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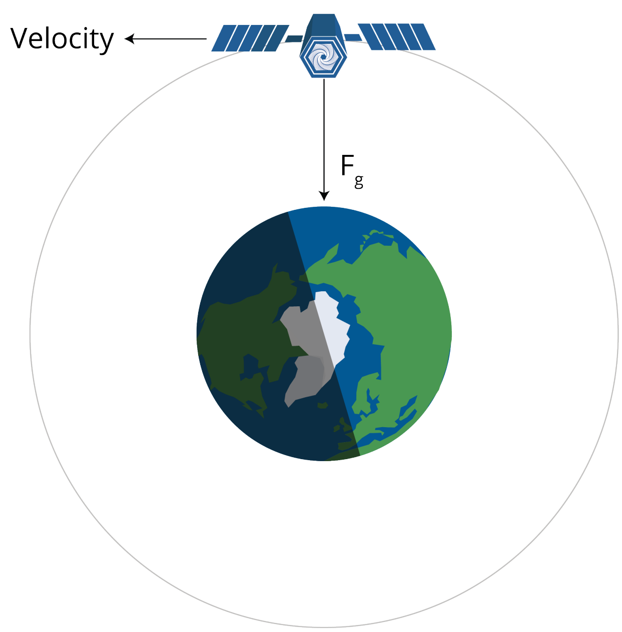
Chapter 1

Orbits

A satellite stays in orbit around the planet because the pull of the planet's gravity causes it to accelerate toward the center of the planet.



The satellite must be moving at a very particular speed to keep a constant distance from the planet — to travel in a circular orbit. If it is moving too slowly, it will get closer to the planet. If it is going too fast, it will get farther from the planet.

The radius of the earth is about 6.37 million meters. A satellite that is in a low orbit is typically about 2 million meters above the ground. At that distance, the acceleration due to gravity is more like $6.8\text{m}/\text{s}^2$, instead of the $9.8\text{m}/\text{s}^2$ that we experience on the surface of the planet.

How fast does the satellite need to be moving in a circle with a radius of 8.37 million meters to have an acceleration of $6.8\text{m}/\text{s}^2$? Real fast.

Recall that the acceleration vector is

$$a = \frac{v^2}{r}$$

Thus the velocity v needs to be:

$$v = \sqrt{ar} = \sqrt{6.8(8.37x10^6} = 7,544 \text{ m/s}$$

(That's 16,875 miles per hour.)

When a satellite falls out of orbit, it enters the atmosphere at that 7,544 m/s. The air rushing by generates so much friction that the satellite gets very, very hot, and usually disintegrates.

1.1 Astronauts are not weightless

Some people see astronauts floating inside an orbiting spacecraft and think there is no gravity: that the astronauts are so far away that the gravity of the planet doesn't affect them. This is incorrect. The gravity might be slightly less (Maybe 6 newtons per kg instead of 9.8 newtons per kg), but the weightless they experience is because they and the spacecraft is in free fall. They are just moving so fast (in a direction perpendicular to gravity) that they don't collide with the planet.

Exercise 1 Mars Orbit

Working Space

The radius of Mars is 3.39 million meters. The atmosphere goes up another 11 km. Let's say you want to put a satellite in a circular orbit around Mars with a radius of 3.4 million meters.

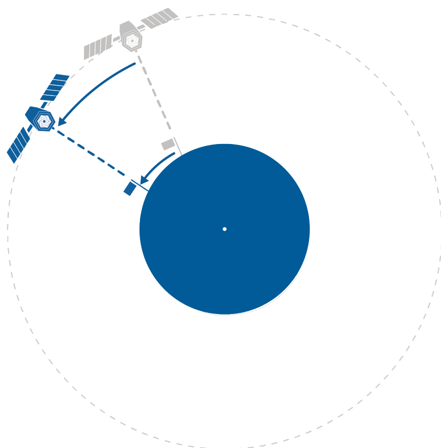
The acceleration due to gravity on the surface of Mars is $3.721 \mathrm{m/s}^2$. We can safely assume that it is approximately the same 11 km above the surface.

How fast does the satellite need to be traveling in its orbit? How long will each orbit take?

Answer on Page 37

1.2 Geosynchronous Orbits

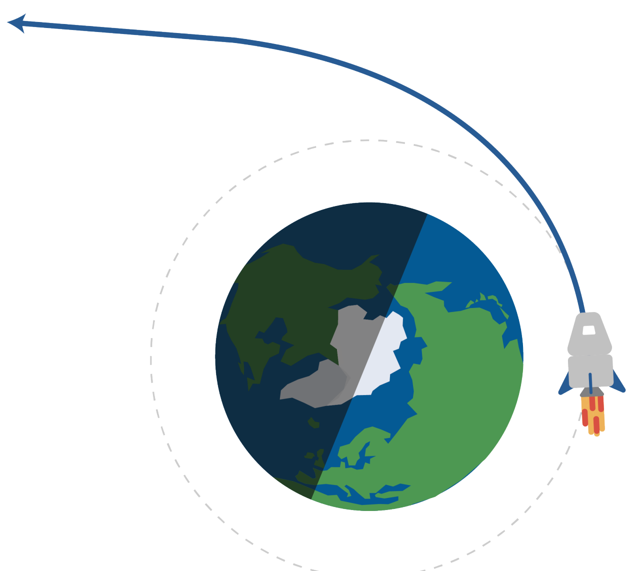
The planet earth rotates once a day. Satellites in low orbits circle the earth many times a day. Satellites in very high orbits circle less than once per day. There is a radius at which a satellite orbits exactly once per day. Satellites at this radius are known as "geosynchronous" or "geostationary", because they are always directly over a place on the planet.



The radius of a circular geosynchronous orbit is 42.164 million meters. (About 36 km above the surface of the earth.)

A geosynchronous satellite travels at a speed of 3,070 m/s.

Geosynchronous satellites are used for the Global Positioning Satellite system, weather monitoring system, and communications system.



FIXME: Add text for escape velocity

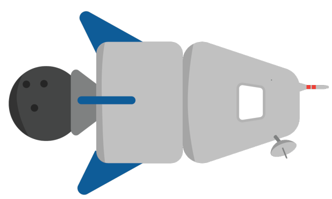
Chapter 2

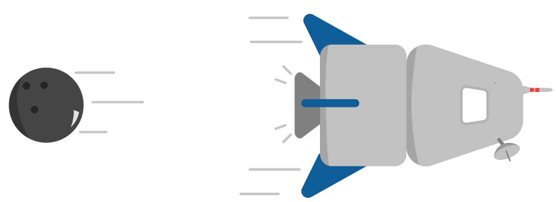
Rocketry

Rockets propel hot gases, which recreates an equal and opposite reaction that pushes it forwards, even in a vacuum.

Even without anything to push against, the rocket can still move forward thanks to Newton's Third Law.

Imagine a spacecraft with a bowling ball attached to the back. If that spacecraft exerts a force to throw the bowling ball backwards, the ball will exert a force on the ship, moving it forwards.



