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Electromagnetic Waves

Your phone is bathed in invisible waves right now: Wi-Fi, cell signals, GPS from satellites 20,000 km overhead, and the radio of every station in range. They are all the same kind of wave as the light from this page, the warmth from a campfire, and the X-rays at the dentist. The only difference among them is frequency.

Sound is a compression wave: to travel, it needs a medium to compress, such as air or water. (Whatever the movies show, there is no sound in the vacuum of space!)

Light is different. It is an *electromagnetic wave*: a ripple in electric and magnetic fields themselves. It needs no medium at all, which is how sunlight crosses 150 million kilometers of empty space to reach us.

1.1 Where Light Comes From

In the induction chapter, you saw that a changing magnetic field creates an electric field. In the 1860s, James Clerk Maxwell realized that the reverse is also true: a changing electric field creates a magnetic field. Put those together, and something amazing happens. Shake a charge back and forth, and its changing electric field makes a changing magnetic field, which makes a changing electric field, which makes a changing magnetic field...The two fields keep regenerating each other and race away from the charge as a wave. The electric and magnetic fields point at right angles to each other and to the direction the wave travels, so it is a transverse wave.

Maxwell calculated how fast this wave should travel, using only two constants measured with magnets and charged plates in the lab:

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \frac{1}{\sqrt{(4\pi \times 10^{-7})(8.854 \times 10^{-12})}} \approx 3.00 \times 10^8 \text{ m/s}$$

That was exactly the measured speed of light. In one stroke, Maxwell showed that light is an electromagnetic wave.

Electromagnetic waves travel at about 300 million meters per second in a vacuum: fast enough to circle Earth seven times in one second. They travel slower through matter. For example, light travels at 225 million meters per second in water.

Electromagnetic waves come in different frequencies. For example, the light coming out of a red laser pointer is usually about 4.75×10^{14} Hz. The wifi data sent by your computer

is carried on an electromagnetic wave too. It is usually close to 2.4×10^9 Hz or 5×10^9 Hz (2.4 or 5 gigahertz).

Because we know how fast the waves are moving, we sometimes talk about their wavelengths instead of their frequencies. Since we know its speed and its oscillation rate, we can calculate its wavelength. Since we want the distance per oscillation, we can divide the speed by the frequency. The light coming out of a laser pointer is $300 \times 10^6 / 4.75 \times 10^{14} = 630 \times 10^{-9}$ m, or 630 nm.

Exercise 1 Wavelengths

A green laser pointer emits light at 5.66×10^{14} Hz. What is its wavelength in a vacuum?

Working Space

Answer on Page 63

In general,

$$\lambda = \frac{c}{f}$$

where $c = 3 \times 10^8$ m/s is the speed of light in a vacuum.

Worked example. A Wi-Fi signal has frequency 2.4 GHz. What is its wavelength?

Solution. $\lambda = \frac{3.0 \times 10^8}{2.4 \times 10^9} = 0.125$ m, or 12.5 cm. That is why Wi-Fi antennas are a few centimeters long.

We have given names to different ranges of the electromagnetic spectrum in Figure 1.1

(You may have heard of “cosmic rays” and wonder why they are not listed in this table. Cosmic rays are not electromagnetic waves at all. They are particles, mostly protons and the nuclei of atoms stripped of their electrons, racing through space at nearly the speed of light. Some come from the Sun, but the most energetic ones come from exploding stars far outside our solar system. They were originally thought to be electromagnetic waves and were named “rays” by mistake.)

Different frequencies pass through different materials. A radio wave can pass through the walls of your house, but visible light cannot. X-rays, at much higher frequency, pass

Name	Hertz	Meters
Gamma rays	$10^{20} - 10^{24}$	$10^{-14} - 10^{-11}$
X-rays	$10^{16} - 10^{20}$	$10^{-11} - 10^{-8}$
Ultraviolet	$10^{15} - 10^{16}$	$10^{-8} - 10^{-7}$
Blue	$\sim 6 \times 10^{14}$	$\sim 5 \times 10^{-7}$
Red	$\sim 4 \times 10^{14}$	$\sim 7 \times 10^{-7}$
Infrared	$10^{12} - 10^{14}$	$10^{-6} - 10^{-3}$
Microwaves	$10^9 - 10^{12}$	$10^{-3} - 10^{-1}$
Radio waves	$10^3 - 10^9$	$10^{-1} - 10^5$

Figure 1.1: A table of different ranges of the electromagnetic spectrum.

through skin and muscle but are stopped by bone, which is why they make such good medical images. Microwave ovens use 2.45 GHz because water in food absorbs those waves well, so they heat the first few centimeters of the food.

1.2 The greenhouse effect

Humans have dug up a bunch of long carbon-based molecules (like oil and coal) and burned them, releasing large amounts of CO_2 into the atmosphere. It may not be obvious why that has made the planet warmer, but the answer is electromagnetic waves.

A warm object gives off infrared electromagnetic waves. That is why, for example, motion detectors in security systems are actually infrared detectors: even in a dark room, your body gives off a lot of infrared radiation.

You may have heard of “heat-seeking missiles.” These are more accurately called “Infrared homing missiles” because they follow objects giving off infrared radiation – hot things like jet engines.

The sun beams a lot of energy to our planet in the form of electromagnetic radiation: visible light, infrared, ultraviolet. (How much? At the top of the atmosphere directly facing the sun, we get 1,360 watts of radiation per square meter. That is a lot of power!)

About 29% of that radiation just reflects back into space: roughly 23% from clouds and the atmosphere, and 6 or 7% from the surface (especially snow and ice).

The other 71% is absorbed. 48% is absorbed by the surface and 23% is absorbed by the atmosphere. All of that energy warms the planet and the atmosphere so that it gives off infrared radiation. The planet lives in equilibrium; the infrared radiation leaving our atmosphere is exactly the same amount of energy as that 71% of the radiation that it absorbs.

(If the planet absorbs more energy than it releases, the planet gets hotter. Hotter things

release more infrared. When the planet is in equilibrium again, it stops getting hotter.)

So, what is the problem with CO_2 and other large molecules in the atmosphere? They absorb the infrared radiation instead of letting it escape into space. This means the planet must be hotter to maintain equilibrium.

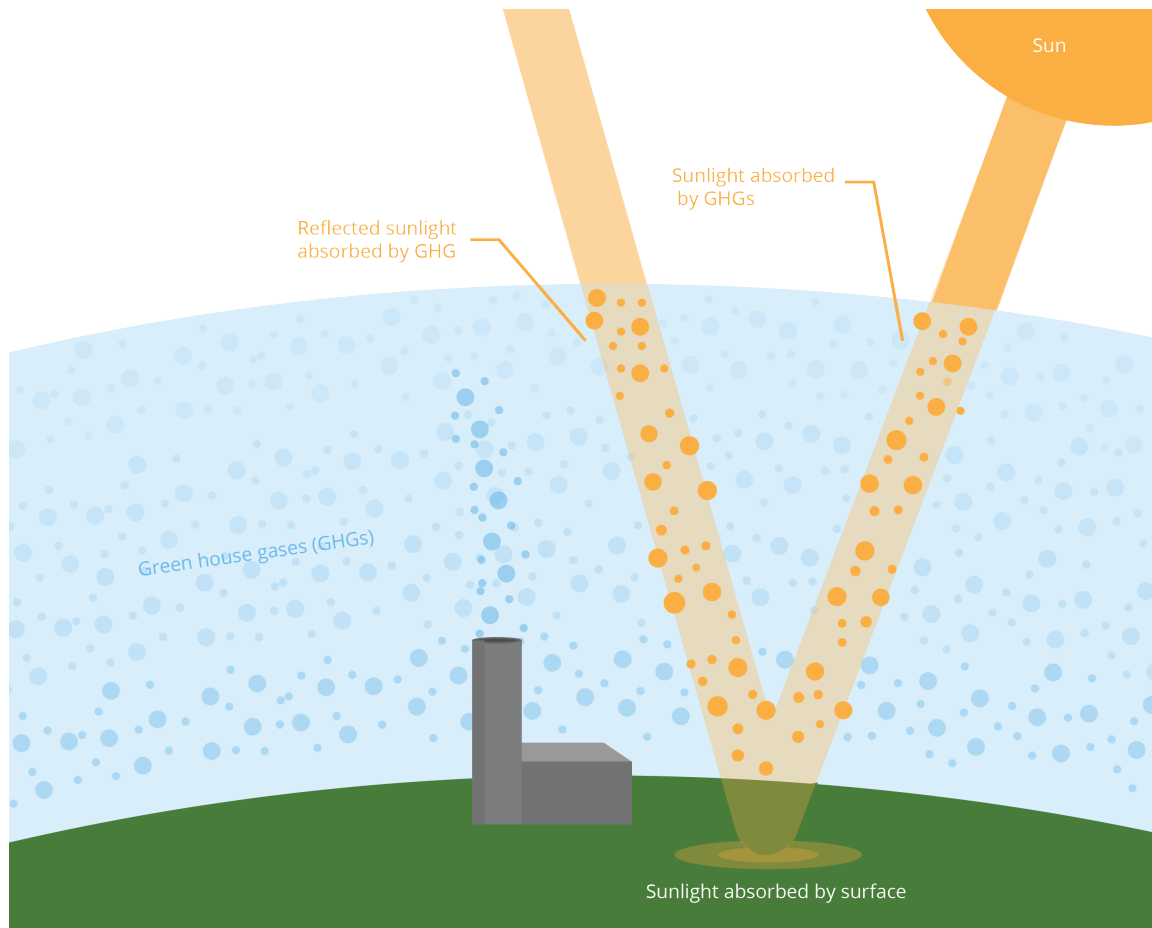


Figure 1.2: Greenhouse gas effects from the solar rays.

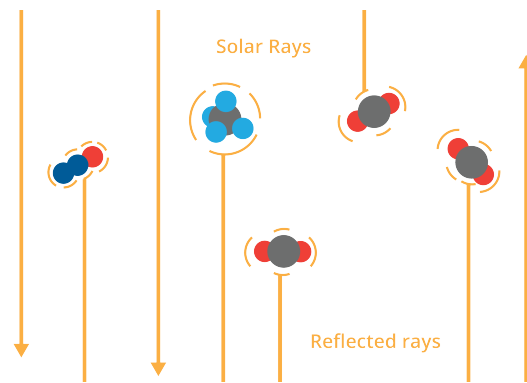


Figure 1.3: A zoomed in version of greenhouse gases.

The planet is getting hotter, and it is creating a multitude of problems:

- Weather patterns are changing, which leads to extreme floods and droughts.
- Ice and snow in places like Greenland are melting and flowing into the oceans. This is raising sea levels.
- Biomes with biodiversity are resilient. Rapidly changing climate is destroying biodiversity everywhere, which is making these ecosystems very fragile.
- In many places, permafrost, which has trapped large amounts of methane in the ground for millennia, is melting.

That last item is particularly scary, because methane is a large gas molecule — it absorbs even more infrared radiation than CO_2 . As it escapes the permafrost, the problem will get worse.

Scientists are working on four kinds of solutions:

- **Stop increasing the amount of greenhouse gases in our atmosphere.** It is hoped that non-carbon based energy systems like solar, wind, hydroelectric, and nuclear could let us stop burning carbon. Given the methane already being released, it may be too late for this solution to work on its own.
- **Take some of greenhouse gases out of our atmosphere and sequester them somewhere.** The trunk of a tree is largely made up of carbon molecules. When you grow a tree where there had not been one before, you are sequestering carbon inside the tree. There are also scientists that are trying to develop a process that pulls greenhouse gases out of the air and turn them into solids.
- **Decrease the amount of solar radiation that is absorbed by our planet and its atmosphere.** Clouds reflect a lot of radiation back into space. Could we increase the cloudiness of our atmosphere? Or maybe launch mirrors into orbit around our planet?

- **Adapt to the changing climate.** These scientists are assuming that global warming will continue, and are working to minimize future human suffering. How will we protect or relocate the hundreds of millions of people who live near today's coastlines as sea levels rise? When massive heat waves occur, how will we keep people from dying? As biodiversity decreases, how can we make sure that species that are important to human existence survive?

What are the greenhouse gases and how much does each contribute to keeping the heat from exiting to space? These numbers are still being debated, but this will give you a feel:

Water vapor	H ₂ O	36 - 72 %
Carbon dioxide	CO ₂	9 - 26 %
Methane	CH ₄	4 - 9 %
Ozone	O ₃	3 - 7 %

Notice that while we talk a lot about carbon dioxide, the most important greenhouse gas is actually water vapor. Why do we not talk about it more? Because we do not control it directly. The amount of water vapor in the air is set by temperature: warm air holds more of it, and any extra rains out within days. Carbon dioxide, on the other hand, stays in the air for centuries. So CO₂ sets the temperature, and the temperature sets the water vapor. In fact, water vapor makes the problem worse: a little warming from CO₂ puts more water vapor in the air, which traps still more heat.

Exercise 2 FM radio

An FM radio station broadcasts at 98.7 MHz. What is its wavelength? A good FM antenna is about half a wavelength long. How long is that?

Working Space

Answer on Page 63

Exercise 3 Frequency from Wavelength

An electromagnetic wave has a wavelength of 0.5 m. What is its frequency?

Working Space

Answer on Page 63

Interference, Diffraction, and the Doppler Effect

An ambulance races past, and its siren drops in pitch the instant it passes you. Noise-cancelling headphones make a jet engine go quiet by *adding* more sound. A soap bubble shimmers with colors that are not in the soap. You can hear someone talking around a corner but cannot see them. And astronomers know the universe is expanding because light from distant galaxies is slightly too red.

All of these come from what happens when waves meet each other or meet obstacles. In the Introduction to Waves and Sound chapters, you studied one wave at a time. This chapter is about waves combining (interference), waves bending around things (diffraction), and waves from moving sources (the Doppler effect). Along the way, you will see the experiment that proved light is a wave.

2.1 Superposition

When two waves overlap, the total disturbance at each point is simply the sum of the two. This is the *principle of superposition*. Two water ripples that cross pass right through each other, and where they overlap, their heights add.

