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CHAPTER 1

Charge

Rub a balloon on your hair and hold it against a wall. It sticks! Your hair stands on end, too. Why? This chapter on **charge** and **electrostatics** explains it.

The balloon has stripped some electrons from your hair, so now the balloon has slightly more electrons than protons. We say it has a *net electrical charge*: an imbalance between protons and electrons. While any **friction** or contact between two objects can cause a difference in electric charge, *net charge cannot be created or destroyed*. In this case, the balloon has a negative electrical charge. Objects with slightly more protons than electrons have a positive charge. Consequently, your hair now has a positive charge. Note that the differences are equal but opposite.

An object with a negative charge and an object with a positive charge will be *attracted* to each other. Two objects with the same charge will be *repelled* by each other. All charge comes from subatomic particles: protons are positive and electrons are negative. An object is charged when its numbers of protons and electrons are unequal.

The charge of one proton is called the *elementary charge*, written e (not to be confused with the number $e \approx 2.718$). An electron's charge is $-e$. Just as you cannot pay with a fraction of a penny, an object's net charge is always a whole number of elementary charges: $Q = ne$, where n is an integer. We say charge is *quantized*.

Robert Millikan measured e in 1909 by watching tiny charged oil drops float between charged plates. We write the charge of a particle as q . In SI units, charge is measured in *coulombs*, and $e \approx 1.6 \times 10^{-19}$ coulombs. So one coulomb is about 6×10^{18} protons' worth of charge.

Two objects with charges are bound to either attract (pull) or repel (push) on each other. This is a type of force referred to as *electric force*.

Coulomb's Law

If two objects with charge q_1 and q_2 (in coulombs) are r meters from each other, the force of attraction or repulsion (electric force) is given by

$$F = k \frac{q_1 q_2}{r^2} \quad (1.1)$$

where F is in newtons and k is Coulomb's constant: about 8.988×10^9 . You may use

$9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}$ in your calculations.

The constant is also written $k = \frac{1}{4\pi\epsilon_0}$, where ϵ_0 is the *permittivity of free space*.¹ The electric force is often written F_e .

Note how similar this looks to Newton's Law of Universal Gravitation:

$$F_g = G \frac{m_1 m_2}{r^2}$$

Both laws follow an inverse-square relationship, meaning the force decreases with the square of the distance. However, gravity is always attractive, while electric force can be either attractive or repulsive depending on the signs of the charges (q_1 and q_2).

If you keep the signs of the charges, a negative F means attraction (the charges have opposite signs) and a positive F means repulsion (same signs). If either charge is zero, there is no electric force.

Worked example. Two charges of $1 \text{ } \mu\text{C}$ (a microcoulomb, 10^{-6} C) are 0.1 m apart. What is the force between them?

Solution. $F = \frac{kq_1q_2}{r^2} = \frac{(8.99 \times 10^9)(10^{-6})(10^{-6})}{(0.1)^2} \approx 0.90 \text{ N}$. That is about the weight of a large apple, from two specks of charge. The electric force is enormously stronger than gravity.

Exercise 1 Coulomb's Law

Working Space

Two balloons are charged with an identical quantity and type of charge: $-5 \times 10^{-9} \text{ coulombs}$. They are held apart at a separation distance of 12 cm . Determine the magnitude of the electrical force of repulsion between them.

Answer on Page 51

¹A note on the permittivity of free space: ϵ_0 (or the electric constant) represents the capability of a vacuum to permit electric field lines. Its exact value in SI units is $\epsilon_0 \approx 8.854 \times 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2)$. Remarkably, ϵ_0 together with its magnetic partner sets the speed of light, as you will see in the electromagnetic waves chapter.

At this point, you might ask “If the wall has zero charge, why is the balloon attracted to it?” The answer: the electrons in the wall move away from the balloon, polarizing the atoms. The negative charge on the balloon pushes electrons away from itself, so the surface of the wall gets a mild positive charge. The negative charge on the balloon is attracted more to the positive charge on the surface of the wall than the negative charge on the inside, thus the balloon sticks. Even though the wall has no net charge, its charges shift, and that is enough. The details depend on whether a material is a *conductor* or an *insulator*.



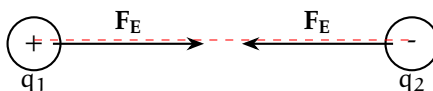
Figure 1.1: The negative balloon repels the wall’s electrons, leaving a net positive surface. The balloon is then attracted to this induced positive charge.