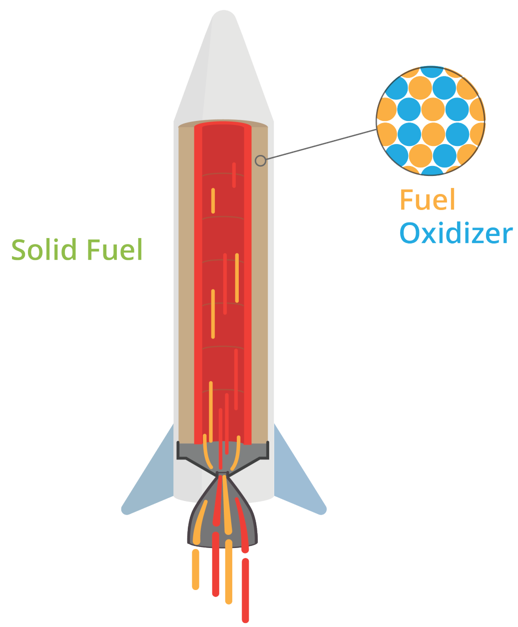


Instead of a bowling ball, real-life rockets usually "throw" particles of hot gas at very high speeds. Rockets carry their own oxidizer to provide oxygen to allow fuel to burn.

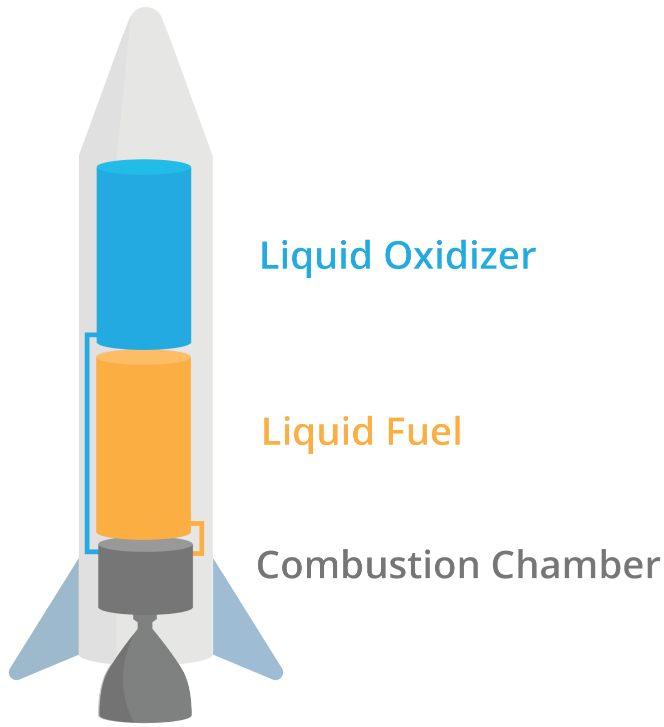
2.1 Types of rocket motors

There are two main types of chemical rockets.

One type is a *Solid Fuel Rocket*, which ignites a solid fuel-oxidizer mix. Once the solid fuel is ignited, it can't be stopped until all of the fuel is exhausted.



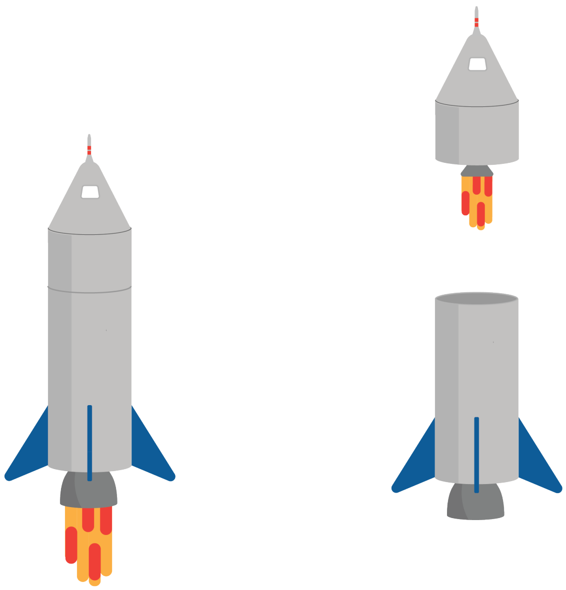
The other main type of chemical rocket is called a *Liquid Fuel Rocket*. Liquid fuel rockets contain separate tanks for liquid fuel and liquid oxygen. Fuel pumps bring them both to a combustion chamber where they ignite and exit the rocket. Most liquid fuel engines can control their thrust.



2.2 Tyranny of the rocket equation

Chemical rockets can only burn the fuel that they bring with them. However, the more fuel you carry, the heavier the vehicle will be.

One way to help reduce this weight is by using *staging*.



Staging allows rockets to drop unnecessary structural mass once they've used up a certain amount of fuel.

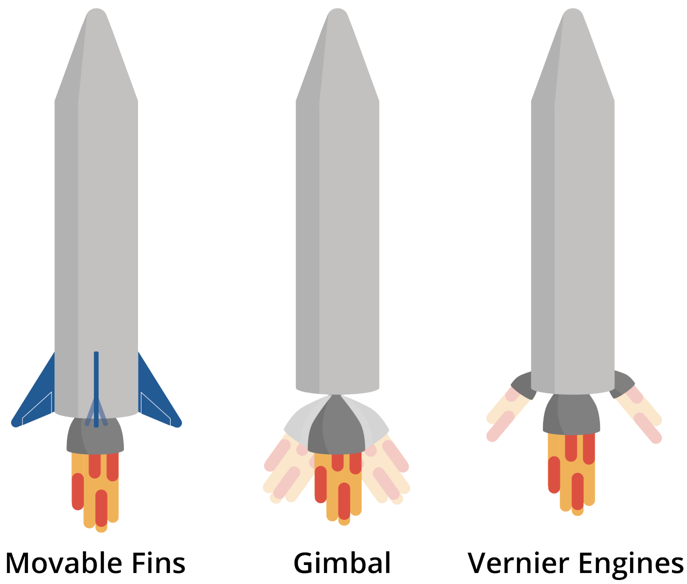
2.3 Control in atmosphere

There are several common ways that engineers have managed to control rockets' direction in the atmosphere. Usually, on-board sensors detect the orientation of the rocket, and can automatically adjust these controls to keep the rocket going the correct direction.

One method is using *movable fins*. The fins work similarly to control surfaces that we covered in the airplanes chapter.