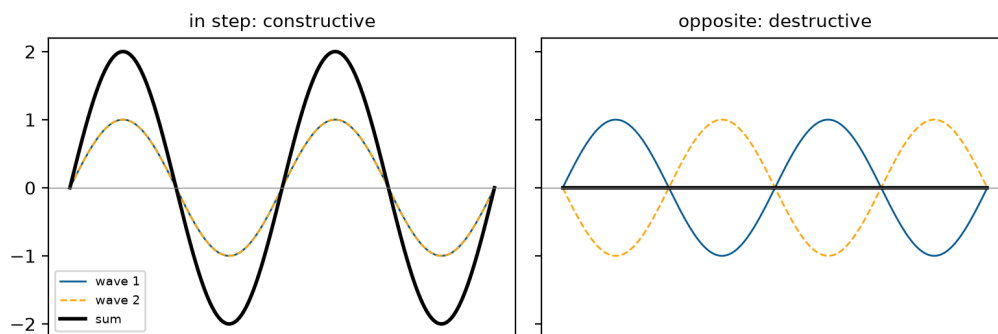


Figure 2.1: Left: two identical waves in step add to a wave twice as tall. Right: two waves exactly out of step cancel completely.

When crests meet crests, the waves reinforce each other: *constructive interference*. When crests meet troughs, they cancel: *destructive interference*. Noise-cancelling headphones use

a microphone to record the noise around you, then play the same sound flipped upside down, crest for trough. The two cancel inside your ear, and the drone of the engine fades.

2.1.1 Beats

What if two sounds have *almost* the same frequency? They drift in and out of step. When they are in step, the sound is loud; a moment later they are out of step, and it nearly vanishes. You hear a pulsing “wah-wah-wah” called *beats*.

The sum formulas from the Trigonometric Identities chapter show exactly what happens. Adding $\sin(a + b)$ and $\sin(a - b)$ gives $\sin(a + b) + \sin(a - b) = 2 \sin a \cos b$. With $a = 2\pi \frac{f_1 + f_2}{2} t$ and $b = 2\pi \frac{f_2 - f_1}{2} t$, this becomes

$$\sin(2\pi f_2 t) + \sin(2\pi f_1 t) = 2 \cos \left(2\pi \frac{f_2 - f_1}{2} t \right) \sin \left(2\pi \frac{f_1 + f_2}{2} t \right)$$

You hear a tone at the average frequency, whose loudness is controlled by the slowly changing cosine. The loudness peaks twice per cycle of that cosine, so

Beat Frequency

$$f_{\text{beat}} = |f_1 - f_2|$$

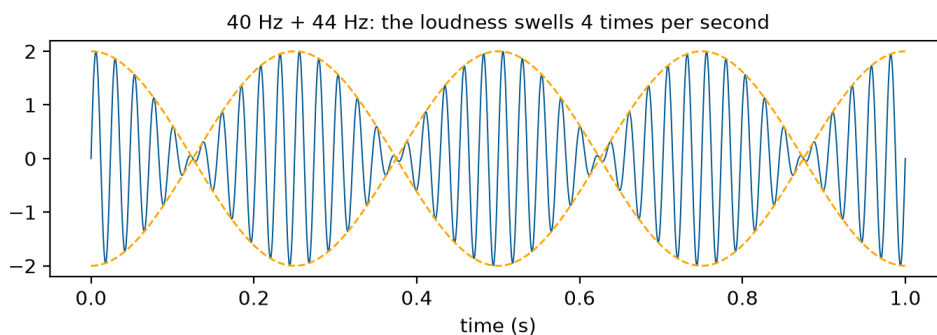


Figure 2.2: Two tones 4 Hz apart produce 4 beats per second.

Musicians tune by ear this way. Play a guitar string along with a 440 Hz tuning note; if you hear 3 beats per second, the string is 3 Hz off. Turn the peg until the beats slow down and disappear.

Exercise 4 Adding Waves

Two waves with amplitudes 3 cm and 2 cm pass through each other. What are the largest and smallest possible amplitudes of the combined wave?

Working Space

Answer on Page 63

Exercise 5 Tuning a Guitar

A guitarist hears 3 beats per second when playing a string together with a 440 Hz tuning note. What are the two possible frequencies of the string? The guitarist tightens the string slightly (raising its pitch), and the beats speed up. Which frequency was it?

Working Space

Answer on Page 63

2.2 Two Sources and the Double Slit

When waves from two sources overlap, whether they add or cancel at a point depends on how much farther one wave traveled. If the *path difference* is a whole number of wavelengths, crests arrive with crests: constructive. If it is a whole number plus a half, crests arrive with troughs: destructive.

Two-Source Interference

constructive: $\Delta = m\lambda$, destructive: $\Delta = (m + \frac{1}{2})\lambda$, $m = 0, 1, 2, \dots$

where Δ is the path difference.

You can hear this with two speakers playing the same tone: walk across the room, and

the sound gets loud and soft, loud and soft, as the path difference changes.

2.2.1 Young's Experiment

For centuries, scientists argued about whether light was a wave or a stream of tiny particles. In 1801, Thomas Young settled it (for a hundred years, anyway) with a simple experiment. He passed light through two narrow slits, very close together, onto a screen. Particles would make two bright stripes. Instead, he saw a row of many bright and dark bands, called *fringes*: interference. Light is a wave.

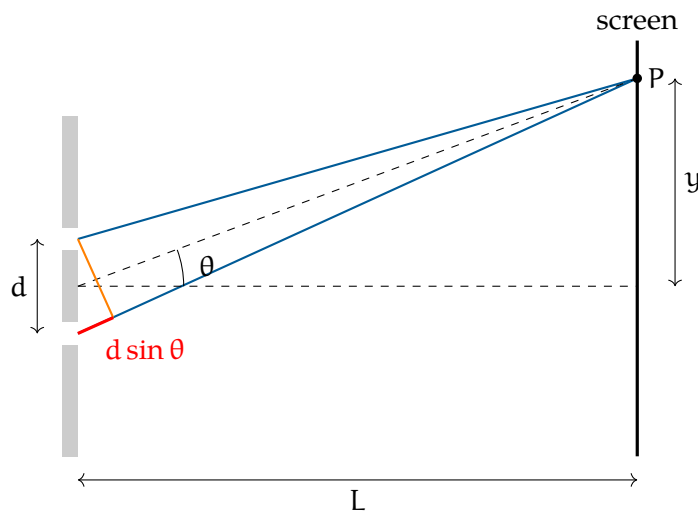


Figure 2.3: Light from the lower slit travels farther to P by the red segment, $d \sin \theta$. The real screen is so far away compared with d that the two rays are almost parallel.

Figure 2.3 shows why the bands appear where they do. The slits are a distance d apart. Light heading toward a point P at angle θ from the lower slit travels farther than light from the upper slit by $d \sin \theta$. So P is bright when

$$d \sin \theta = m\lambda$$

In a real experiment, θ is tiny, so $\sin \theta \approx \tan \theta = \frac{y}{L}$, where y is the distance from the center of the screen and L is the distance to the screen. The m th bright fringe is at $y = \frac{m\lambda L}{d}$, and neighboring bright fringes are

Double-Slit Fringe Spacing

$$\Delta y = \frac{\lambda L}{d}$$

Example: A red laser pointer ($\lambda = 650 \text{ nm}$) shines through two slits 0.25 mm apart onto a wall 2.0 m away. How far apart are the bright fringes?

Solution: $\Delta y = \frac{(650 \times 10^{-9})(2.0)}{0.25 \times 10^{-3}} = 5.2 \times 10^{-3} \text{ m}$, about 5 mm , easy to see and measure. Turn the formula around, and a ruler can measure a wavelength of light, a few ten-millionths of a meter.

The real pattern is shown below, computed in Python. The tall, narrow peaks are the two-slit fringes. They fade toward the sides because each slit also spreads its light out on its own, which is the next topic.

```

1  import numpy as np
2  import matplotlib.pyplot as plt
3
4  wavelength = 650e-9      # red laser, meters
5  d = 0.25e-3              # distance between the slits, meters
6  a = 0.04e-3              # width of each slit, meters
7  L = 2.0                  # distance to the screen, meters
8
9  y = np.linspace(-0.04, 0.04, 4000)      # position on the screen, meters
10 theta = y / L                            # small-angle approximation
11
12 two_slit = np.cos(np.pi * d * theta / wavelength) ** 2
13 # np.sinc(x) means sin(pi x) / (pi x)
14 one_slit = np.sinc(a * theta / wavelength) ** 2
15 intensity = two_slit * one_slit
16
17 print(f"fringe spacing: {wavelength * L / d * 1000:.1f} mm")
18 print(f"central bright band: {2 * wavelength * L / a * 1000:.0f} mm wide")
19
20 plt.figure(figsize=(7, 3), dpi=200)
21 plt.plot(y * 1000, intensity)
22 plt.plot(y * 1000, one_slit, "--", color="orange")
23 plt.xlabel("position on screen (mm)")
24 plt.ylabel("brightness")
25 plt.tight_layout()
26 plt.savefig("double_slit.png")

```

fringe spacing: 5.2 mm
 central bright band: 65 mm wide

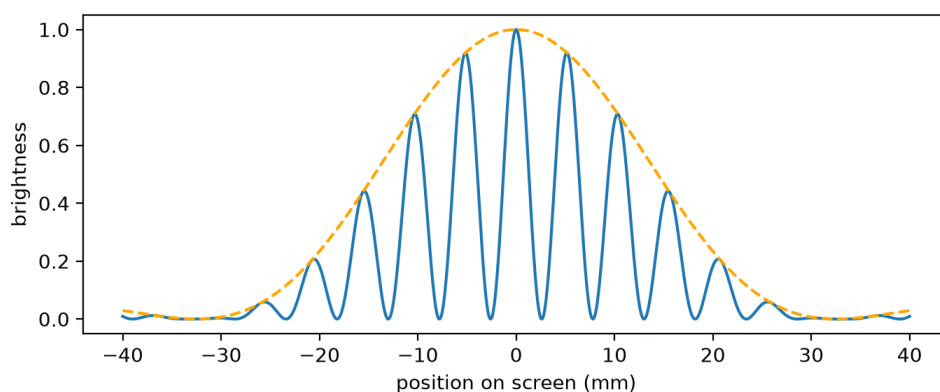


Figure 2.4: The double-slit pattern. The dashed orange curve is the pattern from one slit alone.

A century after Young, the Photons chapter's discovery that light also comes in particles made this experiment mysterious again. If you send photons through the slits *one at a time*, each lands as a single dot, yet over time the dots build up the same striped pattern. As the Paradoxes chapter put it, each photon somehow goes through both slits. Physicists are still arguing about what that means.

Exercise 6 Green Fringes

A green laser ($\lambda = 532 \text{ nm}$) shines through two slits 0.20 mm apart onto a screen 1.5 m away. Find the fringe spacing.

Working Space

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Exercise 7 Measuring Light with a Ruler

Light passes through two slits 0.30 mm apart onto a screen 2.0 m away. The bright fringes are 4.0 mm apart. What is the wavelength? What color is it?

Working Space

Answer on Page 64

2.3 Diffraction

Waves spread out after passing through an opening or around an edge. This is *diffraction*, and how much a wave spreads depends on the size of the opening compared with the wavelength. If the opening is much bigger than the wavelength, the wave goes nearly straight through. If it is about the same size or smaller, the wave spreads widely.

That is why you can hear around a corner but not see around it. A voice at 500 Hz has a wavelength of $\frac{343}{500} \approx 0.7$ m, about the size of a doorway, so it spreads around the corner. Visible light has a wavelength of about 500 nm, more than a million times smaller than the doorway, so it travels in straight lines and casts sharp shadows.

A single slit of width a makes a wide central bright band with dark bands where $a \sin \theta = m\lambda$, for $m = 1, 2, \dots$. That is the orange envelope in figure 2.4. The narrower the slit, the wider the spread.

Diffraction also sets a limit on how sharply anything can see. Light passing through a telescope lens or a microscope spreads by an angle of about $\frac{\lambda}{\text{width}}$, which blurs fine detail. Bigger telescopes and shorter wavelengths see finer detail. That is why Blu-ray discs use a blue-violet laser instead of a DVD's red one: the shorter wavelength focuses to a smaller spot, so the pits that store data can be packed more tightly.

2.3.1 Diffraction Gratings

A *diffraction grating* has thousands of evenly spaced slits. The bright fringes obey the same rule, $d \sin \theta = m\lambda$, but with so many slits they become very sharp, and each color lands at a different angle. The closely spaced tracks on a CD act as a grating, which is why it shines with rainbows. Astronomers attach gratings to telescopes to split starlight into a spectrum and read which elements the star contains.

Exercise 8 A Grating

A grating has 600 slits per millimeter. At what angle does the first-order ($m = 1$) red light ($\lambda = 650$ nm) appear? What is the highest order of red light you can see?

Working Space

Answer on Page 64

Exercise 9 Around the Corner

Explain why you can hear a friend talking in the hallway when you are in a classroom with the door open, even though you cannot see them.

Working Space

Answer on Page 64

2.4 The Doppler Effect

A source of waves sends out crests at a steady rate. If it is moving, each new crest starts from a slightly different place. In front of the source, the crests bunch up; behind it, they spread out (figure 2.5). Bunched-up crests mean a shorter wavelength and a higher frequency. This is the *Doppler effect*, named after Christian Doppler, who predicted it in 1842.

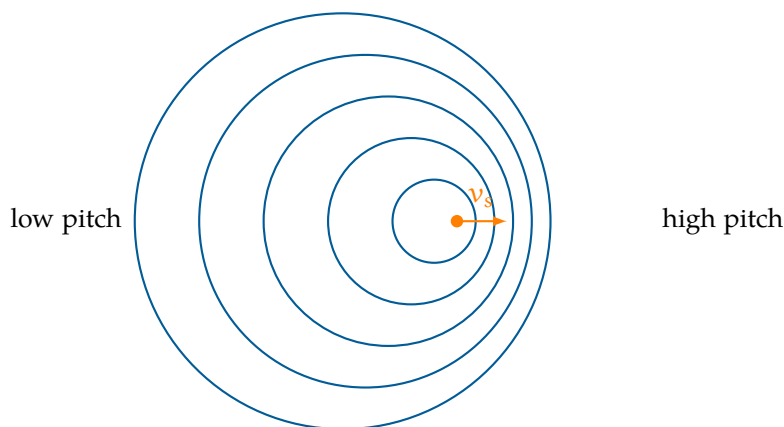


Figure 2.5: Wavefronts from a source moving to the right. Each circle was emitted from where the source was at that time. Crests crowd together ahead of the source and spread apart behind it.

