular geosynchronous orbit is 42.164 million meters. (About 36 km above the surface of the earth.)

A geosynchronous satellite travels at a speed of 3,070 m/s.

Geosynchronous satellites are used for the Global Positioning Satellite system, weather monitoring system, and communications system.

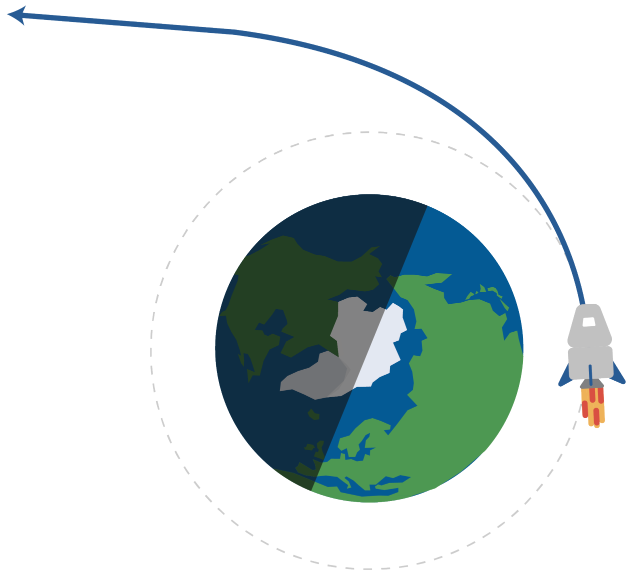


Figure 6.4: A satellite can reach a speed at which it ”escapes” earth’s orbit (centripetal force).

6.3 Kepler’s Laws

There are three laws that describe the motion of planets and their ellipical properties.

1. Every planet moves along an ellipse with the sun at one focus.

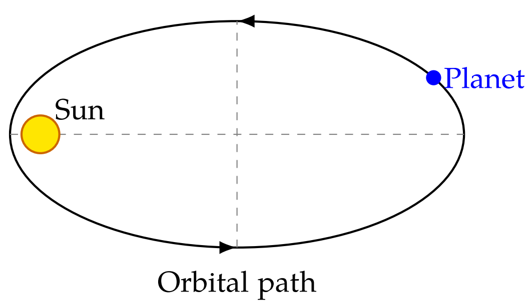


Figure 6.5: A diagram of Kepler’s First Law.

2. As a planet moves around its orbit, it sweeps out equal areas in equal times. This means that a planet moves faster when it is closer to the sun and slower when it is farther from the sun. Note that this directly comes from the *Conservation of Angular Momentum*.

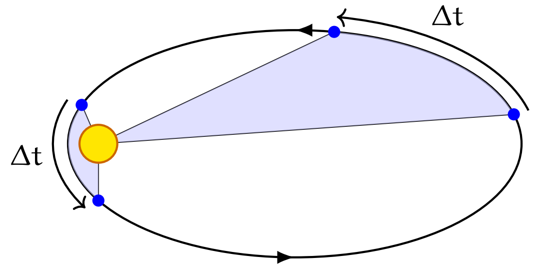


Figure 6.6: A diagram of Kepler's Second Law.

3. If T is the period of a planet's orbit, and a is the length of the semi-major axis of the ellipse, then $\frac{T^2}{a^3}$ is the same for all planets orbiting the same star.

6.4 Escape velocity

The escape velocity is the minimum speed needed for an object to "break free" from the gravitational attraction of a massive body or planet, without further propulsion. Gravity will never pull it back, because we assume no further forces act on the object. This is most commonly associated with a spaceship or satellite escaping from a planet's gravitational pull.

We need to understand that escape velocity relies on energy, rather than a force or acceleration. An object must have enough kinetic energy to overcome the gravitational potential energy pulling it back toward the planet. To escape from an orbit, the object's mechanical energy (the sum of its kinetic and potential energy) must be greater than or equal to zero.

We can derive its initial energy as: $E = K + U = \frac{1}{2}mv^2 - \frac{GMm}{R}$, where m is the mass of the object, M is the mass of the planet, R is the distance from the center of the planet to the object, and G is the gravitational constant.

After its escape, the object will be infinitely far away from the planet, and its potential energy will be zero. If we assume it just barely escapes, its kinetic energy will also be zero. Therefore, we set the initial energy equal to zero:

$$\frac{1}{2}mv^2 - \frac{GMm}{R} = 0 \tag{6.1}$$

$$\frac{1}{2}mv^2 = \frac{GMm}{R} \tag{6.2}$$

$$v^2 = \frac{2GM}{R} \tag{6.3}$$

$$v_{esc} = \sqrt{\frac{2GM}{R}} \tag{6.4}$$

Notice that the mass of the object m cancels out, meaning that the escape velocity is independent of the mass of the escaping object. It only depends on the mass of the planet and the distance from its center.