Another method of control uses a *gimbaled engine*. [pros and cons]

A more outdated method is using *vernier engines*, which are two smaller engines that control attitude. However, this adds a large amount of weight to the rocket, so they are less frequently used today. [pros and cons]



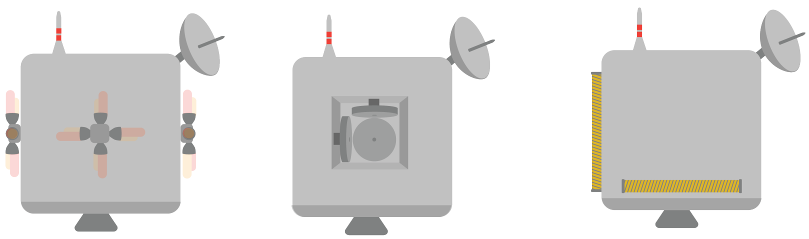
2.4 Control in space

The previous section describes ways that engineers control rockets in the atmosphere, but most rockets will end up in the vacuum of space. There are several common ways to adjust the orientation in space.

One method is using *RCS thrusters*. An RCS, or reaction control system, is a series of small thrusters that are used to change the direction and position of a spacecraft.

Another method called *reaction wheels* uses angular momentum to rotate the spacecraft. By accelerating and decelerating wheels on three axes, the spacecraft can rotate in any direction.

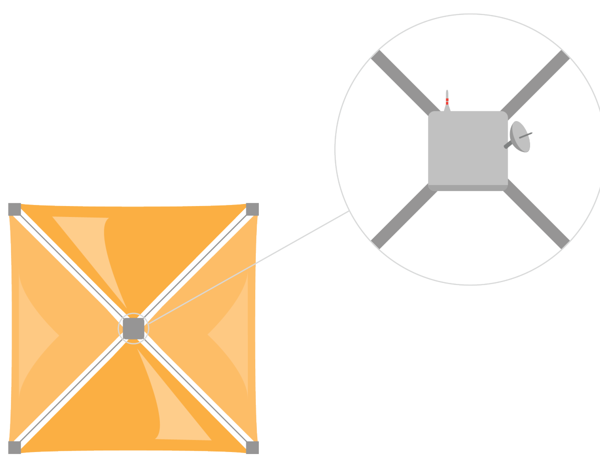
A third common attitude control technology is *magnetorquer*. Magnetorquers use electromagnets and the earth's magnetic field to adjust the orientation of the spacecraft.

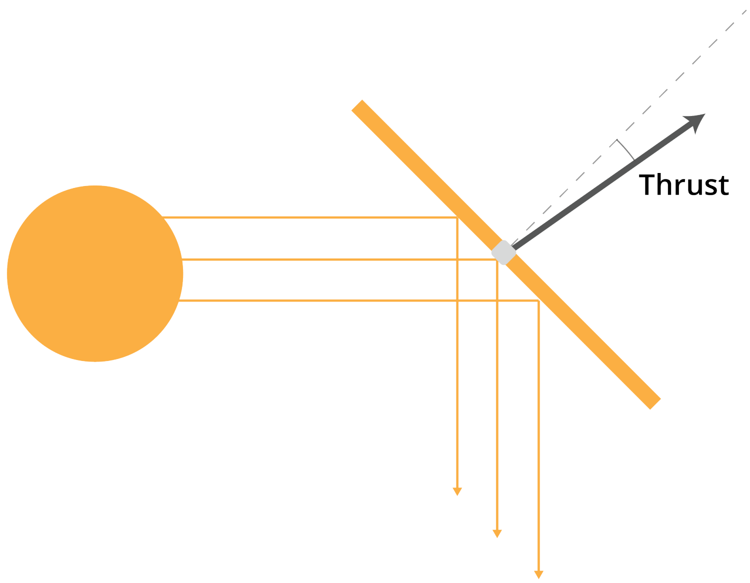


Thrusters Reaction wheels Magnetorquer

2.5 Alternative propulsion

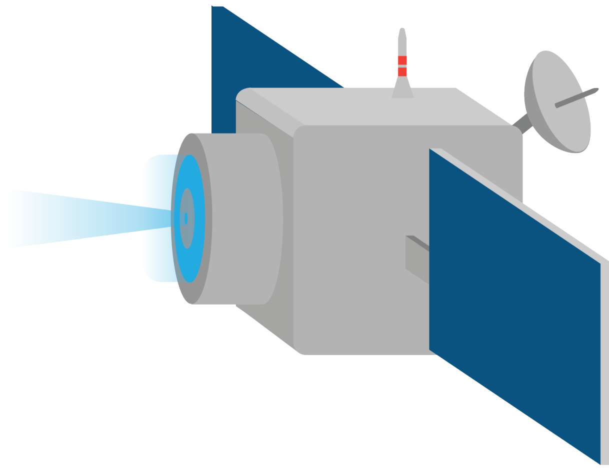
One type of alternate propulsion is called a *solar sail*. Solar sails use lightweight reflective surfaces to use photons in space to propel the spacecraft without on-board fuel.





Photons reflect off of the surface of the sail. However, since the surface is not perfectly reflective, some of those photons are absorbed, and they produce a horizontal equal and opposite reaction. That small force causes the net thrust to be slightly skewed away from a right angle to the sail.

ion propulsion



Chapter 3

Simulation with Vectors

In an earlier chapter, you wrote a python program that simulated the flight of a hammer to predict its altitude. Your simulation dealt only with scalars. Now, you are ready to create simulations of positions, velocities, accelerations, and forces as vectors.

In this chapter, you are going to simulate two moons that, as they wandered through the vast universe, get caught in each other's gravity well. We will assume there are no other forces acting upon the moons.

3.1 Force, Acceleration, Velocity, and Position

We talked about the magnitude of a gravitational attraction between two masses:

$$F = G\frac{m_1 m_2}{r^2}$$

where F is the magnitude of the force in newtons, m_1 and m_2 are the masses in kg, r is the distance between them in meters, and g is the universal gravitational constant: 6.67430×10^{-11}.

What is the direction? For the two moons, the force on moon 1 will pull toward moon 2. Likewise, the force on moon 2 will pull toward moon 1.

Of course, if something is big (like the sun), you need to be more specific: The force points directly at the center of mass of the object that is generating the force.

Each of the moons will start off with a velocity vector. That velocity vector will change over time as the moon is accelerated by the force of gravity. If you have a mass m with an initial velocity vector of $\vec{v_0}$ that is being accelerated with a constant force vector $\vec{F}$, at time t, the new velocity vector will be:

$$\vec{v}_t = \vec{v}_0 + \frac{t}{m}\vec{F}$$

If an object is at an initial position vector of $\vec{p_0}$ and moves with a constant velocity vector $\vec{v}$ for time t, the new position will be given by

$$\vec{p}_t = \vec{p}_0 + t\vec{v}$$

3.2 Simulations and Step Size

As two moons orbit each other, the force, acceleration, velocity, and position are changing smoothly and continuously. It is difficult to simulate truly continuous things on a digital computer.

However, think about how a movie shows you many frames each second. Each frame is a still picture of the state of the system. The more frames per second, the smoother it looks.

We do a similar trick in simulations. We say "We are going run our simulation in 2 hour steps. We will assume that the acceleration and velocity were constant for those two hours. We will update our position vectors accordingly, then we will recalculate our acceleration and velocity vectors."