Recall that a **conductor** is a type of material through which valence electrons (electrons in the outer shell) easily flows, and an **insulator** is a type of material through which charge does not easily flow, if at all. Metals, such as copper or zinc, are the best conductors because the electrons within the atoms are not tied to a nucleus, but rather form a sparse collection or sea in which the charge can flow through. Insulators like plastic, rubber, and glass block the flow of charge because their electrons are tightly bound to their atoms. (In an insulating wall, the electrons cannot leave their atoms, but each atom’s electron cloud shifts slightly away from the balloon. The effect is the same.)

1.0.1 Point Charge

The wall and the balloon all contain a bunch of tiny particles which we can refer to as **point charges**. A point charge is an electric charge that is treated as a concentrated single point in space, with no physical size or volume. Similar to point masses, we use point charges to simulate effects of charge on other objects or points.

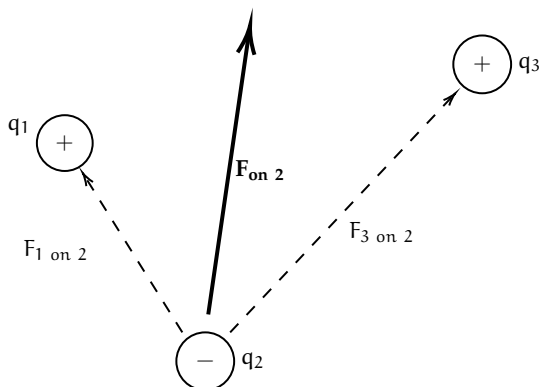
The force between two point charges always points along the straight line joining them.



1.0.2 Superposition

What if several charges act on one point charge? Coulomb's law handles one pair at a time, so we use the **principle of superposition**.

The principle of superposition states that the total electric force on a charge is the vector sum of the individual forces exerted on it by all other charges.²



Exercise 2 Net force at the center of a square

Working Space

This question is taken from the AP Physics C 19th edition practice workbook.

Consider four equal, positive point charges that are situated at the vertices of a square. Find the net electric force on a negative point charge placed at the square's center.

Hint: draw a diagram to help you

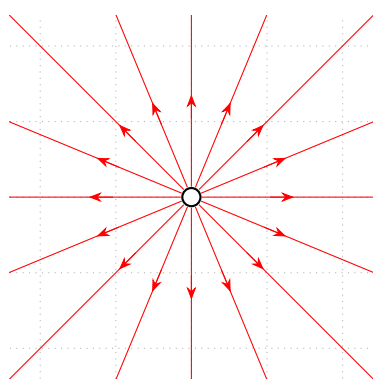
Answer on Page 51

²This is similar how we calculate tensions when there are multiple tensions (or just forces in general) on an object. The net force is the sum of all the forces after vector addition is applied.

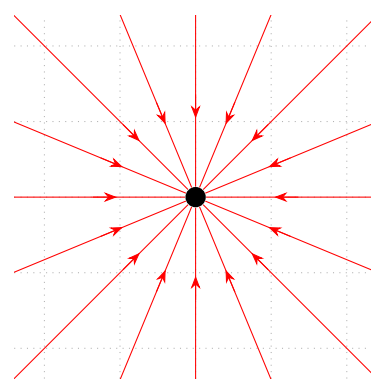
1.1 Electric Field

A gravitational field assigns a force-per-kilogram vector to every point in space around a mass. An electric field does the same thing for charge.

Similar to how the gravitational field extends out from the center of a given point mass, an *electric field* of a point charge has (imaginary) vectors pointing out of it. However, the direction of the vectors³ (and electric force) depends on the charge of the point charge. Around a negative charge, the field lines point *toward* it; around a positive charge, they point *away* from it (figure 1.2). In short, they always originate from positive charges and approach negative charges.



(a) A positively (open circle) charged electric field with corresponding electric field lines.



(b) A negatively (filled circle) charged electric field with corresponding electric field lines.

Figure 1.2: Two electric fields with different charges.