Further, **notice that the escape velocity is $\sqrt{2}$ times the orbital velocity**, $v_{orb} = \sqrt{GM/R}$ at the same distance from the planet's center. This means that to escape from a circular orbit, an object must increase its speed by about 41.4%. For Earth, the escape velocity at the surface is approximately 11.2 km/s (about 25,000 mph). This means that any object, regardless of its mass, must reach this speed to escape Earth's gravitational pull without further propulsion.

Exercise 16 Escape Velocity from Mars

Working Space

The radius of Mars is 3.4×10^6 meters. The mass of Mars is 6.42×10^{23} kg. A spaceship is sitting on the surface of Mars.

1. Find the escape velocity from the surface of Mars.
2. Suppose the spaceship is in a circular orbit 11 km above the surface of Mars. What is the escape velocity from that orbit?

Answer on Page 59

Note that giving an object an extra height h causes the escape velocity to decrease (as escape velocity is inversely proportional to the square root of the radius plus height). This is due to gravitation potential energy being less negative at larger radii. However, the effect is small compared to the overall radius of the planet. Problems may give you a radius plus height, or just the radius of the planet, or a total distance from the center, depending on the context, so read each problem carefully.

Appendix A

Answers to Exercises

Answer to Exercise 1 (on page 3)

$$\frac{300 \times 10^6}{5.66 \times 10^{14}} = 530 \times 10^{-9} = 530 \text{ nm}$$

Answer to Exercise 2 (on page 11)

Assuming the mirror is truly vertical and the floor is truly horizontal, the new cut off should be exactly the same as the old one: It should be below your chin the same amount that your eyes are above your chin.

Illustration Here

Answer to Exercise 3 (on page 12)

Are there white photons? No. What we call "white" is a blend of photons that are several different colors.

Some people like to say white light is the combination of all visible colors of photons in equal amounts. That seems oddly specific and unusual.

Maybe it is better to imagine it from the human experience of white light. In our eyes, we have three different types of color-sensing cones, which generally correspond to the the red, green, and blue regions of the spectrum. When all three are excited about equal amounts, humans experience that as white. On your computer screen, for example, what you see as white is just a blend of three colors of photons: a red, a green, and a blue.

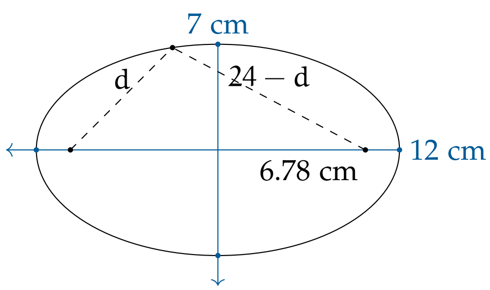
Are there black photons? No. What we call "black" is an absence of photons in the visible range.

Are there yellow photons? Yes! There is a region of the color spectrum that is yellow. It has a wavelength of about 527 nm. Photons at this energy level excite both our green-sensitive and red-sensitive cones (we well expand on this in the eye chapter!). Your computer monitor does not actually create light with a 527 nm wavelength. Instead, it creates red light and green light, which our eyes interpret as yellow.

Answer to Exercise 4 (on page 15)

The length of the string is easy: $2 \times 12 = 24$ cm.

The distance from the center to the focal point is $\sqrt{12^2 - 7^2} \approx 6.78$ cm.



Answer to Exercise 5 (on page 19)

We need the distance from the center out to each of the three axes. We know that $a = \left(\frac{1}{2}\right) 30 = 15$ cm.

We can calculate the b and c (which are equal) using the circumference given: $2b\pi = 60$, so $c = b \approx 9.55$ cm.

The volume, then is

$$V = \frac{4}{3}\pi(15)(9.55)(9.55) \approx 5,730 \text{ cubic centimeters}$$

The mass would be $5,730 \times 11.34 = 64,973$ grams or about 65 kg.

Answer to Exercise 6 (on page 34)

The correct answer is: The frequency would increase by a factor of $\sqrt{2}$.