Generally, as you make the step size smaller, your simulation will get more accurate and take longer to execute.

3.3 Make a Text-based Simulation

To start, you are going to write a Python program that simulates the moons and prints out their position for every time step. Later, we will add graphs and even animation.

We are going to assume the two moons are traveling the same plane so we can do all the math and graphing in 2 dimensions.

Each moon will be represented by a dictionary containing the state of the moon:

- Its mass in kilograms
- Its position — A 2-dimensional vector represent x and y coordinates of the center of the moon.
- Its velocity — A 2-dimensional vector
- Its radius — Each moon has a radius so we know when the centers of the two moons are so close to each other that they must have collided.
- Its color — We will use that when do the plots and animations. One moon will be red, the other blue.

There will then be a loop where we will update the positions of the moons and then recalculate the acceleration and velocities.

How much time will be simulated? 100 days or until the moons collide, whichever comes first.

We will use numpy arrays to represent our vectors.

Create a file called `moons.py`, and type in this code:

```
import numpy as np

# Constants
G = 6.67430e-11              # Gravitational constant (Nm^2/kg^2)
SEC_PER_DAY = 24 * 60 * 60   # How many seconds in a day?
MAX_TIME = 100 * SEC_PER_DAY # 100 days
TIME_STEP = 2 * 60 * 60      # Update every two hours

# Create the inital state of Moon 1
m1 = {
    "mass": 6.0e22,  # kg
    "position": np.array([0.0, 200_000_000]),  # m
    "velocity": np.array([100.0, 25.0]),  # m/s
    "radius": 1_500_000.0,  # m
    "color": "red" # For plotting
}

# Create the inital state of Moon 2
m2 = {
    "mass": 11.0e22,  # kg
    "position": np.array([0.0, -150_000_000]),  # m
    "velocity": np.array([-45.0, 2.0]),  # m/s
    "radius": 2_000_000.0,  # m
    "color": "blue" # For plotting
}

# Lists to hold positions and time
position1_log = []
position2_log = []
time_log = []

# Start at time zero seconds
current_time = 0.0

# Loop until current time exceed Max Time
while current_time <= MAX_TIME:

    # Add time and positions to log
    time_log.append(current_time)
    position1_log.append(m1["position"])
    position2_log.append(m2["position"])
```

```
    # Print the current time and positions
    print(f"Day {current_time/SEC_PER_DAY:.2f}:")
    print(f"\tMoon 1:({m1['position'][0]:,.1f},{m1['position'][1]:,.1f})")
    print(f"\tMoon 2:({m2['position'][0]:,.1f},{m2['position'][1]:,.1f})")

    # Update the positions based on the current velocities
    m1["position"] = m1["position"] + m1["velocity"] * TIME_STEP
    m2["position"] = m2["position"] + m2["velocity"] * TIME_STEP

    # Find the vector from moon1 to moon2
    delta = m2["position"] - m1["position"]

    # What is the distance between the moons?
    distance = np.linalg.norm(delta)

    # Have the moons collided?
    if distance < m1["radius"] + m2["radius"]:
        print(f"*** Collided {current_time:.1f} seconds in!")
        break

    # What is a unit vector that points from moon1 toward moon2?
    direction = delta / distance

    # Calculate the magnitude of the gravitational attraction
    magnitude = G * m1["mass"] * m2["mass"] / (distance**2)

    # Acceleration vector of moon1 (a = f/m)
    acceleration1 = direction * magnitude / m1["mass"]

    # Acceleration vector of moon2
    acceleration2 = (-1 * direction) * magnitude / m2["mass"]

    # Update the velocity vectors
    m1["velocity"] = m1["velocity"] + acceleration1 * TIME_STEP
    m2["velocity"] = m2["velocity"] + acceleration2 * TIME_STEP

    # Update the clock
    current_time += TIME_STEP

print(f"Generated {len(position1_log)} data points.")
```

When your run the simulation, you will see the positions of the moons for 100 days:

```
> python3 moons.py
Day 0.00:
 Moon 1:(0.0,200,000,000.0)
 Moon 2:(0.0,-150,000,000.0)
Day 0.08:
 Moon 1:(720,000.0,200,180,000.0)
 Moon 2:(-324,000.0,-149,985,600.0)
```

```
Day 0.17:
 Moon 1:(1,439,990.7,200,356,896.1)
 Moon 2:(-647,995.0,-149,969,507.0)
...
Day 100.00:
 Moon 1:(119,312,305.5,283,265,313.5)
 Moon 2:(17,393,287.9,-60,319,261.9)
Generated 1201 data points.
```

Look over the code. Make sure you understand what every line does.

3.4 Graph the Paths of the Moons

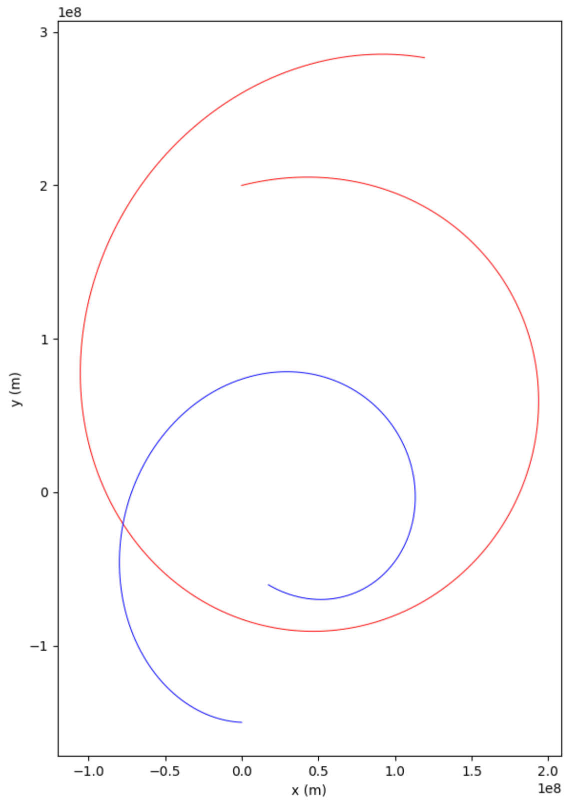
Now, you will use the matplotlib to graph the paths of the moons. Add this line to the beginning of `moons.py`.

```
import matplotlib.pyplot as plt
```

Add this code to the end of your `moons.py`:

```
# Convert lists to np.arrays
positions1 = np.array(position1_log)
positions2 = np.array(position2_log)

# Create a figure with a set of axes
fig, ax = plt.subplots(1, figsize=(7.2, 10))

# Label the axes
ax.set_xlabel("x (m)")
ax.set_ylabel("y (m)")
ax.set_aspect("equal", adjustable='box')

# Draw the path of the two moons
ax.plot(positions1[:, 0], positions1[:, 1], m1["color"], lw=0.7)
ax.plot(positions2[:, 0], positions2[:, 1], m2["color"], lw=0.7)

# Save out the figure
fig.savefig("plotmoons.png")
```

When you run it, your `plotmoons.png` should look like this:



It is nifty to see the paths, but we don't know where each moon was at a particular time. In fact, it is difficult to figure out which end of each curve was the beginning and which was the ending.