To measure the field of a charge Q at some point, imagine placing a tiny positive *test charge* q_t there and measuring the force on it. The electric field is the force per unit of test charge:

$$\vec{E} = \frac{\vec{F}_e}{q_t} \quad (1.2)$$

What are the *units* of an electric field? Well, by this equation, $\vec{F}_e$ is a force which we know is in *Newtons* (N), and the test particle q is in *Coulombs* (C). The resulting units are N/C, newtons per coulomb. This turns out to be the same as volts per meter (V/m), which you will see again when we discuss electric potential. (For example, 1 kV/cm is $\frac{1000 \text{ V}}{0.01 \text{ m}} = 10^5 \text{ V/m} = 10^5 \text{ N/C}$.)

³You may hear the terms electric field lines and electric field vectors used interchangeably

It is important to note that we can simplify this equation by substituting Coulomb's law:

$$\vec{E} = \frac{\vec{F}_e}{q_t} = \frac{\frac{kq_1q_t}{r^2}}{q_t} = \frac{kQ}{r^2} \quad (1.3)$$

where Q is the electric field's source charge.

The field gets weaker the farther you are from the charge. In fact, it decreases by a factor of $\frac{1}{r^2}$. Think to yourself where this factor comes from. (Hint: refer back to Coulomb's law (1.1))

Electric Field Force from a singular charge to a second charge

A charge q_1 creates an electric field at a distance r from it with strength:

$$E = \frac{kq_1}{r^2} \quad (1.4)$$

Introducing a new charge q_2 , the force on it due to the electric field is:

$$F = q_2 \cdot \frac{kq_1}{r^2} \iff F = q_2E \quad (1.5)$$

where E is the electric field strength at the location of q_2 . This is often referred to as the charge within a known field.

Exercise 3 Point Charges and Electric Fields

This problem is adapted from an Example IB Exam. Two isolated point charges, X of charge $+Q$ and Y of charge $+2Q$, are separated by distance $3d$. Point P is distance d from X and distance $2d$ from Y.

Working Space



What is the net electric field strength at P?

1. 0
2. $\frac{2kQ}{d^2}$
3. $\frac{kQ}{2d^2}$

4. $\frac{2kQ}{1.5d^2}$

[Answer on Page 51](#)

The density of the electric force lines at a given point determines the strength of the force at the point. Locations where field lines are more dense are much stronger than areas where the lines are sparse. This goes back to the idea of the force strength of an electric field.

If you recall the properties of conductors: *there can be no electrostatic field within the body of a conductor*. If an electric field were to exist inside the conductor, it would exert a force $F = qE$ on these free charges, causing them to accelerate. This internal motion continues until the charges redistribute themselves along the conductor's outer surface, creating an induced internal field that perfectly opposes and cancels the original field.

Electric field lines always

- Start at positive charges and terminate at negative ones. If it is a single charge, this is indicated by the direction of arrows off to infinity.
- The number of lines is proportional to the magnitude of charge.
- The line approaches a conductive surface perpendicularly.
- No two field lines ever cross in a charge-free region.

Keep these in mind for any sketches you may do.

1.2 Dipoles

An **electric dipole** is a pair of equal and opposite charges, $+q$ and $-q$, a small distance d apart. Many molecules, including water, act like dipoles: one end is slightly positive and the other slightly negative.

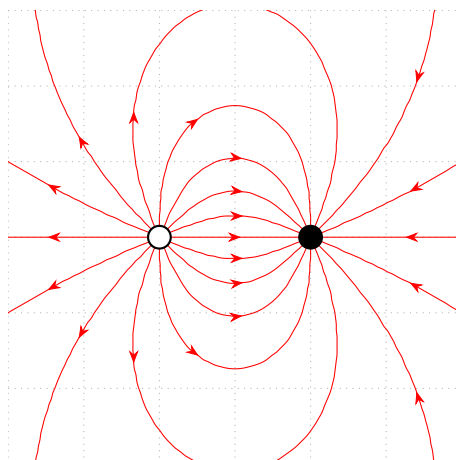


Figure 1.3: A basic dipole diagram.

Field lines run from the positive charge to the negative one. The strength of a dipole is measured by its *dipole moment* $\vec{p} = q\vec{d}$, a vector pointing from the negative charge to the positive charge, with units of coulomb-meters (C·m).

1.3 Electric Flux and Gauss's law

Electric flux, written Φ , measures how much electric field passes through a surface. Picture it as the number of field lines poking through.

The electric flux depends on 3 factors:

- Electric Field Strength (E)
- Surface area (A)
- The angle θ between the field and the line perpendicular to the surface

If a uniform field passes straight through a flat surface, the flux is

$$\Phi = EA \quad (1.6)$$

Think of a sheet of paper held facing a fan: it catches the most wind.