The frequency of oscillation for a mass-spring system is given by the formula:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

where k is the spring constant and m is the mass attached to the spring. Since frequency is inversely proportional to the square root of the mass, if the mass is halved, the new frequency f' can be calculated as follows:

$$\begin{aligned} f' &= \frac{1}{2\pi}\sqrt{\frac{k}{m/2}} \\ &= \frac{1}{2\pi}\sqrt{\frac{2k}{m}} \\ &= \sqrt{2}\cdot\frac{1}{2\pi}\sqrt{\frac{k}{m}} \\ &= \sqrt{2}\cdot f \end{aligned}$$

Therefore, the frequency increases by a factor of $\sqrt{2}$ when the

Answer to Exercise 7 (on page 36)

First, we find the angular frequency ω using the formula:

$$w = 2\pi f = \sqrt{\frac{k}{m}} = \sqrt{\frac{400\,\text{N/m}}{4\,\text{kg}}} = \sqrt{100} = 10\,\text{rad/s}$$

Therefore, the general equation for the position of a mass-spring system undergoing simple harmonic motion is:

$$x(t) = A\cos(\omega t + \phi)$$

becomes $x = 6\sin(10t + \phi)$ since the initial velocity is 0, meaning it starts at maximum displacement. Solving for ϕ using the initial condition $x(0) = 6$ cm:

$$6 = 6\sin(\phi) \implies \sin(\phi) = 1 \implies \phi = \frac{\pi}{2}$$

So the base equation becomes:

$$x(t) = 6\sin(10t + \frac{\pi}{2})$$

Which is option C.

Answer to Exercise 8 (on page 37)

Assessing statement I: The acceleration in simple harmonic motion is not constant; it varies with displacement. Therefore, statement I is false. (b), (c) cannot be correct. Statement II must be true since it appears in both remaining answer choices. Statement III is also true, as the frequency of simple harmonic motion does not depend on amplitude. Therefore, the correct answer is (d), II and III only.

Answer to Exercise 9 (on page 37)

To find the speed of the ball as it leaves the spring, we can use the principle of conservation of mechanical energy. The net energy in the system remains constant, meaning that the potential energy stored in the compressed spring is converted into the kinetic energy of the ball when it is released. The potential energy stored in the spring when compressed is given by:

$$U_s = \frac{1}{2}kx^2 = \frac{1}{2}(500)(-0.24)^2 = 14.4\,\text{J}$$

The kinetic energy of the ball as it leaves the spring is the same amount, so we set the potential energy equal to the kinetic energy:

$$\frac{1}{2}mv^2 = U_s$$

Solving for v:

$$v = \sqrt{\frac{2U_s}{m}} = \sqrt{\frac{2(14.4)}{0.2}} = \sqrt{144} = 12\,\text{m/s}$$

The speed of the ball as it leaves the spring is $12\,\text{m/s}$.

Answer to Exercise 10 (on page 39)

The correct differential equation is $x'' + 100x = 0$. This is derived from the equation of motion for any simple harmonic oscillator: $mx'' + kx = 0$, where $m = 2\,\text{kg}$ and $k = 200\,\text{N/m}$. Dividing by m, we get $x'' + \frac{k}{m}x = 0$, which simplifies to $x'' + 100x = 0$.

Answer to Exercise 11 (on page 40)

- This system is damped. Specifically, it is underdamped because the amplitude of oscillation decreases over time.
- To make this system undergo simple harmonic motion, we would need to remove

the damping term from the equation of motion. This means setting the damping coefficient c to zero in the differential equation, which would eliminate the exponential decay factor in the position function. The equation becomes $x(t) = 0.15\cancel{e^{-0.40t}}\cos(4t + 0.30) \rightarrow x(t) = 0.15\cos(4t + 0.30)$

Answer to Exercise 12 (on page 41)

1. Notice that the system equation is linear as it is defined with an x'' and x.The general solution to the differential equation $x'' + 16x = 0$ is of the form $x(t) = A\cos(4t) + B\sin(4t)$. This comes from the idea that the characteristic equation $r^2 + 16 = 0$ has roots $r = \pm 4i$, where the β term is 4. Using the initial conditions $x(0) = 0.20$ and $v(0) = x'(0) = 0$, we find our velocity equation to be $v(t) = x'(t) = -4A\sin(4t) + 4B\cos(4t)$.

   From $x(0) = A\cos 0 + B\sin 0 = A(1) + B(0) = A = 0.20$, we find that $A = 0.20$.

   From $v(0) = -4A\sin 0 + 4B\cos 0 = -4(0.20)(0) + 4B(1) = 4B = 0$, we find that $B = 0$.

   Therefore, the solution is:
   $$x(t) = 0.20\cos(4t)$$

2. To find the acceleration, we double differentiate $x(t)$ with respect to time: To find the velocity, we differentiate $x(t)$ with respect to time:

   $$v(t) = x'(t) = -0.80\sin(4t)$$

   $$a(t) = v'(t) = -3.20\cos(4t)$$

   Since cosine and sine functions oscillate between -1 and 1 (regardless of angular frequency $\omega = 4$), the maximum velocity is 0.80 m/s and the maximum acceleration is $3.20\,\text{m/s}^2$.