What if we added some lines and labels every 300 steps to put a sense of time into the plot? Add one more constant after the import statements:

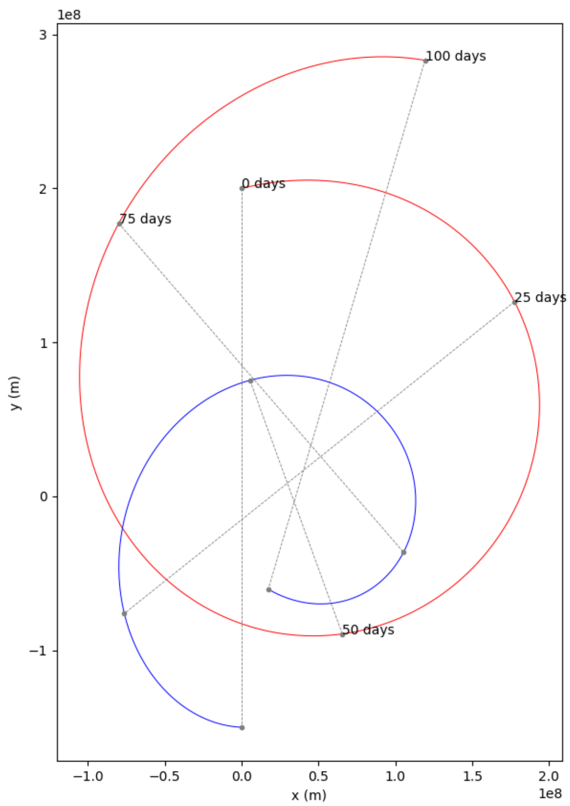
```
PAIR_LINE_STEP = 300  # How time steps between pair lines
```

Immediately before you save the figure to the file, add the following code:

```
# Draw some pair lines that help the
# viewer understand time in the graph
i = 0
while i < len(positions1):

    # Where are the moons at the ith entry?
    a = positions1[i, :]
    b = positions2[i, :]
    ax.plot([a[0], b[0]], [a[1], b[1]], "--", c="gray", lw=0.6, marker=".")

    # What is the time at the ith entry?
    t = time_log[i]
```

```
    # Label the location of moon 1 with the day
    ax.text(a[0], a[1], f"{t/SEC_PER_DAY:.0f} days")
    i += PAIR_LINE_STEP
```

When you run it, your plot should look like this:

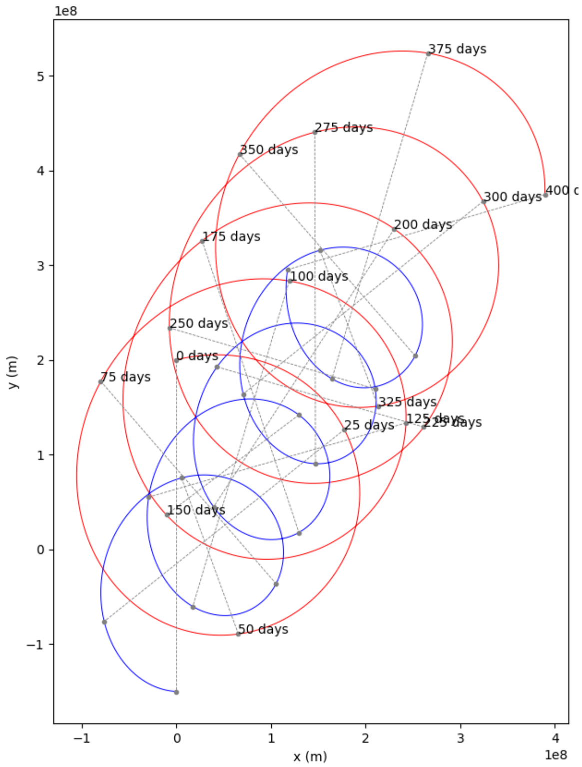


Now you can get a feel for what happened. The moons were attracted to each other by gravity and started to circle each other. The heavier moon accelerates less quickly, so it makes a smaller loop.

Maybe we will get a better feel for what is happening if we look at more time. Let's increase it to 400 days. Change the relevant constant:

```
MAX_TIME = 400 * SEC_PER_DAY # 100 days
```

Now it should look like this:



Now you can see the pattern. They are rotating around each other and the pair is gradually migrating up and to the right.

3.5 Conservation of Momentum

You are observing an extra important idea: the momentum of a system will be conserved. That is, absent forces from outside the system, the velocity of the center of mass will not change.

We can compute the initial center of mass and its velocity. In both cases, we just do a weighted average using the mass of the moon as the weight.

Immediately after you initialize the state of two moons, calculate the initial center of mass and its velocity:

```
# Calculate the initial position and velocity of the center of mass
tm = m1["mass"] + m2["mass"]  # Total mass
cm_position = (m1["mass"] * m1["position"] + m2["mass"] * m2["position"]) / tm
cm_velocity = (m1["mass"] * m1["velocity"] + m2["mass"] * m2["velocity"]) / tm
```

Let's record the center of mass for each time. Before the loop starts, create a list to hold them:

```
cm_log = []
```

Inside the loop (before any calculations), append the current center of mass position to the log:

```
    cm_log.append(cm_position)
```

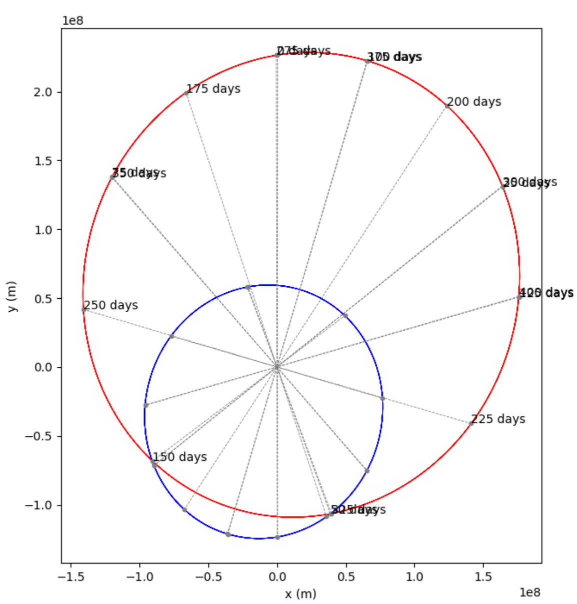
Anywhere later in the loop (after you update the positions of the moon), update `cm_position`:

```
    # Update the center of mass
    cm_position = cm_position + cm_velocity * TIME_STEP
```

Now, let's look at the positions of the moons relative to the center of mass. Before you do any plotting, convert the list to a numpy array and subtract it from the positions:

```
cms = np.array(cm_log)

# Make positions relative to the center of mass
positions1 = positions1 - cms
positions2 = positions2 - cms
```

When you run it, you can really see what is happening:



The moons are tracing elliptical paths. The center of mass is the focus point for both of them.