Here is the size of the effect. A source emits waves of frequency f (period $T = \frac{1}{f}$), and the waves move at speed v . If the source moves toward you at speed v_s , then in one period a crest travels vT while the source follows it by v_sT , so the next crest is only $(v - v_s)T$ behind. The wavelength you receive is $(v - v_s)T$, and the frequency you hear is $\frac{v}{(v - v_s)T}$. For a source moving away, the minus sign becomes a plus.

The Doppler Effect for a Moving Source

$$f_{\text{heard}} = f \frac{v}{v - v_s} \text{ (approaching),} \quad f_{\text{heard}} = f \frac{v}{v + v_s} \text{ (receding)}$$

Example: An ambulance siren sounds at 700 Hz, and the ambulance drives at 30 m/s. The speed of sound is 343 m/s. What do you hear as it approaches, and as it drives away?

Solution: Approaching: $700 \cdot \frac{343}{343-30} \approx 767$ Hz. Receding: $700 \cdot \frac{343}{343+30} \approx 644$ Hz. As it passes, the pitch drops by more than 120 Hz, about three musical half steps (the Sound chapter explained half steps).

The same idea works when you move instead of the source; then $f_{\text{heard}} = f \frac{v+v_o}{v}$ for an observer moving toward the source at speed v_o . Police radar guns and weather radar bounce waves off moving cars or raindrops and measure the frequency shift to find their speed. Doctors use Doppler ultrasound to watch blood flow through the heart.

2.4.1 Sonic Booms

What happens as v_s approaches the speed of sound v ? The denominator $v - v_s$ shrinks to zero: the crests pile up on top of each other in front of the source. A jet flying faster than sound outruns its own sound waves, which merge into a cone-shaped shock wave trailing behind it. When the cone sweeps over you, you hear a single loud *sonic boom*. A whip's crack is a tiny sonic boom: the tip moves faster than sound.

2.4.2 Redshift

Light shows a Doppler effect too. For a source moving away at a speed v much less than the speed of light c , the wavelength stretches by

$$\frac{\Delta\lambda}{\lambda} \approx \frac{v}{c}$$

so the light looks a little redder: a *redshift*. In the 1920s, Edwin Hubble found that nearly every distant galaxy is redshifted, and the farther away it is, the faster it is receding. The universe is expanding.

Example: Hydrogen gives off red light at 656.3 nm. In a distant galaxy, that line is observed at 662.9 nm. How fast is the galaxy moving away?

Solution: $v \approx c \frac{\Delta\lambda}{\lambda} = (3.00 \times 10^5 \text{ km/s}) \frac{662.9-656.3}{656.3} \approx 3,000 \text{ km/s.}$

Exercise 10 A Passing Train

A train horn sounds at 400 Hz, and the train moves at 25 m/s. The speed of sound is 343 m/s. What frequencies do you hear as the train approaches and as it moves away?

Working Space

Answer on Page 64

Exercise 11 A Faster Galaxy

In another galaxy, the 656.3 nm hydrogen line is observed at 669.4 nm. About how fast is the galaxy receding?

Working Space

Answer on Page 64

Optics

How does a flashlight throw a straight beam? How do eyeglasses fix blurry vision? Why does a straw look bent in a glass of water? Optics, the study of how light travels, bounces, and bends, answers all of these. Most of it comes down to two simple rules: light bounces off mirrors at equal angles, and it bends when it changes speed.

3.1 Reflection

No surface reflects all the light that hits it, but a mirror comes close. A mirror is a sheet of glass with a thin layer of metal, usually silver or aluminum, on its back. Metals contain many electrons that are free to move. When light reaches the metal, these electrons vibrate and send nearly all of the light back out.

The smoothness of the surface decides what you see. On a mirror, the metal layer is so flat that every ray in a parallel beam meets it at the same angle and leaves in the same direction. This is called *specular reflection*, and it is why a mirror shows a clear image. Paper and painted walls reflect light too, but their surfaces are covered in tiny bumps that face many directions. Each ray leaves at a different angle, which is called *diffuse reflection* (Figure 3.1). You can see a wall clearly, yet you cannot see your face in it.

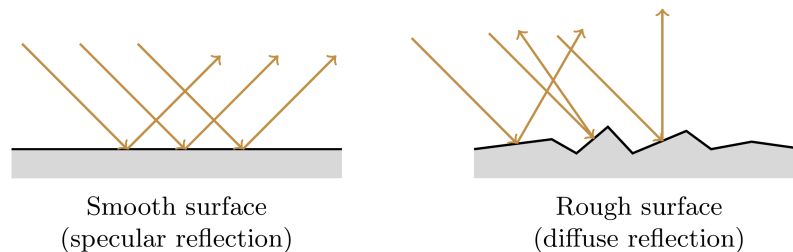


Figure 3.1: Parallel rays leave a smooth surface in one direction (specular reflection) and leave a rough surface in many directions (diffuse reflection).

When a beam of light hits a mirror, it bounces off the mirror at the same angle it approached from. That is, if it approaches nearly perpendicular to the mirror, it departs nearly perpendicular to the mirror. If it hits the mirror at a glancing angle, it departs at an angle close to the mirror's surface. A good video to see this in action can be found here: https://www.youtube.com/shorts/qA_VQZfTiUA

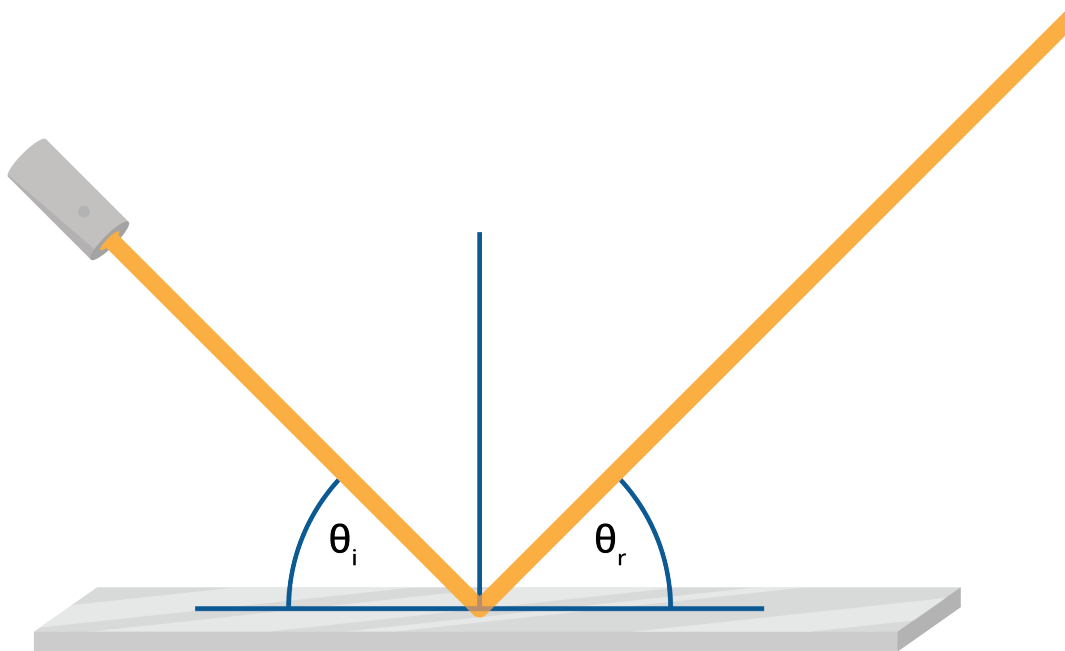
Law of Reflection

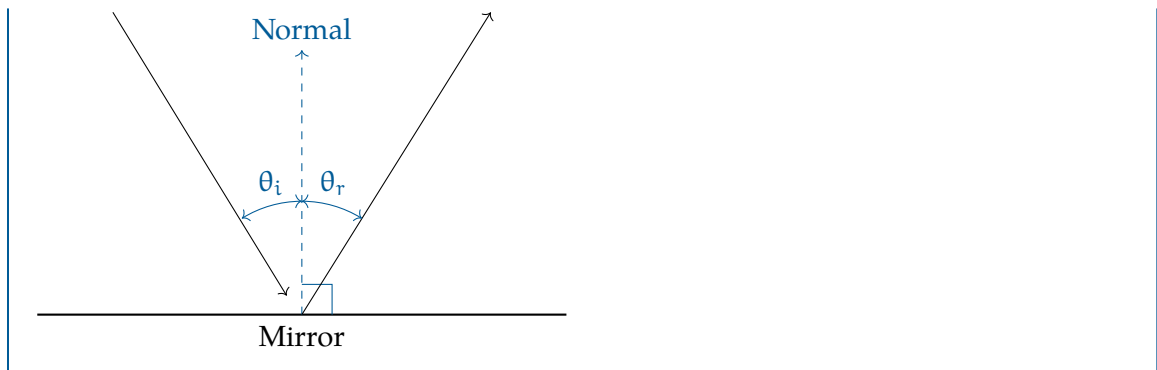
Figure 3.2: The angle of *incidence* is equal to the angle of *reflection*.

Law of Reflection

The angle of incidence, denoted as θ_i , is equal to the angle of reflection, denoted as θ_r . This law can be mathematically expressed as:

$$\theta_i = \theta_r$$

where θ_i is the angle between the incident light ray and the normal to the surface, and θ_r is the angle between the reflected light ray and the normal.



Exercise 12 Law of Reflection

Working Space

You are standing 4 meters from a mirror hung on a wall. The bottom of the mirror is the same height as your chin, so you cannot see your whole body. You stick a piece of masking tape to your body.

You walk forward until you are only 3 meters from the mirror, then put a piece of masking tape on your body at the new cut-off point. Is the new masking tape higher or lower on your body?

Answer on Page 64

3.1.1 Ellipses and Curved Mirrors

Flat mirrors are common and useful, but things get more interesting once you bend the mirrors. In this section, we are going to talk about a few different kinds of curved mirrors.

The Reflective Properties of Circles and Spheres

For example, if you were inside a circular room (a cylinder, actually), you could imagine standing in the center and pointing a flash light in any horizontal direction. The beam of light would bounce right back to you. See Figure 3.3

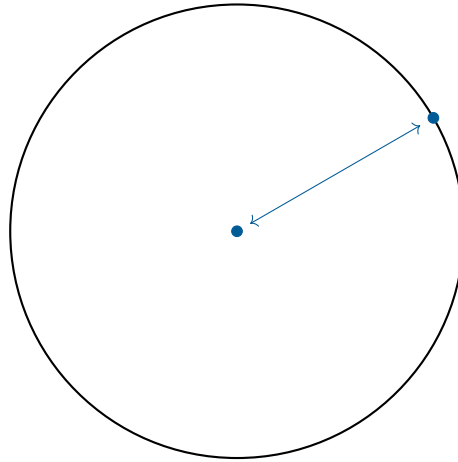


Figure 3.3: A circle would reflect back onto itself.

How do you know this? Because the tangent line is always perpendicular to the radius to the point of tangency:

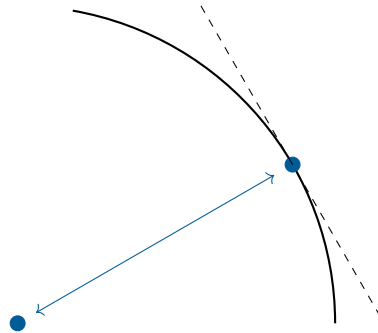


Figure 3.4: The reflection on a circle uses a radius which hits a tangent line as a perpendicular bisector.

You could create a spherical room with mirror walls. You would create a platform in the center where you could stand, and if you pointed your flashlight in any direction, its beam of light would shine back at you.

Ellipses and Ellipsoids

You met ellipses in the conic sections chapter. Now we will see why they matter for light.

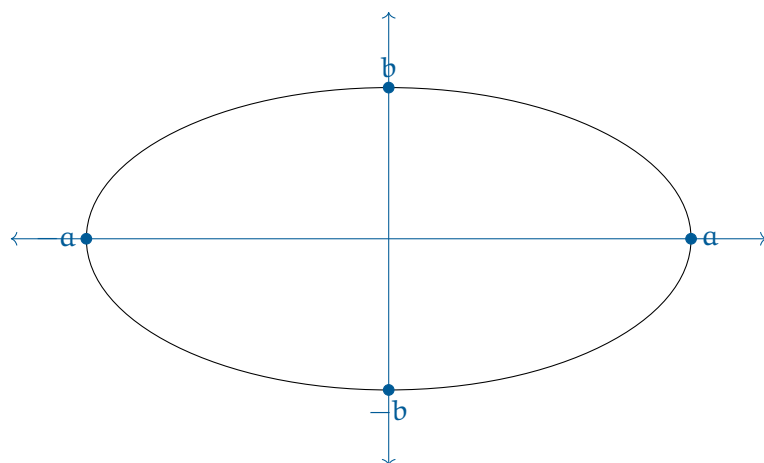
Mathematicians talk about a *standard* ellipse. A standard ellipse is centered on the origin $(0, 0)$ and its long axis is parallel with the x -axis or the y -axis.

Equation for a Standard Ellipse

To be precise, a standard ellipse is the set of points (x, y) that are solutions to the equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Note that $(a, 0)$, $(-a, 0)$, $(0, b)$, $(0, -b)$ are all part of the set. The complete set looks like this:



The area contained inside this ellipse is given by

$$A = \pi ab$$

We can now talk about two special points: the *foci*. Each focal point is on the long axis of the ellipse. Let us assume for a moment that $a > b$. (Everything works the same if $b > a$, but it gets confusing if we try to deal with both cases simultaneously.)