Answer to Exercise 13 (on page 43)

We know that the restoring force for a pendulum is given by $F = -mg\sin\theta$. For small angles, we can approximate $\sin\theta \approx \theta$ (in radians), leading to $F \approx -mg\theta$. For any point on the circle, we can relate the angular displacement θ to the arc length x using $x = L\theta$, where L is the length of the pendulum. Thus, we can rewrite the restoring force in terms of the second derivative, which represents angular acceleration:

$$a_{tan} = \frac{d^2x}{dt^2} = L\frac{d^2\theta}{dt^2} \tag{5.9}$$

Using Newton's Second Law, we have:

$$mL\frac{d^2\theta}{dt^2} = -mg\sin\theta \tag{5.10}$$

$$L\frac{d^2\theta}{dt^2} + g\sin\theta = 0 \tag{5.11}$$

$$\frac{d^2\theta}{dt^2} + \frac{g}{L}\sin\theta = 0 \tag{5.12}$$

But for small angles, we can approximate $\sin\theta \approx \theta$, leading to:

$$[label = eq : linearpendulum]\frac{d^2\theta}{dt^2} + \frac{g}{L}\theta = 0 \tag{5.13}$$

This is a second-order linear differential equation with constant coefficients, similar to the mass-spring system problems discussed earlier. The angular frequency ω of the pendulum can be identified as:

$$\omega = \sqrt{\frac{g}{L}} \tag{5.14}$$

Therefore, the period T of the pendulum is given by:

$$T = 2\pi\sqrt{\frac{L}{g}} \tag{5.15}$$

Answer to Exercise 14 (on page 44)

The correct answer is: Approximately 4.4 seconds.

The period T of a simple pendulum is given by the formula:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

where L is the length of the pendulum and g is the acceleration due to gravity.

On Earth, the period is 2 seconds, so we can set up the equation:

$$2 = 2\pi\sqrt{\frac{L}{g_{\text{Earth}}}}$$

This shows that T is inversely proportional to the square root of g. We can rearrange this equation to solve for L. So if g decreases by a factor of 6, then T will increase by a factor of $\sqrt{6}$.

Now, on the Moon, the acceleration due to gravity is approximately $\frac{1}{6}g_{\text{Earth}}$. We can substitute this into the period formula:

$$T_{\text{Moon}} = 2\pi\sqrt{\frac{L}{g_{\text{Moon}}}} = 2\pi\sqrt{\frac{L}{\frac{1}{6}g_{\text{Earth}}}} = 2\pi\sqrt{\frac{6L}{g_{\text{Earth}}}} = \sqrt{6}\cdot 2\pi\sqrt{\frac{L}{g_{\text{Earth}}}} = \sqrt{6}\cdot T_{\text{Earth}}$$

Since our original period on Earth is 2 seconds, we have:

$$T_{\text{Moon}} = \sqrt{6}\cdot 2 \approx 4.9 \text{ seconds}$$

Which is equal to option (c).

Answer to Exercise ?? (on page 48)

$$v = \sqrt{3.721(3.4\times 10^6)} = 3,557 \text{ m/s}$$

The circular orbit is $2\pi(3.4\times 10^6) = 21.4\times 10^6$ meters in circumference.

The period of the orbit is $(21.4\times 10^6)/3,557 \approx 6,000$ seconds.

Answer to Exercise 16 (on page 52)

Using equation **??**:

1. $$\begin{aligned} v_{\text{esc}} &= \sqrt{\frac{2GM}{R}} \\ &= \sqrt{\frac{2(6.674\times 10^{-11} \text{ N m}^2/\text{kg}^2)(6.42\times 10^{23} \text{ kg})}{3.4\times 10^6 \text{ m}}} \\ &\approx 5,027 \text{ m/s} \end{aligned}$$

2. Now the radius is $3.4\times 10^6 + 11,000 = 3.411\times 10^6$ meters, because the satellite is 11

km above the surface. Recalculating:

$$
\begin{aligned}
v_{esc} &= \sqrt{\frac{2GM}{R}} \\
&= \sqrt{\frac{2(6.674 \times 10^{-11}\ \mathrm{N\ m^2/kg^2})(6.42 \times 10^{23}\ \mathrm{kg})}{3.411 \times 10^{6}\ \mathrm{m}}} \\
&\approx 5,013\ \mathrm{m/s}
\end{aligned}
$$

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