3.6 Animation

One of the features of matplotlib that not a lot of people understand is how to make animations with it. This seems like a really great opportunity to make an animation showing the position, velocity, acceleration of the moons. We will also show the center of mass.

The trick to animations is that you create a bunch ”artist” objects. You create a function that updates the artists. matplotlib will call your functions, tell the artists to draw themselves, and make a movie out of that.

Make a copy of `moons.py` called `animate_moons.py`.

Edit it to look like this:

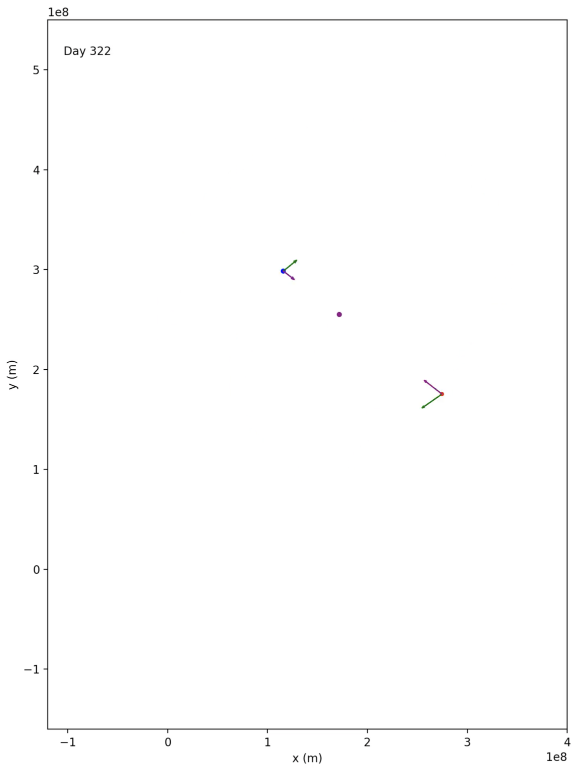
```
import numpy as np
import matplotlib.pyplot as plt

# Import animation support and artists
from matplotlib.animation import FuncAnimation
from matplotlib.patches import Circle, FancyArrow
from matplotlib.text import Text

# Constants
G = 6.67430e-11  # Gravitational constant (Nm^2/kg^2)
SEC_PER_DAY = 24 * 60 * 60  # How many seconds in a day?
MAX_TIME = 400 * SEC_PER_DAY  # 100 days
TIME_STEP = 12 * 60 * 60  # Update every 12 hours
FRAMECOUNT = MAX_TIME / TIME_STEP  # How many frames in animation
ANI_INTERVAL = 1000 / 50  # ms for each frame in animation

# The velocity and acceleration vectors are invisible
# unless we scale them up.  A lot.
VSCALE = 140000.0
ASCALE = VSCALE * 800000.0

# Create the inital state of Moon 1
m1 = {
    "mass": 6.0e22,  # kg
    "position": np.array([0.0, 200_000_000]),  # m
    "velocity": np.array([100.0, 25.0]),  # m/s
    "radius": 1_500_000.0,  # m
    "color": "red",  # For plotting
}

# Create the inital state of Moon 2
m2 = {
    "mass": 11.0e22,  # kg
    "position": np.array([0.0, -150_000_000]),  # m
    "velocity": np.array([-45.0, 2.0]),  # m/s
```

```
    "radius": 2_000_000.0,  # m
    "color": "blue",  # For plotting
}

# Calculate the initial position and velocity of the center of mass
tm = m1["mass"] + m2["mass"]  # Total mass
cm_position = (m1["mass"] * m1["position"] + m2["mass"] * m2["position"]) / tm
cm_velocity = (m1["mass"] * m1["velocity"] + m2["mass"] * m2["velocity"]) / tm

# Start at time zero seconds
current_time = 0.0

# Create the figure and axis
fig, ax = plt.subplots(1, figsize=(7.2, 10))

# Set up the axes
ax.set_xlabel("x (m)")
ax.set_xlim((-1.2e8, 4e8))
ax.set_ylabel("y (m)")
ax.set_ylim((-1.6e8, 5.5e8))
ax.set_aspect("equal", adjustable="box")
fig.tight_layout()

# Create artists that will be edited in animation
time_text = ax.add_artist(Text(0.03, 0.95, "", transform=ax.transAxes))
circle1 = ax.add_artist(Circle((0, 0), radius=m1["radius"], color=m1["color"]))
circle2 = ax.add_artist(Circle((0, 0), radius=m2["radius"], color=m2["color"]))
circle_cm = ax.add_artist(Circle((0, 0), radius=m2["radius"], color="purple"))
varrow1 = ax.add_artist(FancyArrow(0, 0, 0, 0, color="green", head_width=m1["radius"]))
varrow2 = ax.add_artist(FancyArrow(0, 0, 0, 0, color="green", head_width=m2["radius"]))
acc_arrow1 = ax.add_artist(
    FancyArrow(0, 0, 0, 0, color="purple", head_width=m1["radius"])
)
acc_arrow2 = ax.add_artist(
    FancyArrow(0, 0, 0, 0, color="purple", head_width=m2["radius"])
)


# This function will get called for every frame
def animate(frame):

    # Global variables needed in scope from the model
    global cm_position, cm_velocity, current_time, m1, m2

    # Global variables needed in scope from the artists
    global time_text, varrow1, varrow2, acc_arrow1, acc_arrow2, circle1, circle2, circle_cm

    print(f"Updating artists for day {current_time/SEC_PER_DAY:.1f}.")

    # Update the positions based on the current velocities
    m1["position"] = m1["position"] + m1["velocity"] * TIME_STEP
    m2["position"] = m2["position"] + m2["velocity"] * TIME_STEP
```

```
    # Update day label
    time_text.set_text(f"Day {current_time/SEC_PER_DAY:.0f}")

    # Update positions of circles
    circle1.set_center(m1["position"])
    circle2.set_center(m2["position"])

    # Update velocity arrows
    varrow1.set_data(
        x=m1["position"][0],
        y=m1["position"][1],
        dx=VSCALE * m1["velocity"][0],
        dy=VSCALE * m1["velocity"][1],
    )
    varrow2.set_data(
        x=m2["position"][0],
        y=m2["position"][1],
        dx=VSCALE * m2["velocity"][0],
        dy=VSCALE * m2["velocity"][1],
    )

    # Update the center of mass
    cm_position = cm_position + cm_velocity * TIME_STEP
    circle_cm.set_center(cm_position)

    # Find the vector from moon1 to moon2
    delta = m2["position"] - m1["position"]

    # What is the distance between the moons?
    distance = np.linalg.norm(delta)

    # Have the moons collided?
    if distance < m1["radius"] + m2["radius"]:
        print(f"*** Collided {current_time:.1f} seconds in!")

    # What is a unit vector that points from moon1 toward moon2?
    direction = delta / distance

    # Calculate the magnitude of the gravitational attraction
    magnitude = G * m1["mass"] * m2["mass"] / (distance**2)

    # Acceleration vector of moons (a = f/m)
    acceleration1 = direction * magnitude / m1["mass"]
    acceleration2 = (-1 * direction) * magnitude / m2["mass"]

    # Update the acceleration arrows
    acc_arrow1.set_data(
        x=m1["position"][0],
        y=m1["position"][1],
        dx=ASCALE * acceleration1[0],
        dy=ASCALE * acceleration1[1],
```

```
    )
    acc_arrow2.set_data(
        x=m2["position"][0],
        y=m2["position"][1],
        dx=ASCALE * acceleration2[0],
        dy=ASCALE * acceleration2[1],
    )

    # Update the velocity vectors
    m1["velocity"] = m1["velocity"] + acceleration1 * TIME_STEP
    m2["velocity"] = m2["velocity"] + acceleration2 * TIME_STEP

    # Update the clock
    current_time += TIME_STEP

    # Return the artists that need to be redrawn
    return (
        time_text,
        varrow1,
        varrow2,
        acc_arrow1,
        acc_arrow2,
        circle1,
        circle2,
        circle_cm,
    )


# Make the rendering happen
animation = FuncAnimation(
    fig,
    animate,
    np.arange(FRAMECOUNT),
    interval=ANI_INTERVAL
)

# Save the rendering to a video file
animation.save("moonmovie.mp4")
```