If p is a point on the ellipse, the distance from p to focal point 1 plus the distance from p to focal point 2 is always $2a$.

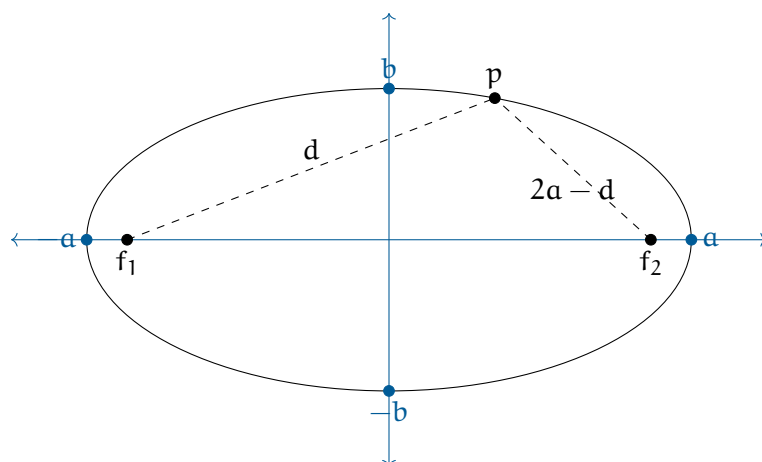


Figure 3.5: The both foci and point p forms a triangle with legs $2a - d$ or d .

How do we find the foci? We know they are on the long axis and that they are symmetrical across the short axis. All we need to know is how far are they from the short axis.

Distance from Center to the Foci

If you have an ellipse with a long axis that extends a from the center, and a short axis that extends b from the center, the foci lie on the long axis and are c from the center. Where

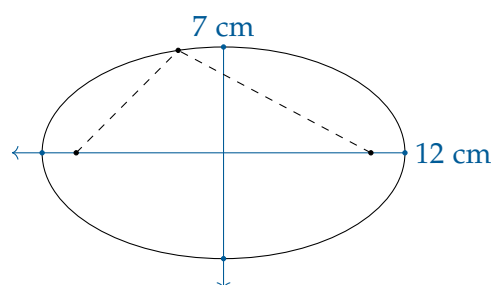
$$c = \sqrt{a^2 - b^2}$$

Exercise 13 Foci of an ellipse

Working Space

You need to draw an ellipse that is 24 cm long and 14 cm wide. You have a string, two pushpins, a ruler, and a pencil. Using the ruler, you draw two perpendicular axes.

You will stick one pin at each focal point. Each end of the string will be tied to a push pin. Using the pencil to keep the string taut, you will draw an ellipse.



How far from the short axis are the pushpins placed?

How long is the string between them?

Answer on Page 65

The Reflective Property of Ellipses

Here is something else that is wonderful about an ellipse: Pick any point p on the ellipse. Draw a line from p to each focal point. Draw the line tangent to the ellipse at p . You will see that the angle between the tangent and the line to focal point 1 is equal to the angle between the tangent and the line to focal point 2. See Figure 3.6

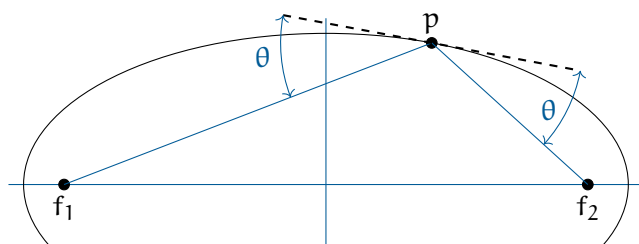


Figure 3.6: Reflective Property of Ellipses.

This is known as “The Reflective Property of Ellipses”. Imagine you and your friend Fred are at an ellipse-shaped skating rink, and the edge of the rink is mirrored. You sit at one focal point and your friend sits at the other. If you point a flashlight at the mirror (in any direction!), the beam will bounce off the wall and head directly for Fred.

If Fred ducks out of the way, the beam will bounce again and head back to you.

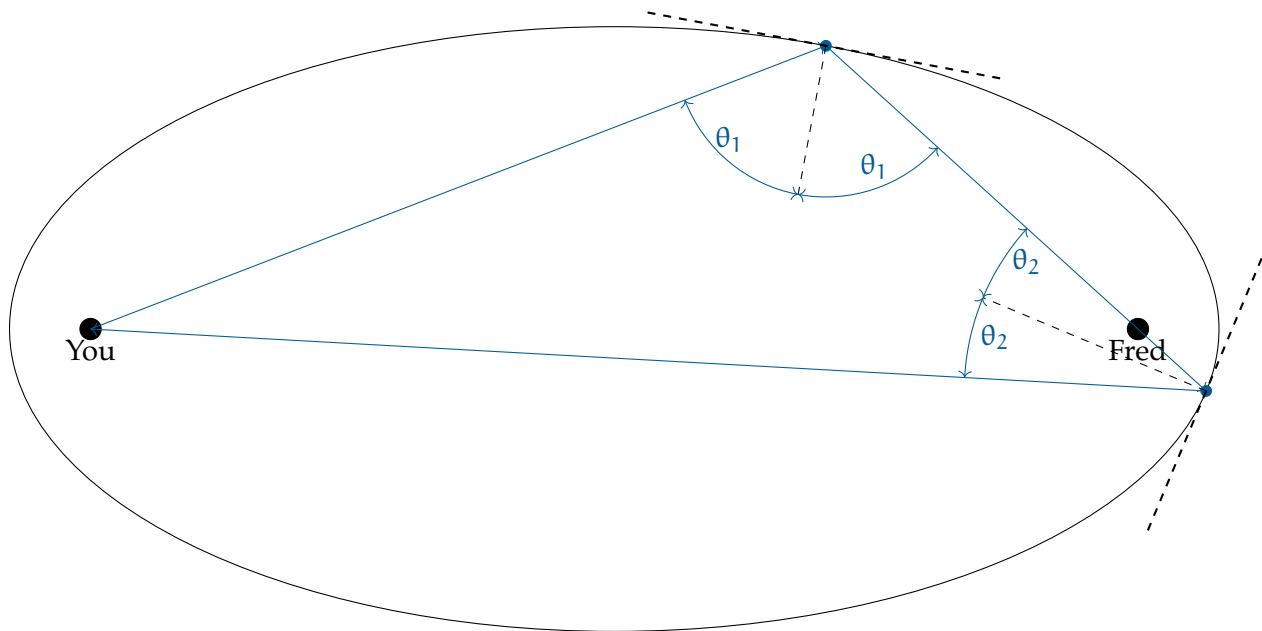


Figure 3.7: Your laser intersects Fred.

This will work for sound as well. If you whisper while on the focal point, Fred (at the other focal point) will hear you surprisingly well, because all the soundwaves that hit the wall will bounce (just like the light) straight at Fred.

Elliptical Orbits

One more fun fact about ellipses: We often imagine the planets traveling in circular orbits with the sun at the center — they actually travel in elliptical orbits, with the sun as one of the focal points.

The Earth is closest to the sun around January 3rd: 147 million km.

The Earth is farthest from the sun around July 3rd: 152 million km.

(Note that these dates are not the same as the solstices: The southern hemisphere is tilted the most toward the sun around December 21 and tilted most away around June 21.)

Ellipsoids

Just as we can pull the ideas of a circle into three dimensions to make a sphere, we can extend the ideas of the ellipse into three dimensions to talk about ellipsoids. Ellipsoids are like blimps.

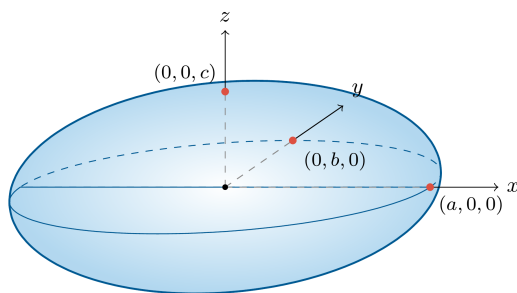
The standard ellipsoids are centered at the origin and aligned with the three axes.

Equation for a Standard Ellipsoid

To be precise, a standard ellipsoid is the set of points (x, y, z) that satisfy the equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

Note that $(a, 0, 0)$, $(0, b, 0)$, $(0, 0, c)$ are all part of the set. The complete set looks like this:



The volume bounded by this ellipsoid is

$$V = \frac{4}{3}\pi abc$$

Of course, a , b , and c can be any positive number, but in the real world, we find ourselves working regularly with ellipsoids where two of the numbers are the same.

Oblate Spheroid

If two axes have the same length and one is shorter, you get something that looks like a sphere compressed in one direction — like a pumpkin. These are called *oblate spheroids*.

The Earth is actually an oblate spheroid; the axis that goes through the north and south pole is shorter than the axes that pass through the equator. How much shorter? Just a little, relatively speaking. The equator is 6,378 km from the center of the Earth; the north pole is 21 km closer.

Prolate Spheroid

If two axes have the same length and one is longer, you get something that looks like a sphere stretched in one direction — like a rugby ball. It is called a *prolate spheroid*.

Like an ellipse, prolate spheroids have two focal points.

Focal Points of a Prolate Spheroid

If the long axis has a radial length of a and the two shorter axes have radial length b , then the focal points are on the long axis. The distance from the center to the focal point is given by

$$c = \sqrt{a^2 - b^2}$$

For any point p on the prolate spheroid, the sum of the distances from p to the focus points will always be $2a$.

It has the reflective property: A photon shot in any direction from one focal point will bounce off the wall and head directly at the other.

Exercise 14 Volume of a Rugby Ball

Working Space

Some jokesters once thought it would be fun to make something that looked like a rugby ball, but made out of lead.

A rugby ball is about 30 cm long and has a circumference of 60 cm at its midpoint. A cubic centimeter of lead has a mass of 11.34 grams.

How much would a solid (not hollow)

lead rugby ball weigh?

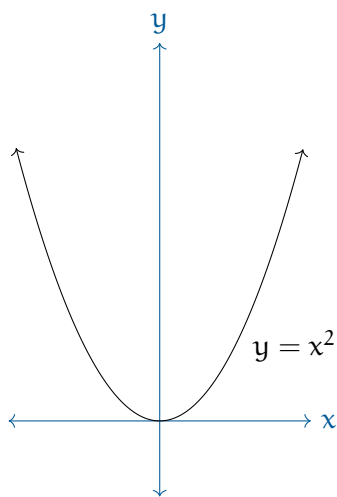
Answer on Page 65

Parabolas and Parabolic Reflectors

You are familiar with quadratic functions:

$$y = ax^2 + bx + c$$

If a is not zero, the graph of a quadratic is a curved line called a *parabola*. The first parabola that most mathematicians think of is the graph of $y = x^2$:



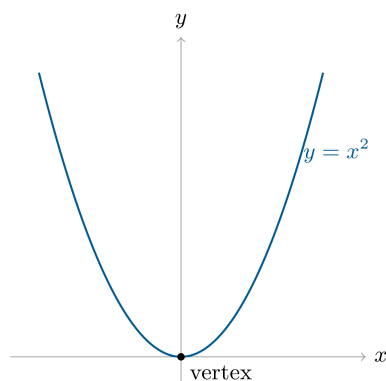


Figure 3.8: The graph of $y = x^2$ is a parabola with its vertex at the origin.

Every parabola has a *focus* and a *directrix*. The focus is a point on the parabola's axis of symmetry. The directrix is a line perpendicular to the axis of symmetry. Every point on the parabola is equal distance from the focus and the directrix.

For the graph of $y = x^2$, the focus is the point $(0, \frac{1}{4})$. The directrix is the line $y = -\frac{1}{4}$:

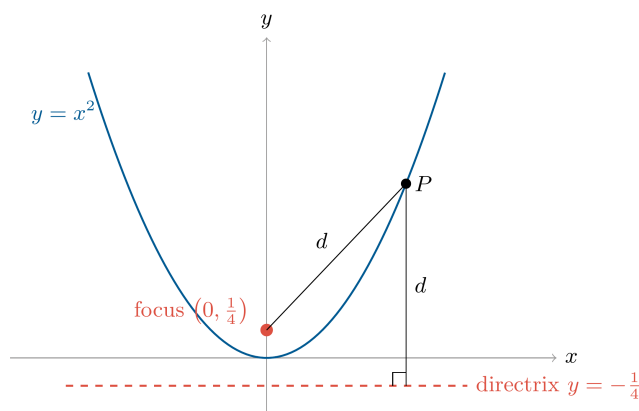


Figure 3.9: Every point P on $y = x^2$ is the same distance d from the focus $(0, \frac{1}{4})$ and from the directrix $y = -\frac{1}{4}$.

For example, the point $(1, 1)$ is on this parabola. It is $\frac{5}{4}$ from the directrix. How far is it from the focus? 1 horizontally and $\frac{3}{4}$ vertically.

$$\sqrt{1^2 + \left(\frac{3}{4}\right)^2} = \frac{5}{4}$$

Thus, we have confirmed that $(1, 1)$ is equal distances from the focus and the directrix.

When we think about a parabola and its properties, we usually rotate and translate it to be symmetric around the y -axis, flip it so that it is low in the middle and rising on both sides, and push it up or down until the low point is on the x -axis.

Then, they can all be written:

$$y = \frac{a}{4}x^2$$

where $a > 0$. If a is small, the parabola opens wider.

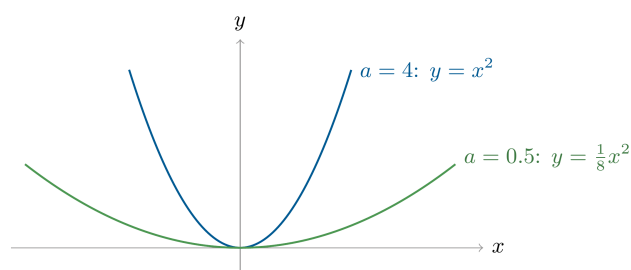


Figure 3.10: A smaller value of a gives a wider parabola.