If the paper is at an angle to the fan, less flow from the fan affects the paper. If you tilt it until it is parallel to the wind, zero wind affects the paper. Tilting the surface reduces the flux in the same way:

$$\Phi = EA \cos(\theta) \quad (1.7)$$

For curved surfaces or changing fields, you chop the surface into tiny pieces, find $E \, dA \cos \theta$

for each, and add them all up. In calculus, that sum becomes an integral, written $\Phi = \oint \vec{E} \cdot d\vec{A}$ (the dot product supplies the $\cos \theta$).

1.3.1 Gauss's law

Now imagine a closed surface, like a balloon, around some charges. Count the field lines coming out of it. Every line starts on a positive charge and ends on a negative one, so the net number of lines poking out depends only on the charge inside. That is *Gauss's law*:

Gauss's Law

The total electric flux out of any closed surface equals the charge enclosed divided by ϵ_0 :

$$\Phi = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0} \quad (1.8)$$

For a point charge q , choose a sphere of radius r centered on it. By symmetry, the field has the same strength everywhere on the sphere and points straight out, so the flux is just E times the sphere's area, $4\pi r^2$. Gauss's law then gives

$$E = \frac{q}{4\pi r^2 \epsilon_0} \quad (1.9)$$

where q is the source charge, r is the distance from that source charge, and $4\pi r^2$ is the surface area of the electric field. That is exactly Coulomb's law again, since $\frac{1}{4\pi\epsilon_0} = k$. Gauss's law explains *why* the field falls off as $\frac{1}{r^2}$: the same field lines spread out over a sphere whose area grows as r^2 .

1.4 Continuous Charge Distributions

In the previous sections, we treated charges as discrete "dots" in space. However, in applications of charges such as a long charged wire, a flat metal plate, or a solid plastic sphere, charge is spread out over a macroscopic object. When we have a collection of so many point charges that they appear to form a smooth, "fluid-like" surface or volume, we call it a **continuous charge distribution**. This means that the charges, while not truly continuous, can be considered continuous as a long single stream.

To analyze these, we stop summing individual particles ($\sum$) and instead use calculus to integrate ($\int$) over the entire object. While we have not covered calculus yet, keep in mind that this is just a way of calculating the area or volume of a continuous solid.

1.4.1 Charge Densities

The first step in any continuous distribution problem is defining how the charge is spread. We use three Greek letters to represent the “concentration” of charge based on the object’s geometry:

Type	Symbol	Definition	SI Units
Linear	λ (lambda)	Charge per unit length	C/m
Surface	σ (sigma)	Charge per unit area	C/m ²
Volume	ρ (rho)	Charge per unit volume	C/m ³

Table 1.1: The three types of charge density used in electrostatics.

If the charge is distributed **uniformly**, these values stay constant. For example, for a rod of length L and total charge Q , the linear density is simply $\lambda = Q/L$. However, if these the charge is not constant per the unit of length, we must use calculus to find the total charge density. To find the total electric field $\vec{E}$ at a point P near a charged object, we imagine breaking the object into many infinitesimal “pieces” of charge, dq . Each chunk acts like a tiny point charge and contributes a tiny bit of electric field, $d\vec{E}$.

Recall our point charge formula:

$$d\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{dq}{r^2} \quad (1.10)$$

To find the total field, we integrate over the entire distribution:

$$\vec{E} = \int d\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r^2} \quad (1.11)$$

The key to solving these integrals is replacing dq with the appropriate density:

- Lines: $dq = \lambda dl$
- Surfaces: $dq = \sigma dA$
- Volumes: $dq = \rho dV$

You will learn to evaluate integrals like these in the calculus volumes. For now, the key idea is that a big charged object acts like a huge number of tiny point charges, each adding its own small piece of field.

Exercise 4 Belt being charged

Working Space

This question is from an IB Electromagnetics problem. A continuous conveyor belt of width 0.60 m travels at a constant speed of 2.5 ms^{-1} . The belt has a uniform distribution of charge of 5.0 mCm^{-2} on the surface. As the belt passes a point A, all of its charge is transferred to a wire.

What is the current in the wire? *Hint: recall that current is $\frac{\Delta Q}{\Delta t}$.*

Answer on Page 52

1.5 But...

The idea that like charges repel and opposite charges attract raises some big questions that you do not yet have the tools to answer fully.