When you run this, it will take longer than the previous versions. You should have a video file that shows a simulation of the moons tracing their elliptical paths around their center of mass:



3.7 Challenge: The Three-Body Problem

It is time to stretch a little as a physicist and programmer: You are going to make a new version of `moons.py` that handles three moons instead of just two.

This is known as ”The Three-Body Problem”, and people have tried for centuries to come up with a way to figure out (from the initial conditions) where the three moons would be at time t without doing a simulation. And no one has.

For a lot of problems, the outcome is not very sensitive to the initial conditions. For example, consider the flight of a cannonball: If it leaves the muzzle of the cannon a little faster, it will go a little farther.

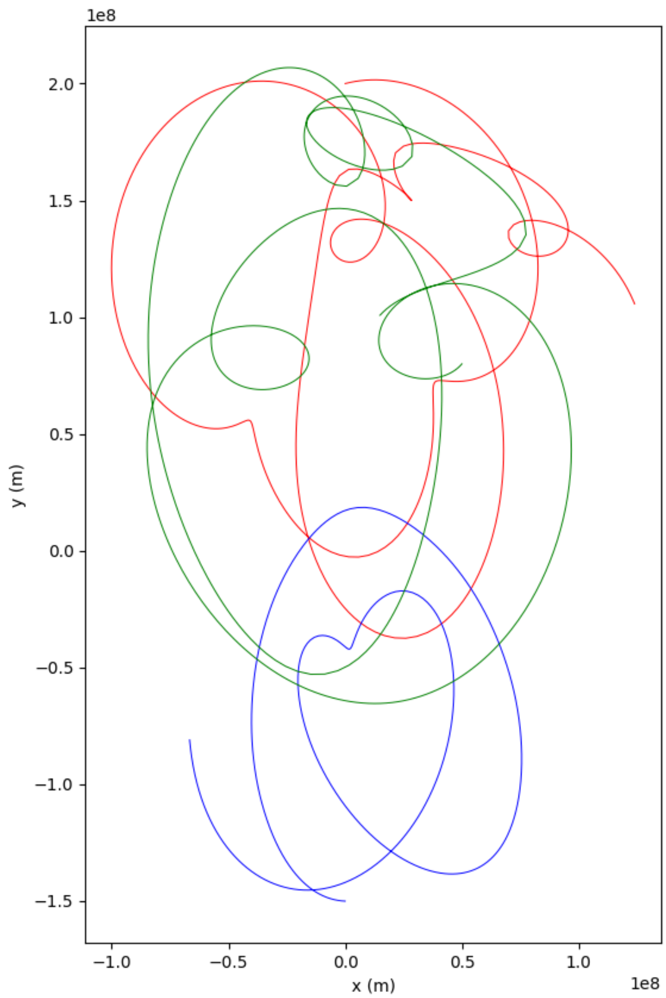
For the three-body problem, the outcome can be radically different even if the initial conditions are very similar.

(There is a whole field of mathematics studying systems that are very sensitive to initial conditions. It is known as *dynamical systems* or *chaos theory*.

Copy `moons.py` to `3moons.py`. Here is a reasonable initial state for your third moon:

```
m3 = {
    "mass": 4.0e22,  # kg
    "position": np.array([50_000_000, 80_000_000]),  # m
    "velocity": np.array([-30.0, -35.0]),  # m/s
    "radius": 1_700_000.0,  # m
    "color": "green"
}
```

If we run that simulation for 100 days, we get a plot like this:



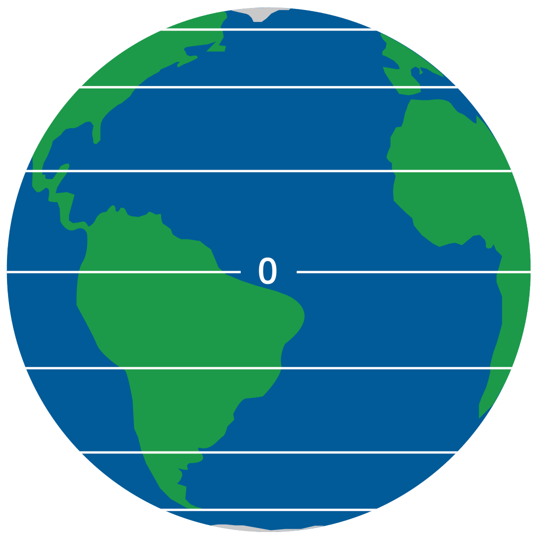
Visibly, you can see this is very different from the two-body problem that just traced ellipses around the center of mass.