The focus is at $(0, \frac{1}{a})$ and the directrix is the line $y = -\frac{1}{a}$.

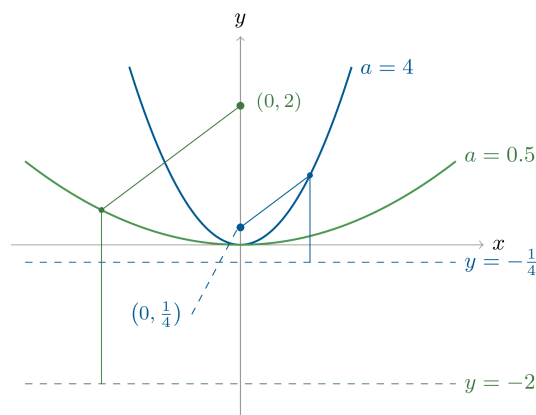


Figure 3.11: Each parabola has its own focus and directrix: $(0, \frac{1}{4})$ and $y = -\frac{1}{4}$ for $a = 4$, and $(0, 2)$ and $y = -2$ for $a = 0.5$.

Reflective Property of a Parabola

Assume you have a parabola-shaped mirror. A beam of light shot from the focus will bounce off the mirror in the direction of the axis of symmetry:

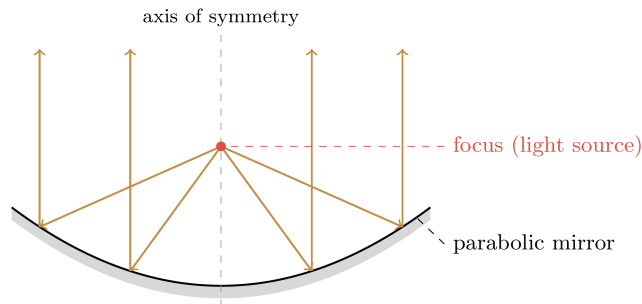


Figure 3.12: Light leaving the focus of a parabolic mirror reflects parallel to the axis of symmetry.

This is why your flashlight has a parabolic mirror. The lightbulb is at the focus, so any photons that hit the mirror are redirected straight forward.

(Note that in the real world, we use parabolic dishes: a parabola rotated around its axis of symmetry.)

The reflection works exactly the same in reverse. There are solar cookers that are big parabolic mirrors. They let you put a pot on the focus point. You move the dish until its axis of symmetry is pointed at the sun.

You will also see a lot of antennas have parabolic dishes. Note that photons that come in parallel to the axis of symmetry are redirected to a single point: where the receiver is.



Figure 3.13: A real life parabolic dish as a satellite.

Sometimes in a science museum, you will see two parabolic dishes far apart and pointed at each other. One person speaks with their mouth at the focus of one. The other person listens with their ear at the focus of the other. Even though you are very far apart, it sounds like they are really, really close.



Figure 3.14: Two parabolic dishes send waves between each other.

This is because the speaker's parabolic wall focuses the sound energy in a nice beam the size of the wall pointed straight at the listener's parabolic wall. The listener's wall focuses

the energy of that beam at the listener's ear.

3.2 Refraction

3.2.1 Bending Light at a Boundary

Put a straw in a glass of water, and it looks broken at the surface. That is *refraction*: light changes direction when it passes from one material into another, because it changes speed.

Here is a way to picture it. Imagine a lawn mower rolling at an angle off a sidewalk onto thick grass. The wheel that hits the grass first slows down first, so the mower swings around toward the grass before the other wheel catches up. Light does the same thing when it enters a slower material: it bends toward the line perpendicular to the surface. (Do not confuse *refraction* with *rarefaction*, the spread-out regions of a sound wave.)

3.2.2 Snell's Law

This phenomenon is explained by Snell's law, which states:

$$n_1 \cdot \sin(\theta_1) = n_2 \cdot \sin(\theta_2) \quad (3.1)$$

where:

- n_1 and n_2 are the indices of refraction for the first and second media, respectively. The index of refraction is the ratio of, c the speed of light in a vacuum to, v the speed of light in the medium, $n = \frac{c}{v}$. It is a dimensionless quantity. $n \geq 1$ for all materials, such that $n_{\text{vacuum}} = 1$ and $n_{\text{air}} \approx 1$. See a list of all mediums here: https://en.wikipedia.org/wiki/List_of_refractive_indices
- θ_1 and θ_2 are the angles of incidence and refraction, respectively. These angles are measured from the normal (perpendicular line) to the surface at the point where light hits the boundary.

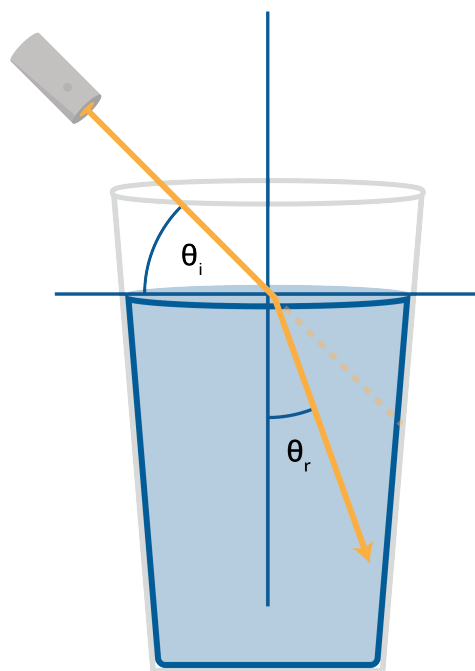


Figure 3.15: Refraction occurs when light changes the medium it is in.

The angle of incidence (θ_1) is the angle between the incident ray and the normal to the interface at the point of incidence. Similarly, the angle of refraction (θ_2) is the angle between the refracted ray and the normal.

When light travels from a medium with a lower refractive index to a medium with a higher refractive index, it bends towards the normal. Conversely, when light travels from a medium with a higher refractive index to one with a lower refractive index, it bends away from the normal.

Worked example. A beam of light in air hits a pool of water ($n = 1.33$) at 45° from the normal. At what angle does it travel in the water?

Solution. $(1.00) \sin 45^\circ = 1.33 \sin \theta_2$, so $\sin \theta_2 = \frac{0.707}{1.33} \approx 0.532$ and $\theta_2 \approx 32^\circ$. The beam bends toward the normal.

3.2.3 Total Internal Reflection

Going the other way, from water into air, light bends *away* from the normal. Tilt the beam far enough, and the refracted angle would have to be more than 90° , which is impossible. Then the light does not leave at all: it reflects perfectly back into the water. This is *total internal reflection*. The smallest angle where it happens, the *critical angle*, satisfies $\sin \theta_c = \frac{n_2}{n_1}$. For water into air, $\sin \theta_c = \frac{1}{1.33}$, so $\theta_c \approx 49^\circ$.

Fiber optic cables carry internet data this way. Light zigzags down a thin glass fiber, bouncing off the walls by total internal reflection, and travels for kilometers without escaping.

Exercise 15 Into glass

Light in air enters a glass block ($n = 1.5$) at 30° from the normal. At what angle does it travel in the glass? What is the critical angle for light trying to leave the glass into air?

Working Space

Answer on Page 65

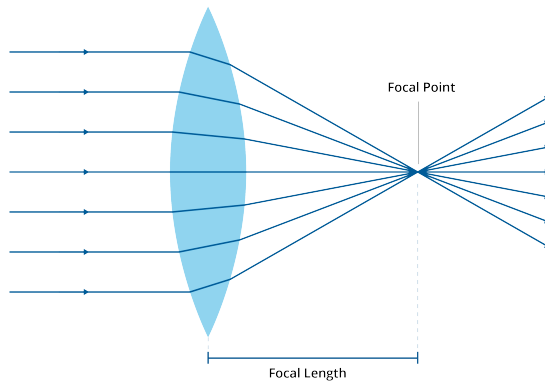
3.2.4 Diffraction

Light can also bend around obstacles. When a wave passes through a gap or around an edge, it spreads out. This is *diffraction*. You hear it all the time: you can hear a friend talking around a corner, because sound waves (about a meter long) spread through the doorway. Light diffracts too, but its wavelength is less than a millionth of a meter, so you only notice it with very small openings. Look at a bright light through the gap between two fingers held almost touching, and you will see faint dark lines: diffraction. The rainbow on the back of a DVD comes from light diffracting off its tightly packed tracks.

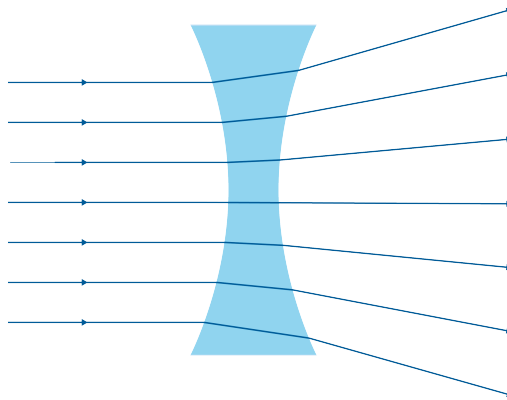
3.3 Lenses

A lens is a curved piece of glass or plastic that refracts light to bend a whole beam at once. There are two main types:

- **Converging (or Convex) Lenses:** These are thicker at the center than at the edges. When parallel light rays enter a convex lens, they converge to a point called the focal point. Examples of converging lenses include magnifying glasses and camera lenses.



- **Diverging (or Concave) Lenses:** These are thinner at the center than at the edges. When parallel light rays enter a concave lens, they diverge or spread out. These lenses are often used in glasses to correct nearsightedness.



3.3.1 Focal Length

The focal length of a lens is the distance between the center of the lens and the focal point. It is determined by the lens shape and the refractive index of the lens material. For a converging lens, the focal length is positive; for a diverging lens, the focal length is negative.

3.3.2 The Lens Equation

For a thin lens,

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

where d_o is the distance from the object to the lens and d_i is the distance from the lens to the image. A converging lens has $f > 0$, and a diverging lens has $f < 0$.

Worked example: a converging lens. A converging lens has $f = 10$ cm. An object is 30 cm from the lens. Then

$$\frac{1}{d_i} = \frac{1}{10} - \frac{1}{30} = \frac{2}{30}, \quad d_i = 15 \text{ cm}$$

The image is real (you could catch it on a screen), upside down, on the other side of the lens, and half the size of the object.

The same equation works for curved mirrors. A concave (caved-in) mirror acts like a converging lens; its focal point is where parallel rays meet after reflecting. A makeup mirror is concave, which is why it magnifies your face when you are close to it.

3.3.3 Refractive Index

Recall that $n = \frac{c}{v}$. The higher the refractive index of the lens material, the more it bends light, so a high-index plastic lens can be thinner than a glass one with the same focal length. That is why strong eyeglasses are often made of special high-index plastic.

Lenses work by refracting light at their two surfaces. By choosing the right lens shape and material, lenses can be designed to bring light to a focus, spread it out, or perform more complex transformations.

Exercise 16 Object Moved Closer

A concave mirror has focal length $f = 10$ cm. An object is placed 20 cm in front of the mirror. Where is its image?

Working Space

Answer on Page 66

Photons

4.1 What Is a Photon?

Why can a sunburn come from invisible ultraviolet light but never from the bright red glow of a heat lamp? Why do night-vision goggles work? The answer is one of the strangest discoveries in physics: light comes in lumps.

In the last chapters, we treated light as a wave, with each color a different wavelength. But light also behaves like a stream of particles, called *photons*. The term photons was coined by physicist Gilbert Newton Lewis, the same Lewis known for the Lewis dot structures we have talked about at the beginning of the sequence!

Every photon comes with an amount of energy that determines what color it is. When we are talking about light interacting with objects, your intuition will be right more often if you think of light as a beam of photons. Photons travel at the speed of light, $c \approx 2.998 \times 10^8$ m/s in a vacuum. Photons

- behave both like particles and like waves;
- have no mass and no charge;
- are created when matter gives off light and destroyed when matter absorbs it.

Formula for energy of a photon

For any given photon, its energy, E , is given by

$$E = hf = \frac{hc}{\lambda}$$

where c is the speed of light, λ is the wavelength of the electromagnetic wave, and h is Planck's constant: 6.63×10^{-34} J·s. (Recall that $f = \frac{c}{\lambda}$.) Light energy comes in packets, or *quanta*, of size hf , just as charge comes in packets of e . A beam can carry 1 photon or 2 photons, but never 1.5.

For example, a red laser light has a wavelength of about 630 nm. So, the energy in each photon is:

$$\frac{(300 \times 10^6)(6.63 \times 10^{-34})}{630 \times 10^{-9}} \approx 3.2 \times 10^{-19} \text{ joules}$$

Joules are awkward for such tiny energies, so physicists use the *electronvolt* (eV): the energy an electron gains crossing 1 volt, 1.602×10^{-19} J. In those units, $hc \approx 1240$ eV·nm, so $E = \frac{1240}{\lambda}$, with λ in nanometers and E in eV.

Worked example. What is the energy of a photon of green light with wavelength 500 nm?

Solution. $E = \frac{1240}{500} = 2.48$ eV.

Blue light has more energy per photon than red light; ultraviolet has more still. That is why UV can damage the molecules in your skin and cause a sunburn, while red light cannot, no matter how bright: each red photon is too weak to break a molecular bond.

The Photoelectric Effect

Around 1900, physicists shined light on metal plates and measured the electrons knocked loose. If light were just a wave, brighter light should eventually shake electrons free, whatever its color, like waves slowly pushing a boat to shore. But that is not what happened. Below a certain frequency, no electrons came out, no matter how bright the light. Above it, electrons came out instantly, even in dim light. This is the *photoelectric effect*.

In 1905, Albert Einstein explained it (and later won the Nobel Prize for it). Light arrives in photons, and each electron is knocked loose by a single photon. If one photon does not have enough energy, a million of them will not help; brighter light only means more photons, not stronger ones.