However, you probably have these questions, so we will point you in the direction of the answers.

The first is “In any atom bigger than hydrogen, there are multiple protons in the nucleus. Why do not the protons push each other out of the nucleus?”

We are not ready to talk about it, but there is a force called *the strong nuclear force*, which pulls the protons and neutrons in the nucleus of the atom toward each other. At very, very small distances, it is strong enough to overpower the repulsive force due to the protons’ charges.

Another question is “Why do the electrons whiz around in a cloud so far from the nucleus of the atom? Negatively charged electrons should cling to the protons in the center, right?”

We are not ready to talk about this either, but quantum mechanics tells us that electrons like to live in a certain specific energy level. Hugging protons is not one of those levels.

Exercise 5 Doubling the Distance

The two charges in the example are moved to 0.2 m apart. What is the force

Working Space

now?

[Answer on Page 52](#)

The next chapter, Electric Potential, looks at electric forces through the lens of energy.

Electric Potential

Shuffle across a carpet in socks and touch a doorknob, and a spark jumps. A thunderstorm does the same thing on a scale of a billion volts. In the Charge chapter, you learned how charges push and pull on each other through electric fields. This chapter looks at the same forces through the lens of energy: how much energy a charge gains or loses as it moves through a field, and what a “volt” really measures.

2.1 Electric Potential

A ball held above the ground has gravitational potential energy; let it go, and gravity turns that into kinetic energy. Charges work the same way. A charge in an electric field has **electric potential energy** U_E . When the field does work W_E on the charge, its potential energy drops by the same amount:

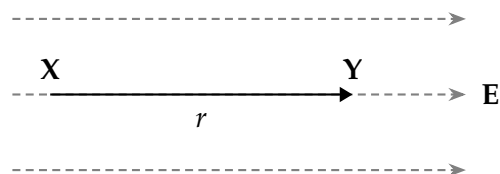
$$\Delta U_E = -W_E \quad (2.1)$$

Notice how this is similar to an object of mass m undergoing work done by a gravitational field $\Delta U_G = -W_G$.

Exercise 6 Change in Electric Potential Energy

Working Space

A positive charge $+q$ moves a distance r from position X to position Y in a **uniform** electric field E .



What is its change in electric potential energy? Assume moving in direction r

moves in the direction of E .

Answer on Page 52

Electric potential V is the electric potential energy per unit charge: $V = \frac{U_E}{q}$. It is measured in volts (1 volt = 1 joule per coulomb). Potential belongs to a *place*, whether or not a charge is there, the same way height belongs to a spot on a hill whether or not a ball is sitting there.

Just as moving a mass up in height against gravity increases its gravitational potential, moving a positive charge “up” against an electric field increases its electric potential. In both cases, the field *tries* to push the object toward a state of lower potential energy. A positive charge naturally wants to move with the electric field toward lower potential; in this case, its electric potential energy is converted into kinetic energy. Conversely, to move a positive charge against an electric field, work must be done on the charge, causing its electric potential to increase.

Moving from two points not on the same electric field line introduces an angle component θ :

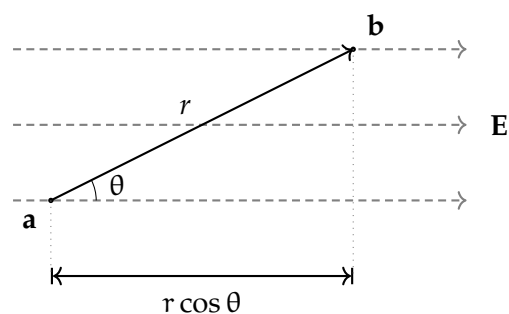


Figure 2.1: A gain of potential energy across an angle.

The electric potential difference between two points A and B is defined by the line integral of the electric field:

$$\Delta V = V_B - V_A = - \int_A^B \vec{E} \cdot d\vec{l} \quad (2.2)$$

The negative sign indicates that the potential decreases in the direction of the electric field.

We can easily define two point charges q_1 and q_2 sep-

arated by distance r to have potential energy

$$U = k \frac{q_1 q_2}{r} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r} \quad (2.3)$$

Exercise 7 Accelerating an Electron

This is a sample problem from an IB Physics Exam. An elec-

tron is accelerated from rest by an electric potential difference V . The electron reaches a speed of 9.4×10^6 m/s. Using the charge of an electron as $e = 1.6 \times 10^{-19}$ C and the mass of an electron as $m_e = 9.11 \times 10^{-31}$ kg, find the potential difference V that accelerated the electron to this speed. You may use a calculator to simplify computations.