Chapter 4

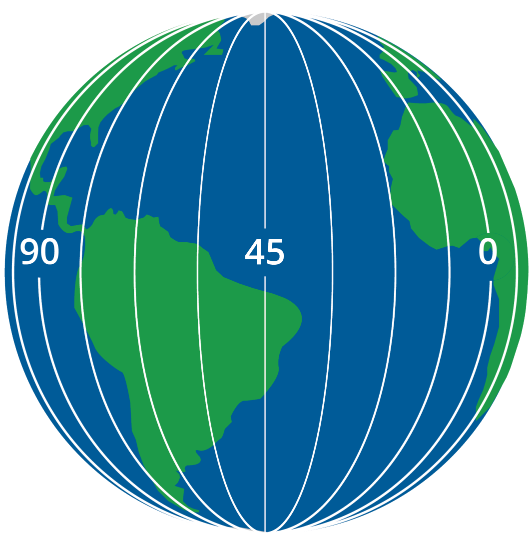
Longitude and Latitude

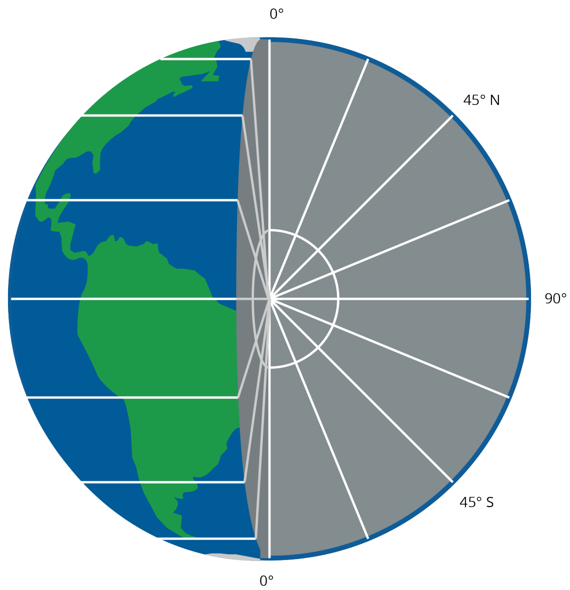
The Earth can be represented as a sphere, and the position of a point on its surface can be described using two coordinates: latitude and longitude.

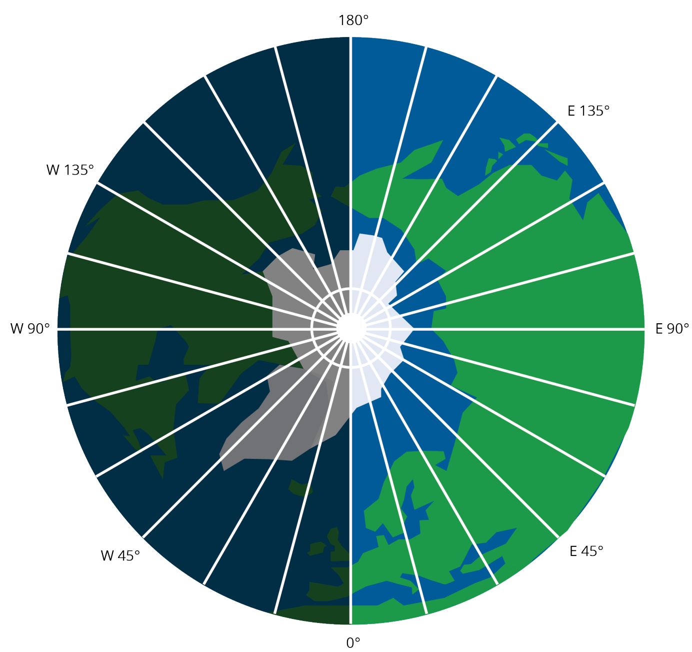
Latitude is a measure of a point's distance north or south of the equator, expressed in degrees. It ranges from $-90°$ at the South Pole to $+90°$ at the North Pole, with $0°$ representing the Equator.



Longitude, on the other hand, measures a point's distance east or west of the Prime Meridian (which passes through Greenwich, England). It ranges from $-180°$ to $+180°$, with the Prime Meridian represented as $0°$.





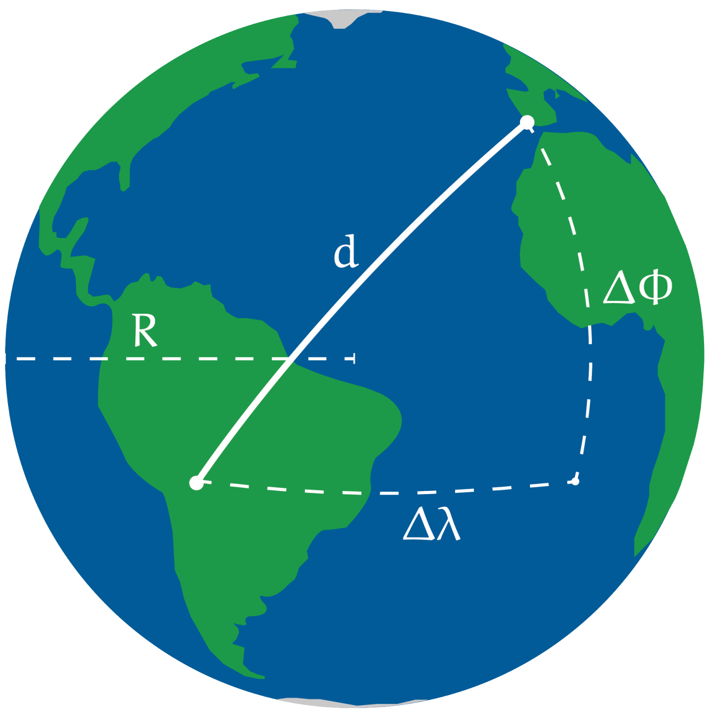


4.1 Nautical Mile

A nautical mile is a unit of measurement used primarily in aviation and maritime contexts. It is based on the circumference of the Earth, and is defined as one minute $(1/60°)$ of latitude. This makes it directly related to the Earth's geometry, unlike a kilometer or a mile, which are arbitrary in nature. The exact value of a nautical mile can vary slightly depending on which type of latitude you use (e.g., geodetic, geocentric, etc.), but for practical purposes, it iss often approximated as 1.852 kilometers or 1.15078 statute miles.

4.2 Haversine Formula

The haversine formula is an important equation in navigation for giving great-circle distances between two points on a sphere from their longitudes and latitudes. It is especially useful when it comes to calculating distances between points on the surface of the Earth, which we represent as a sphere for simplicity.



In its basic form, the haversine formula is as follows:

$$a = \sin^2\left(\frac{\Delta\phi}{2}\right) + \cos(\phi_1)\cos(\phi_2)\sin^2\left(\frac{\Delta\lambda}{2}\right)$$

$$c = 2 \cdot \operatorname{atan2}\left(\sqrt{a}, \sqrt{1-a}\right)$$

$$d = R \cdot c$$

Here, ϕ represents the latitudes of the two points (in radians), $\Delta\phi$ and $\Delta\lambda$ represent the differences in latitude and longitude (also in radians), and R is the radius of the Earth. The result, d, is the distance between the two points along the surface of the sphere.

Appendix A

Answers to Exercises

Answer to Exercise ?? (on page 5)

$$v = \sqrt{3.721(3.4 \times 10^6)} = 3,557 \text{ m/s}$$

The circular orbit is $2\pi(3.4 \times 10^6) = 21.4 \times 10^6$ meters in circumference.

The period of the orbit is $(21.4 \times 10^6)/3,557 \approx 6,000$ seconds.

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