The effect is governed by the conservation of energy. The energy of an incoming photon (E_{photon}) must be at least the energy holding the electron to the metal. This is described by

$$E_{\text{photon}} = \Phi + K_{\text{max}}$$

where

- $E_{\text{photon}} = hf$: The energy of the incoming photon, where h is Planck's constant and f is the frequency.
- Φ (*Work Function*): The minimum energy required to eject an electron from a specific metal. Each material has a different "cost" to let an electron go.
- f_0 (*Threshold Frequency*): The specific frequency where the photon has exactly enough

energy to match the work function ($\Phi = hf_0$). If $f < f_0$, no electrons are ejected. If $f = f_0$, electrons are released but have zero kinetic energy ($K_{\max} = 0$).

- $K_{\max}$: The “leftover” kinetic energy of the electron once released from the metal.

A closely related effect is how solar panels and digital camera sensors turn light into electricity. In a solar cell, each photon of sunlight can knock an electron loose inside the silicon. A built-in electric field in the cell pushes the loose electrons in one direction, out through the wires and around the circuit, creating a direct current. You will look at camera sensors in depth in the camera chapter.

Exercise 17 Ejected electrons

Light with a wavelength of 400 nm shines on a metal with a work function of 2.3 eV. What is the maximum kinetic energy of the ejected electrons?

Working Space

Answer on Page 66

Photons and Momentum

In classical mechanics, we define momentum as the product of mass and velocity ($p = mv$).

This leads to a common question: if a photon has zero rest mass, how can it possess momentum? Energy and momentum are linked, even for massless particles. For a photon, the momentum (p) is inversely proportional to its wavelength (λ):

$$p = \frac{h}{\lambda} \quad \text{or} \quad p = \frac{E}{c} \quad (4.1)$$

Application: For example, solar sails are spacecraft propulsion systems in space. These large, mirror-like sails in space can intercept millions of photons. As these photons reflect off the surface, they undergo a change in momentum (Δp), which, according to Newton’s Second Law ($F = \frac{\Delta p}{\Delta t}$), exerts a constant force on the spacecraft, allowing for fuel-free propulsion.

Photonic Interactions

When a photon comes from the sun and hits an object, one of several things can happen:

- The energy of the photon is absorbed by the object. It makes the object a little warmer. If a large proportion of photons hitting the mass are absorbed like this, we say the object is “black”.
- The photon bounces off the object. If the surface is very smooth, the photons bounce in a predictable manner, and we call this *reflection* and we say the object is “shiny”.
- If the surface is rough and the photons are not absorbed, the photons are scattered in random directions. We call this *diffuse reflection*. If most of the photons hitting an object are bounced in random directions, we say that the object is “white”.
- The photon passes through the mass. If the mass has smooth surfaces and a consistent composition, the photons will pass through the mass in a predictable manner. We say that the mass is “transparent”.
- If the photons pass through, but in an unpredictable, scattering manner, we say the mass is “translucent”.

No object absorbs every photon, but chemists are always coming up with “blackier” materials. Vantablack, for example, is a super-black coating made of tiny carbon tubes that absorbs 99.965% of visible light. A crumpled sheet of foil coated in it looks like a flat black hole.

Exercise 18 Photons and Color

Working Space

There are red photons.

Are there black photons?

Are there white photons?

Are there yellow photons?

Answer on Page 66

4.2 Exercises

Exercise 19 Energy of a photon

Find the energy, in electron volts, of a photon with wavelength 400 nm. Use $hc = 1240 \text{ eV nm}$.

Working Space

Answer on Page 66

Exercise 20 Momentum of a photon

Find the momentum of a photon with wavelength 630 nm. Use $h = 6.626 \times 10^{-34} \text{ J s}$.

Working Space

Answer on Page 67

Exercise 21 Threshold wavelength

A metal has a work function of 2.3 eV. What is the longest wavelength of light that can eject electrons from it? Use $hc = 1240 \text{ eV nm}$.

Working Space

Answer on Page 67

Exercise 22 Red Light

What is the energy of a photon with

Working Space

wavelength 700 nm?

Answer on Page 67

Special Relativity

Your phone's GPS works by timing radio signals from satellites. The satellites carry atomic clocks accurate to a billionth of a second. Yet engineers have to set those clocks to tick at the wrong rate before launch, because once in orbit they run fast, by 38 millionths of a second every day. Without that correction, your map would drift by about 11 kilometers a day.

Why would a perfect clock run fast in orbit? Because time itself does not pass at the same rate for everyone. Moving clocks run slow. This was Albert Einstein's discovery in 1905, and it sounds like science fiction. But you can derive it yourself in half a page, using nothing but the Pythagorean theorem.

5.1 A Puzzle About Light

In the Electromagnetic Waves chapter, you saw that Maxwell's equations predict the speed of light: $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \approx 3.00 \times 10^8$ m/s. That raised a strange question: 300 million meters per second *relative to what*? If you throw a ball forward at 20 m/s from a train moving at 30 m/s, the ball moves at 50 m/s relative to the ground. You would expect light from a moving flashlight to work the same way. But Maxwell's formula contains no speed of the source or the observer, just two constants of nature.

Experiments agreed with Maxwell. Light from the front and the back of the moving Earth, and from fast-moving stars, always arrived at the same speed. Einstein took the result seriously and built a new physics on two principles, called *postulates*:

1. **The laws of physics are the same for everyone moving at a steady velocity.** No experiment inside a smoothly moving train, with the windows covered, can tell you how fast the train is going.
2. **Everyone measures the same speed of light, c ,** no matter how fast they or the source are moving.

Each sounds reasonable. Together, they force time to stretch.

5.2 The Light Clock

Imagine the simplest possible clock: a pulse of light bouncing between two mirrors a distance L apart. Each round trip is one “tick.” For someone holding the clock, the light travels $2L$ at speed c , so one tick takes

$$\Delta t_0 = \frac{2L}{c}$$

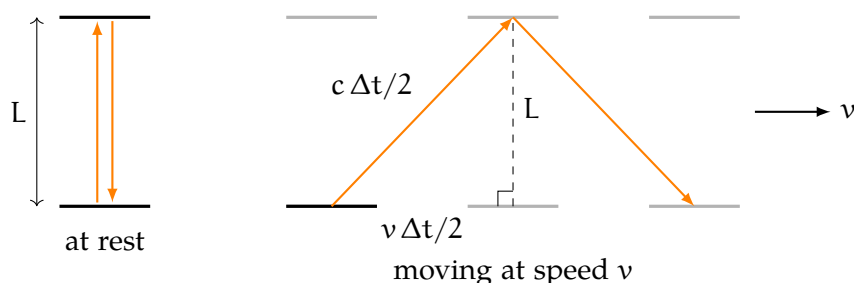


Figure 5.1: A light clock. Left: for someone holding it, the light bounces straight up and down. Right: watched from the ground as the clock moves, the light travels a longer, slanted path.

Now put the clock on a spaceship moving past you at speed v . From the ground, the mirrors move sideways while the light is in flight, so the light travels along a slanted path (figure 5.1). The slanted path is longer. But by the second postulate, the light still moves at exactly c . A longer path at the same speed takes more time.

Let Δt be the time of one tick as measured from the ground. During half a tick, the light travels $c \frac{\Delta t}{2}$ along the slant, while the clock moves $v \frac{\Delta t}{2}$ sideways. Together with the height L , these form a right triangle, and the Pythagorean theorem gives

$$\left(\frac{c\Delta t}{2} \right)^2 = \left(\frac{v\Delta t}{2} \right)^2 + L^2$$

Solve for Δt :

$$\frac{\Delta t^2}{4} (c^2 - v^2) = L^2 \quad \Rightarrow \quad \Delta t = \frac{2L}{\sqrt{c^2 - v^2}} = \frac{2L/c}{\sqrt{1 - v^2/c^2}}$$

Since $\frac{2L}{c} = \Delta t_0$, this is

Time Dilation

$$\Delta t = \gamma \Delta t_0, \quad \text{where } \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

Here Δt_0 is the time between two events measured by a clock that is present at both (the *proper time*), and Δt is the time measured by someone watching that clock move past at speed v .

The *Lorentz factor* γ (the Greek letter gamma) is always at least 1, so moving clocks run slow. Nothing about the argument depends on light clocks in particular. By the first postulate, every clock on the ship, including the astronauts' heartbeats, must keep pace with the light clock. Otherwise the astronauts could tell they were moving. *Time itself* runs slow on the ship.

Here is γ for a range of speeds:

Speed	v/c	γ
A car on the highway (30 m/s)	10^{-7}	1.000000000000005
The International Space Station (7.7 km/s)	2.6×10^{-5}	1.0000000003
Fast spaceship	0.5	1.155
Faster spaceship	0.9	2.294
Cosmic-ray muon	0.995	10.0
Particle in an accelerator	0.999	22.4

At everyday speeds, γ is so close to 1 that nobody noticed relativity for all of human history. Near the speed of light, it is dramatic.

5.3 The Evidence: Muons

Time dilation is not a thought experiment; it is measured every day. Cosmic rays (from the Electromagnetic Waves chapter) slam into the upper atmosphere about 15 km up and create particles called *muons*. Muons are unstable, with a half-life of 1.52 microseconds: of any group, half decay in that time, half of the rest in the next 1.52 μs , and so on. They move at about $0.995c$.

Without relativity, the trip down takes $\frac{15,000 \text{ m}}{0.995c} \approx 50.3 \mu\text{s}$, about 33 half-lives. The fraction surviving would be $(\frac{1}{2})^{33} \approx 10^{-10}$: essentially none would reach the ground.

But the muons' own clocks run slow by $\gamma \approx 10.0$. For them, the trip takes only $\frac{50.3}{10.0} \approx 5.0 \mu\text{s}$, about 3.3 half-lives, and $(\frac{1}{2})^{3.3} \approx 10\%$ survive. Detectors at sea level count about as many muons as relativity predicts, roughly a billion times more than the old physics allows. Thousands pass through your body every minute.

```

1 from math import sqrt
2
3 C = 299_792_458           # speed of light, m/s

```

```

4
5 def gamma(v):
6     """The time-dilation factor for speed v in m/s."""
7     return 1 / sqrt(1 - (v / C) ** 2)
8
9 for fraction in [0.1, 0.5, 0.9, 0.99, 0.999]:
10     print(f"{fraction:>5} c: gamma = {gamma(fraction * C):.3f}")
11
12 # Muons made 15 km up, moving at 0.995 c, with a half-life of 1.52 microseconds
13 v = 0.995 * C
14 earth_time = 15_000 / v                # seconds, on Earth's clocks
15 muon_time = earth_time / gamma(v)      # seconds, on the muons' own clocks
16 half_life = 1.52e-6
17 print(f"surviving with relativity: {0.5 ** (muon_time / half_life):.3f}")
18 print(f"surviving without relativity: {0.5 ** (earth_time / half_life):.1e}")

```

```

0.1 c: gamma = 1.005
0.5 c: gamma = 1.155
0.9 c: gamma = 2.294
0.99 c: gamma = 7.089
0.999 c: gamma = 22.366
surviving with relativity: 0.101
surviving without relativity: 1.1e-10

```

5.4 Length Contraction

Here is a puzzle. From the muon's point of view, it is sitting still, and the Earth is rushing up at 0.995c. Its clock is not slow to itself. So how does it survive? The answer: for the muon, the atmosphere is shorter. Distances along the direction of motion shrink by the same factor:

Length Contraction

$$L = \frac{L_0}{\gamma}$$

where L_0 is the length measured by someone at rest relative to the object (the *proper length*).

To the muon, the 15 km of atmosphere is only $\frac{15}{10.0} \approx 1.5$ km thick, a trip of 5 μ s. The ground and the muon disagree about time and about distance, but they agree on the measurable outcome: about 10% of muons arrive.

5.5 The Twin Paradox

One twin stays on Earth. The other flies to Proxima Centauri, the nearest star, 4.24 light-years away, at $0.8c$, and immediately comes back. On Earth, the round trip takes $\frac{2 \times 4.24}{0.8} = 10.6$ years. With $\gamma = \frac{1}{\sqrt{1-0.64}} = \frac{1}{0.6} \approx 1.667$, the traveler's clocks, and body, record only $10.6 \times 0.6 \approx 6.4$ years. The twin who left returns 4 years younger than the one who stayed.

"But wait," you might say. "To the traveler, Earth was moving. Shouldn't the Earth twin be younger?" The two twins are not in the same situation. The Earth twin stays in one steady frame the whole time. The traveler fires rockets to turn around and changes from an outbound frame to an inbound one. That switch breaks the symmetry, and a careful calculation shows that the traveler really does age less. Experiments have confirmed this with atomic clocks flown around the world on airplanes: on landing, they disagreed with identical clocks left on the ground by billionths of a second, just as predicted.

5.6 GPS: Relativity in Your Pocket

A GPS receiver finds its position by measuring how long signals take to arrive from several satellites. Light travels about 30 cm in a billionth of a second, so the clocks must agree to within a few billionths of a second.

The satellites orbit at about 3.9 km/s, so special relativity slows their clocks by about 7 microseconds per day. But there is a second, larger effect from Einstein's *general* theory of relativity: clocks run faster where gravity is weaker. Twenty thousand kilometers up, that speeds the satellite clocks by about 46 microseconds per day. The net effect is about $46 - 7 = 38$ microseconds per day fast. In 38 microseconds, light travels $38 \times 10^{-6} \times 3 \times 10^8 \approx 11,000$ meters. So without relativity, GPS positions would drift by about 11 km every day. The satellite clocks are deliberately built to tick slightly slow on the ground, so that in orbit they keep perfect time.

5.7 The Cosmic Speed Limit and $E = mc^2$

As v approaches c , γ grows without bound: $1 - \frac{v^2}{c^2}$ approaches 0, and you are dividing by a number approaching 0. Einstein showed that the energy of a moving object of mass m is γmc^2 . Pushing something to the speed of light would take infinite energy, so nothing with mass can reach it. Particle accelerators push protons to $0.99999999c$, and each extra 9 costs enormously more energy.

When the object is at rest, $\gamma = 1$, and the energy is $E = mc^2$: the energy locked up in mass itself, which you met in the Work and Energy chapter. That formula, which powers the Sun and nuclear reactors, is a consequence of the same two postulates.