Answer on Page 53

2.1.1 Point Charges, Uniform Fields, and Spherical Shells

In a **uniform electric field** (such as the space between two large parallel plates), the field strength E is constant. The potential difference between two points separated by a distance d along the field lines is:

$$\Delta V = -Ed \quad (2.4)$$

For a **point charge** Q , we typically set the reference point ($V = 0$) at infinity. The potential at a distance r from the charge is:

$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r} \quad (2.5)$$

Unlike the electric field, which is a vector and follows an $1/r^2$ factor, electric potential is a **scalar** and follows an $1/r$ factor.

Surface Spherical Charge

For all spherical objects of radius R (surface spheres - assume hollow interior), it has potential inside and outside of the shell. For all points outside the shell, it is the same for any point charge Q , as seen in equation 2.5. For all points *inside* the shell, the potential is

$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{R}$$

This means that moving a test charge q from the surface of the shell to the center of the shell, the potential does not change. The potential is constant within the shell.

If the charge is spread evenly through a *solid* sphere instead, the potential inside at distance r from the center is

$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{2R} \left(3 - \frac{r^2}{R^2} \right)$$

2.1.2 Potential due to Configurations of Charge

Because potential is a scalar, finding the total potential from multiple charges is often simpler than finding the total electric field. We do not need to worry about components, instead, we simply sum the potentials:

$$V = \sum_i \frac{kq_i}{r_i} \quad \text{or for continuous distributions} \quad V = k \int \frac{dq}{r} \quad (2.6)$$

2.1.3 Conservation of Energy

When a charge q moves through a potential difference ΔV , the change in its electric potential energy is $\Delta U_E = q\Delta V$. If no other forces are acting on the charge, the total mechanical energy is conserved:

$$K_a + U_{E,a} = K_b + U_{E,b} \implies \frac{1}{2}mv_a^2 + qV_a = \frac{1}{2}mv_b^2 + qV_b \quad (2.7)$$

2.1.4 Summary

Particle	wants to move toward...	Potential Change (ΔV)	Potential Energy Change (U_E)
Positive (+ q)	Lower Potential	Negative (Decreases)	Negative (Decreases)
Negative ($-q$)	Higher Potential	Positive (Increases)	Negative (Decreases)

Exercise 8 Electron between Field Plates

This is a sample problem from an IB Physics Exam. An electron that is travelling at a horizontal speed $v_x = 9.4 \times 10^6 \text{ms}^{-1}$ now enters a region of a *uniform electric field* between two parallel charged plates, separated by a distance of 4.0 cm. The

Working Space

electron is initially halfway between the plates and its initial velocity is parallel to the plates. The upper plate is positively charged and the lower plate is negatively charged. There is a potential difference between the plates of $V = 30 \text{ V}$.

Find the horizontal distance travelled before hitting a plate. Which plate does the electron hit?

Answer on Page 53

2.2 Lightning

A thundercloud is a tower of water droplets, ice crystals, and soft hail (called graupel), churned by powerful updrafts. When small ice crystals collide with heavier graupel, electrons are rubbed off, just like the balloon on your hair. The heavy, negatively charged graupel sinks toward the bottom of the cloud, while the light, positively charged ice crystals are carried to the top.

Air is a good insulator, so at first the charge has nowhere to go. But it keeps building until the electric field is strong enough to rip electrons off air molecules, turning a path of air into a conductor. Then the charge rushes through in a giant spark: lightning. It often strikes the tallest object around, because that is the shortest path. A great deal of lightning moves within a cloud or between clouds. However, sometimes it jumps to the earth. These bolts of lightning vary in the amount of electrons they carry, but the average is about 15 coulombs.