Exercise 23 Computing γ

Find γ for $v = 0.6c$ and for $v = 0.8c$.
(These two are worth remembering.)

Working Space

Answer on Page 67

Exercise 24 Movie Night in Space

Astronauts on a ship moving at $0.6c$ watch a 2-hour movie. How long does the movie last according to clocks on Earth?

Working Space

Answer on Page 67

Exercise 25 A Shrinking Spaceship

A spaceship is 100 m long when parked. How long is it, as measured from Earth, when it flies past at $0.8c$?

Working Space

Answer on Page 67

Exercise 26 A Longer Trip

A traveler flies to a star 10 light-years away at $0.95c$ and comes straight back. How much time passes on Earth? How

Working Space

much does the traveler age?

Answer on Page 67

Exercise 27 **Who Is Slow?**

Earth sees the astronauts' clocks running slow. The astronauts see Earth moving past at the same speed. Whose clocks do the astronauts see running slow? How can both be right?

Working Space

Answer on Page 68

Exercise 28 **Highway Relativity**

For a car at 30 m/s, $\gamma - 1 \approx 5 \times 10^{-15}$. About how many years would you have to drive at that speed for your watch to fall 1 second behind a watch at home?

Working Space

Answer on Page 68

Nuclear Physics and Radioactivity

The smoke detector on your ceiling contains a speck of radioactive americium. The banana in your lunch gives off about 15 tiny bursts of radiation every second. The Sun has been shining for 4.6 billion years by squeezing hydrogen into helium, and a uranium pellet the size of a pencil eraser can release as much energy as a ton of coal. All of these happen in the *nucleus*, the tiny core of the atom, which is 100,000 times smaller than the atom itself but holds almost all of its mass.

In the Atomic Mass and Chemical Reactions chapter, you met isotopes and the idea of a half-life. In the Work and Energy chapter, you used $E = mc^2$ to find the energy released when hydrogen fuses into helium. This chapter puts those pieces together: what holds a nucleus together, why some nuclei fall apart, how to calculate how much of a radioactive sample is left, and why both splitting heavy nuclei and joining light ones release enormous energy.

6.1 Inside the Nucleus

A nucleus is made of protons and neutrons, together called *nucleons*. We write a nucleus as

$${}^A_ZX$$

where X is the element's symbol, Z is the *atomic number* (the number of protons), and A is the *mass number* (protons plus neutrons). So ${}^{235}_{92}\text{U}$, uranium-235, has 92 protons and $235 - 92 = 143$ neutrons.

That should surprise you. Protons are positively charged, and the Charge chapter's Coulomb's law says like charges repel. Two protons 1 femtometer (10^{-15} m) apart, about the width of a nucleus, push each other apart with a force of

$$F = k \frac{q^2}{r^2} = (8.99 \times 10^9) \frac{(1.60 \times 10^{-19})^2}{(10^{-15})^2} \approx 230 \text{ N}$$

That is about the weight of a large dog, acting on a particle with a mass of only 1.67×10^{-27} kg. Something far stronger must be holding the nucleus together.

That something is the *strong nuclear force*. It attracts nucleons to each other, protons and neutrons alike, and at short range it is about 100 times stronger than the electric repulsion.

But it only reaches about the width of a couple of nucleons; beyond that it fades to nothing. Neutrons are the nucleus's glue: they add strong-force attraction without adding electric repulsion. That is why heavier nuclei need more and more neutrons to hold together, and why, past lead, no amount of neutrons is enough to make a nucleus permanently stable.

6.2 Radioactive Decay

An unstable nucleus eventually changes into a more stable one by emitting radiation. The three most common kinds are:

Decay	What comes out	What happens to the nucleus	Stopped by
Alpha (α)	a helium-4 nucleus, ${}^4_2\text{He}$	A drops by 4, Z drops by 2	a sheet of paper, or skin
Beta (β^-)	an electron, ${}^0_{-1}e$	a neutron becomes a proton: Z rises by 1	a few mm of aluminum
Gamma (γ)	a high-energy photon	loses energy; A and Z unchanged	thick lead or concrete

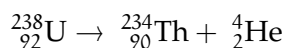
Gamma rays are the highest-energy light in the electromagnetic spectrum from the Electromagnetic Waves chapter, and each one is a single photon, as in the Photons chapter.

6.2.1 Nuclear Equations

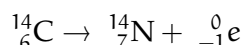
In every nuclear reaction, the total mass number A and the total charge Z are the same before and after. That lets you fill in a missing particle by bookkeeping.

Example: Uranium-238 undergoes alpha decay. What does it become?

Solution: ${}^{238}_{92}\text{U} \rightarrow {}^A_Z\text{X} + {}^4_2\text{He}$. Balancing the top numbers, $238 = A + 4$, so $A = 234$. Balancing the bottom, $92 = Z + 2$, so $Z = 90$. Element 90 is thorium:



Example: Carbon-14 undergoes beta decay:



A neutron turned into a proton, so the carbon became nitrogen, and the mass number stayed 14. (A nearly massless particle called an antineutrino also escapes, but it does not change the bookkeeping.)

Your smoke detector uses the alpha decay of americium-241. The alpha particles knock electrons off air molecules, letting a tiny current flow between two plates. Smoke particles soak up the alpha particles, the current drops, and the alarm sounds. The alpha particles cannot even get through the plastic case.

6.3 Half-Life

You cannot predict when one particular nucleus will decay; it is truly random. But for a large number of them, the randomness averages out (just as in the Monte Carlo Simulation chapter), and every radioactive isotope has a *half-life* T : the time for half of any sample to decay.

Isotope	Half-life
Polonium-214	164 microseconds
Iodine-131 (used to treat thyroid disease)	8.02 days
Cobalt-60 (used in cancer radiation therapy)	5.27 years
Americium-241 (smoke detectors)	432 years
Carbon-14	5,730 years
Plutonium-239	24,100 years
Potassium-40 (in bananas, and in you)	1.25 billion years
Uranium-238	4.47 billion years

After one half-life, half the sample remains; after two, a quarter; after three, an eighth. After t years, $\frac{t}{T}$ half-lives have passed, so

Radioactive Decay

$$N(t) = N_0 \left(\frac{1}{2}\right)^{t/T}$$

where N_0 is the starting amount and T is the half-life.

This is the exponential decay from the Exponentials and Logarithms chapter. Using $\left(\frac{1}{2}\right)^{t/T} = e^{-(\ln 2)t/T}$, physicists often write it as $N = N_0 e^{-\lambda t}$, with *decay constant* $\lambda = \frac{\ln 2}{T}$.

Here is a simulation of 1,000 atoms. In each tenth of a half-life, every surviving atom independently “flips a weighted coin” to decide whether it decays:

```

1 import random
2
3 random.seed(8)
4
5 atoms = 1000
```

```

6 STEPS_PER_HALF_LIFE = 10
7 # chance that one atom decays during one step (a tenth of a half-life)
8 p = 1 - 0.5 ** (1 / STEPS_PER_HALF_LIFE)
9
10 for half_life in range(1, 6):
11     for _ in range(STEPS_PER_HALF_LIFE):
12         # each surviving atom independently decays with probability p
13         atoms = sum(1 for _ in range(atoms) if random.random() > p)
14     print(f"after {half_life} half-lives: {atoms:4} atoms left "
15           f"(formula: {1000 * 0.5 ** half_life:.1f})")

```

```

after 1 half-lives:  494 atoms left (formula: 500.0)
after 2 half-lives:  249 atoms left (formula: 250.0)
after 3 half-lives:  133 atoms left (formula: 125.0)
after 4 half-lives:   72 atoms left (formula:  62.5)
after 5 half-lives:   34 atoms left (formula:  31.2)

```

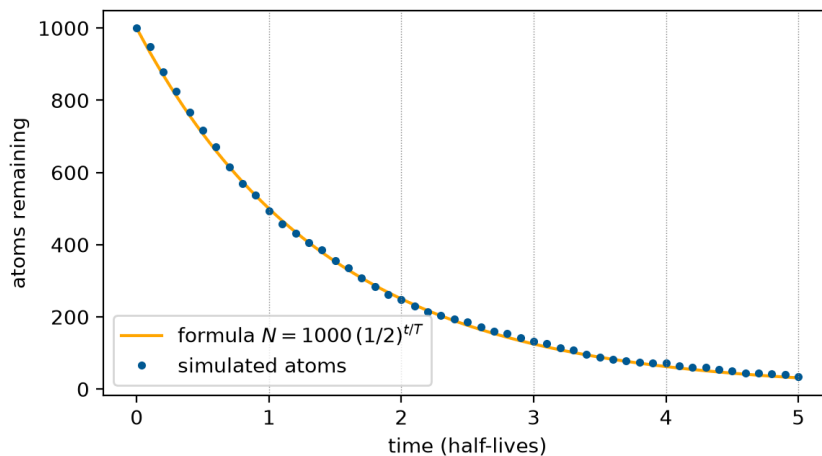


Figure 6.1: Each atom decays at random, yet together they follow the exponential curve.

6.3.1 Radiocarbon Dating

Cosmic rays constantly make carbon-14 in the atmosphere, and living things take it in, so every living thing has the same small fraction of carbon-14. When an organism dies, it stops taking in carbon, and its carbon-14 decays. Measure how much is left, and you know how long ago it died.

Example: In 1991, hikers in the Alps found a frozen man, now called Ötzi. His tissue had about 53% of the carbon-14 of a living person. How long ago did he die?

Solution: Set $(\frac{1}{2})^{t/5730} = 0.53$ and take logarithms of both sides:

$$\frac{t}{5730} \ln \frac{1}{2} = \ln 0.53 \implies t = 5730 \cdot \frac{\ln 0.53}{\ln 0.5} \approx 5,250 \text{ years}$$

Ötzi lived more than 5,000 years ago, before the Egyptian pyramids were built.

6.4 Mass Defect and Binding Energy

Here is a strange fact. A helium-4 nucleus weighs *less* than the two protons and two neutrons it is made of. Nuclear masses are measured in *atomic mass units* (u), where $1 \text{ u} = 1.6605 \times 10^{-27} \text{ kg}$. Using hydrogen-1 atoms for the protons (so the electrons balance out):

$$2(1.007825 \text{ u}) + 2(1.008665 \text{ u}) - 4.002602 \text{ u} = 0.030378 \text{ u}$$

This missing mass, the *mass defect*, is the energy that was released when the nucleus formed, converted by $E = mc^2$. It is also the energy you would have to supply to pull the nucleus apart, so it is called the *binding energy*. Nuclear physicists measure it in *mega-electron-volts* (MeV), where $1 \text{ MeV} = 1.602 \times 10^{-13} \text{ J}$, and $E = mc^2$ works out to $1 \text{ u} \rightarrow 931.5 \text{ MeV}$. So helium-4's binding energy is

$$0.030378 \times 931.5 \approx 28.3 \text{ MeV}, \quad \text{or about } 7.07 \text{ MeV per nucleon}$$

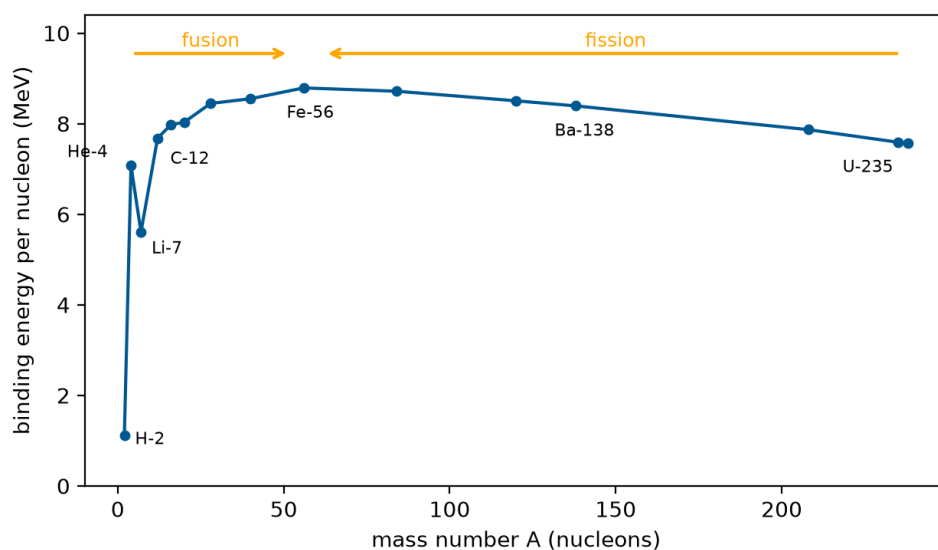
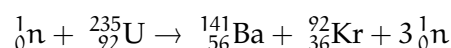


Figure 6.2: Binding energy per nucleon. Iron-56 is near the top: its nucleons are the most tightly bound. Joining light nuclei (fusion) or splitting heavy ones (fission) moves toward the peak and releases energy.

Figure 6.2 is one of the most important graphs in physics. The higher a nucleus sits on it, the more tightly each nucleon is held, and the more energy was released to make it. The curve peaks near iron. So there are two ways to release nuclear energy: *join* light nuclei together (fusion), or *split* heavy nuclei apart (fission). Either way, the products sit higher on the curve than what you started with, and the difference comes out as energy.

6.5 Fission

When uranium-235 absorbs a slow neutron, it becomes so unstable that it splits in two. One common outcome is



(Check the bookkeeping: $1 + 235 = 141 + 92 + 3$ and $0 + 92 = 56 + 36 + 0$.) Comparing masses,

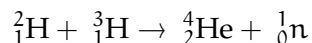
$$(235.043930 + 1.008665) - (140.914411 + 91.926156 + 3 \times 1.008665) = 0.186033 \text{ u}$$

so each fission releases about $0.186 \times 931.5 \approx 173$ MeV right away, and more as the fragments decay. Per kilogram, fissioning uranium-235 releases about 7×10^{13} J, roughly 3 million times more than burning a kilogram of coal.

The key detail is the 3 neutrons. Each can split another uranium nucleus, which releases more neutrons, and so on: a *chain reaction*. In a bomb, the chain reaction runs away in a millionth of a second. In a nuclear power plant, *control rods* made of neutron-absorbing material soak up just enough neutrons that, on average, each fission causes exactly one more. The reaction then runs steadily, heating water to make steam that turns generators, without burning anything or releasing carbon dioxide. The challenge is the waste: the fission fragments are intensely radioactive, and some stay dangerous for thousands of years.

6.6 Fusion

At the other end of the curve, light nuclei release energy by joining. The reaction that engineers are trying to tame is deuterium (hydrogen-2) plus tritium (hydrogen-3):



The mass defect is $(2.014102 + 3.016049) - (4.002602 + 1.008665) = 0.018884 \text{ u}$, which is about 17.6 MeV. Per kilogram of fuel, that is about 5 times more than fission, and the main product is harmless helium.

So why do we not have fusion power plants? Both nuclei are positively charged, and they must get within the strong force's tiny range before it can grab them. Pushing them that

close against the electric repulsion takes temperatures above 100 million degrees, hotter than the center of the Sun. The Sun manages at a mere 15 million degrees because its enormous gravity squeezes its core to great density, and because it has billions of years to wait for the rare collisions that succeed. On Earth, we have to do it in a machine. In 2022, the National Ignition Facility in California crushed a tiny fuel pellet with lasers and, for the first time, got more fusion energy out than the laser energy that went in. Turning that into a working power plant is one of the great engineering challenges of your lifetime.

6.7 Radiation and You

Radiation is not something that only exists in power plants. You receive a natural *background* dose every day: radon gas seeping from rocks, cosmic rays, and the radioactive potassium and carbon inside your own body. Doses are measured in *sieverts* (Sv); natural background gives a typical person about 3 millisieverts per year. A chest X-ray adds about 0.1 mSv. Large doses, many sieverts at once, damage cells and can be deadly, which is why radiation workers wear badges to track their exposure. The same cell-killing power, carefully aimed, is used to destroy tumors.

Exercise 29 Counting Nucleons

How many protons and neutrons are in each nucleus?

1. ${}^{56}_{26}\text{Fe}$
2. ${}^{235}_{92}\text{U}$
3. ${}^{14}_6\text{C}$

Working Space

Answer on Page 68

Exercise 30 Complete the Decay

Write the product of each decay.

1. Radium-226 (${}^{226}_{88}\text{Ra}$) undergoes alpha decay.
2. Cobalt-60 (${}^{60}_{27}\text{Co}$) undergoes beta de-

Working Space

cay.

Answer on Page 68

Exercise 31 A Medical Isotope

A hospital receives 100 mg of iodine-131, which has a half-life of 8.02 days.

Working Space

1. How much is left after about 24 days (three half-lives)?
2. How much is left after 30 days?

Answer on Page 68

Exercise 32 How Old Is It?

Working Space

1. A bone has 25% of the carbon-14 of living bone. How old is it?
2. A piece of charcoal has 10% of the original carbon-14. How old is it?

Answer on Page 68

Exercise 33 Deuterium's Binding Energy

A deuterium nucleus is one proton and one neutron. The atomic masses are hydrogen-1, 1.007825 u; neutron,

Working Space

1.008665 u; and deuterium, 2.014102 u. Find deuterium's binding energy, and its binding energy per nucleon. Compare with figure 6.2.

Answer on Page 69

Exercise 34 The End of a Star

Large stars fuse lighter elements into heavier ones, all the way up to iron. Use figure 6.2 to explain why fusing iron into anything heavier cannot keep a star shining.

Working Space

Answer on Page 69

Answers to Exercises

Answer to Exercise 1 (on page 4)

$$\frac{300 \times 10^6}{5.66 \times 10^{14}} = 530 \times 10^{-9} = 530 \text{ nm}$$

Answer to Exercise 2 (on page 8)

$$\lambda = \frac{3 \times 10^8}{98.7 \times 10^6} \approx 3.04 \text{ m. Half a wavelength is about 1.5 m.}$$

Answer to Exercise 3 (on page 8)

$$f = \frac{c}{\lambda} = \frac{3.0 \times 10^8}{0.5} = 6.0 \times 10^8 \text{ Hz, or 600 MHz.}$$

Answer to Exercise 4 (on page 11)

In step, $3 + 2 = 5$ cm (constructive). Exactly out of step, $3 - 2 = 1$ cm (destructive). Unequal waves cannot cancel completely.

Answer to Exercise 5 (on page 11)

The string is at 440 ± 3 : 437 Hz or 443 Hz. Tightening raises the pitch. If the beats speed up, the string moved *farther* from 440, so it was already above: 443 Hz. The guitarist should loosen it.

Answer to Exercise 6 (on page 14)

$$\Delta y = \frac{(532 \times 10^{-9})(1.5)}{0.20 \times 10^{-3}} \approx 4.0 \times 10^{-3} \text{ m, about 4.0 mm.}$$

Answer to Exercise 7 (on page 14)

$$\lambda = \frac{d \Delta y}{L} = \frac{(0.30 \times 10^{-3})(4.0 \times 10^{-3})}{2.0} = 6.0 \times 10^{-7} \text{ m} = 600 \text{ nm, which is orange light.}$$

Answer to Exercise 8 (on page 15)

$d = \frac{1}{600} \text{ mm} \approx 1.667 \times 10^{-6} \text{ m}$, so $\sin \theta = \frac{650 \times 10^{-9}}{1.667 \times 10^{-6}} \approx 0.39$ and $\theta \approx 23^\circ$. Since $\sin \theta$ cannot exceed 1, we need $m \leq \frac{d}{\lambda} \approx 2.56$, so the highest order is $m = 2$.

Answer to Exercise 9 (on page 16)

Sound waves from a voice have wavelengths around a meter, comparable to the width of the doorway, so they diffract, spreading out around the corner into the room. Light has wavelengths about a million times smaller than the doorway, so it barely diffracts and travels in straight lines.

Answer to Exercise 10 (on page 18)

Approaching: $400 \cdot \frac{343}{318} \approx 431 \text{ Hz}$. Receding: $400 \cdot \frac{343}{368} \approx 373 \text{ Hz}$.

Answer to Exercise 11 (on page 18)

$v \approx (3.00 \times 10^5) \frac{669.4 - 656.3}{656.3} \approx 6,000 \text{ km/s}$, about twice as fast as the galaxy in the example. By Hubble's discovery, it is also about twice as far away.

Answer to Exercise 12 (on page 21)

Assuming the mirror is truly vertical and the floor is truly horizontal, the new cut off should be exactly the same as the old one: It should be below your chin the same amount that your eyes are above your chin.

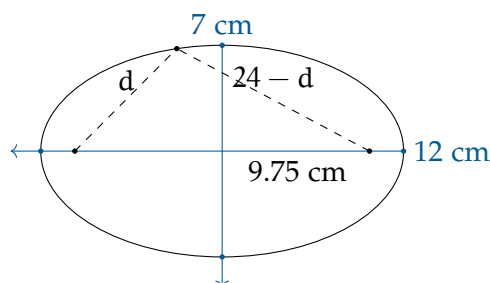
Here is why. Follow the ray from your eyes to the bottom edge of the mirror. It reflects at an equal angle and heads down toward your body. The ray drops from your eyes to the mirror's edge (a height h , from eye level to chin) over a horizontal distance d , and then, by the law of reflection, drops another h over the same distance d on the way back. So

it always lands h below your chin, no matter what d is. Walking closer to a mirror never lets you see more of yourself!

Answer to Exercise 13 (on page 24)

The ellipse extends $a = 12$ cm from the center along the long axis and $b = 7$ cm along the short axis. The string is the sum of the distances from any point to the two foci, which is $2a = 24$ cm. (Check it at the end of the long axis.)

The distance from the center to each focal point is $\sqrt{12^2 - 7^2} = \sqrt{95} \approx 9.75$ cm, so the pushpins go 9.75 cm from the short axis.



Answer to Exercise 14 (on page 28)

We need the distance from the center out to each of the three axes. We know that $a = \left(\frac{1}{2}\right) 30 = 15$ cm.

We can calculate the b and c (which are equal) using the circumference given: $2b\pi = 60$, so $c = b \approx 9.55$ cm.

The volume, then is

$$V = \frac{4}{3}\pi(15)(9.55)(9.55) \approx 5,730 \text{ cubic centimeters}$$

The mass would be $5,730 \times 11.34 = 64,973$ grams or about 65 kg.

Answer to Exercise 15 (on page 36)

$\sin \theta_2 = \frac{\sin 30^\circ}{1.5} = \frac{0.5}{1.5} \approx 0.333$, so $\theta_2 \approx 19.5^\circ$. The critical angle is $\arcsin \frac{1}{1.5} \approx 41.8^\circ$.

Answer to Exercise 16 (on page 38)

$$\frac{1}{d_i} = \frac{1}{10} - \frac{1}{20} = \frac{1}{20}, \text{ so } d_i = 20 \text{ cm.}$$

Answer to Exercise 17 (on page 41)

Each photon carries $\frac{1240}{400} = 3.10$ eV. After paying the 2.3 eV work function, the electron keeps at most $3.10 - 2.3 = 0.80$ eV of kinetic energy.

Answer to Exercise 18 (on page 42)

Are there white photons? No. What we call “white” is a blend of photons that are several different colors.

Some people like to say white light is the combination of all visible colors of photons in equal amounts. That seems oddly specific and unusual.

Maybe it is better to imagine it from the human experience of white light. In our eyes, we have three different types of color-sensing cones, which generally correspond to the red, green, and blue regions of the spectrum. When all three are excited about equal amounts, humans experience that as white. On your computer screen, for example, what you see as white is just a blend of three colors of photons: a red, a green, and a blue.

Are there black photons? No. What we call “black” is an absence of photons in the visible range.

Are there yellow photons? Yes! There is a region of the color spectrum that is yellow. It has a wavelength of about 580 nm. Photons at this energy level excite both our green-sensitive and red-sensitive cones (we will expand on this in the eye chapter!). Your computer monitor does not actually create light with a 580 nm wavelength. Instead, it creates red light and green light, which our eyes interpret as yellow.

Answer to Exercise 19 (on page 43)

$$E = \frac{hc}{\lambda} = \frac{1240 \text{ eV nm}}{400 \text{ nm}} = 3.10 \text{ eV.}$$

Answer to Exercise 20 (on page 43)

$$p = \frac{h}{\lambda} = \frac{6.626 \times 10^{-34}}{630 \times 10^{-9}} \approx 1.05 \times 10^{-27} \text{ kg m/s.}$$

Answer to Exercise 21 (on page 43)

The threshold wavelength is $\lambda_0 = \frac{hc}{\phi} = \frac{1240}{2.3} \approx 539 \text{ nm}$. Light with a longer wavelength has too little energy per photon.

Answer to Exercise 22 (on page 43)

$$E = \frac{1240}{700} \approx 1.77 \text{ eV.}$$

Answer to Exercise 23 (on page 50)

$$\text{For } 0.6c: \gamma = \frac{1}{\sqrt{1-0.36}} = \frac{1}{0.8} = 1.25. \text{ For } 0.8c: \gamma = \frac{1}{\sqrt{1-0.64}} = \frac{1}{0.6} \approx 1.667.$$

Answer to Exercise 24 (on page 50)

The movie's proper time is 2 hours (measured on the ship, where it plays). Earth measures $\Delta t = \gamma \Delta t_0 = 1.25 \times 2 = 2.5 \text{ hours}$.

Answer to Exercise 25 (on page 50)

$$L = \frac{L_0}{\gamma} = \frac{100}{1.667} = 60 \text{ m.}$$

Answer to Exercise 26 (on page 50)

On Earth, the trip takes $\frac{20}{0.95} \approx 21.1 \text{ years}$. With $\gamma = \frac{1}{\sqrt{1-0.9025}} \approx 3.20$, the traveler ages $\frac{21.1}{3.20} \approx 6.6 \text{ years}$.

Answer to Exercise 27 (on page 51)

The astronauts see *Earth's* clocks running slow, by the same factor. Both are right because each observer measures the other's clock as it moves past; neither is "really" at rest (that is the first postulate). The disagreement only becomes a definite age difference if one of them turns around and comes back to compare clocks side by side, as in the twin paradox.

Answer to Exercise 28 (on page 51)

The moving watch loses about 5×10^{-15} seconds per second. To lose 1 second takes $\frac{1}{5 \times 10^{-15}} = 2 \times 10^{14}$ seconds, about 6 million years.

Answer to Exercise 29 (on page 59)

1. 26 protons, $56 - 26 = 30$ neutrons.
2. 92 protons, 143 neutrons.
3. 6 protons, 8 neutrons.

Answer to Exercise 30 (on page 59)

1. ${}_{88}^{226}\text{Ra} \rightarrow {}_{86}^{222}\text{Rn} + {}_2^4\text{He}$. Element 86 is radon, the radioactive gas that seeps into basements.
2. ${}_{27}^{60}\text{Co} \rightarrow {}_{28}^{60}\text{Ni} + {}_{-1}^0\text{e}$. Element 28 is nickel.

Answer to Exercise 31 (on page 60)

1. $100 \times \left(\frac{1}{2}\right)^3 = 12.5$ mg.
2. $100 \times \left(\frac{1}{2}\right)^{30/8.02} \approx 7.5$ mg.

Answer to Exercise 32 (on page 60)

1. 25% is two half-lives: $2 \times 5730 = 11,460$ years.
2. $t = 5730 \cdot \frac{\ln 0.1}{\ln 0.5} \approx 19,000$ years.

Answer to Exercise 33 (on page 60)

The mass defect is $1.007825 + 1.008665 - 2.014102 = 0.002388$ u, so the binding energy is $0.002388 \times 931.5 \approx 2.22$ MeV, or about 1.11 MeV per nucleon. That is the lowest point on the curve: deuterium is barely held together, which is one reason it makes a good fusion fuel.

Answer to Exercise 34 (on page 61)

Iron is at the peak of the curve. Fusing iron into heavier nuclei moves *down* the curve, so it absorbs energy instead of releasing it. When a massive star's core fills with iron, it loses its energy source and collapses, often triggering a supernova explosion.



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