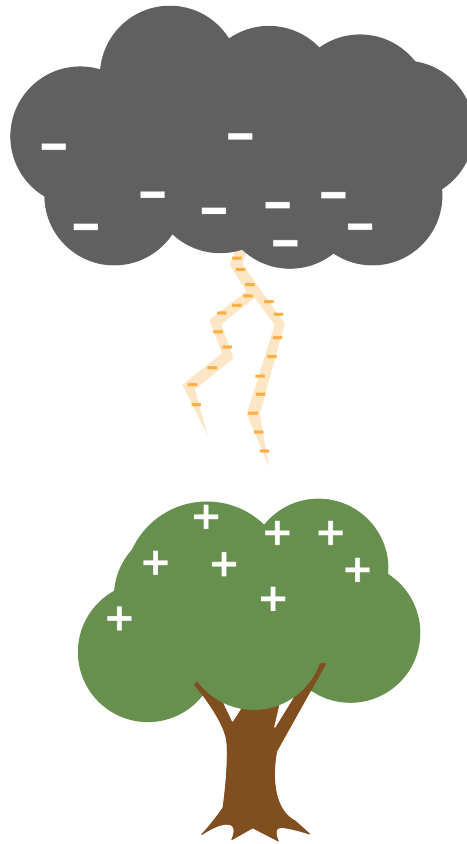


Figure 2.2: A difference of charge causes a spark of lightning to generate and transfer.

Thunder occurs because the lightning heats the air it passes through to about 30,000 kelvins, five times hotter than the surface of the Sun, so the air expands explosively. The resulting shockwave is the sound we know as thunder.

Magnetism and Forces on Charges

A compass needle swings to point north. A fridge magnet holds up your report card. An MRI machine looks inside your body without a single cut. And every electric motor, from a phone's vibrator to a Tesla, spins because of the same force. That force is magnetism, and it turns out to be electricity in motion.

3.1 Magnets

Every magnet has two ends, called its *north pole* and *south pole*. Like poles repel and opposite poles attract, just as with charges. But there is one big difference: you can never get a north pole by itself. Cut a bar magnet in half, and each half grows a new pair of poles.

Earth itself is a giant magnet. A compass needle is a tiny magnet free to turn, and its north pole points toward Earth's geographic north. (So, oddly, the magnetic pole near Earth's North Pole is actually a magnetic *south* pole: it attracts the north end of the needle.)

Where does a magnet's magnetism come from? Every electron acts like a tiny magnet. In most materials, these tiny magnets point every which way and cancel out. In iron, nickel, and cobalt, neighboring atoms line up in regions called *domains*. When most of the domains point the same way, the whole piece of metal becomes a magnet.

3.2 Magnetic Fields

A *magnetic field* $\vec{B}$ is a vector field that exerts forces on moving charges and on currents. Its unit is the *tesla* (T). Magnets and currents both produce magnetic fields. The electric field of the charge chapter pushes on a charge whether or not it moves. A magnetic field pushes only on a charge that is moving, and the push is perpendicular to both the velocity and the field.

3.3 The Field of a Long Wire

A long straight wire carrying a current I produces a magnetic field that circles the wire. At distance r from the wire, its magnitude is

$$B = \frac{\mu_0 I}{2\pi r}, \quad \mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$$

The direction is given by a right-hand rule: point your right thumb along the current, and your fingers curl in the direction of $\vec{B}$. Hans Christian Ørsted discovered this field in 1820 when he noticed a compass needle twitch every time he switched on a nearby current. It was the first clue that electricity and magnetism are connected.

3.3.1 A Worked Example

A wire carries $I = 10 \text{ A}$. At $r = 0.05 \text{ m}$,

$$B = \frac{(4\pi \times 10^{-7})(10)}{2\pi(0.05)} = 4 \times 10^{-5} \text{ T}$$

For comparison, Earth's magnetic field at its surface is roughly $5 \times 10^{-5} \text{ T}$, so the field of this wire at 5 cm is about the same size as Earth's.

3.4 Force on a Moving Charge

A charge q moving with velocity $\vec{v}$ in a magnetic field feels the *magnetic force*

$$\vec{F} = q\vec{v} \times \vec{B}, \quad |\vec{F}| = |q|vB \sin \theta$$

where θ is the angle between $\vec{v}$ and $\vec{B}$. The force is zero when the charge moves parallel to the field, and largest when the motion is perpendicular to it. The cross product means the force is perpendicular to both $\vec{v}$ and $\vec{B}$, so the magnetic force changes the direction of motion but does no work and does not change the speed.

To find the direction for a positive charge, use another right-hand rule: point your fingers along $\vec{v}$, curl them toward $\vec{B}$, and your thumb points along $\vec{F}$. For a negative charge, such as an electron, the force points the opposite way.

3.4.1 A Worked Example

A proton ($q = 1.6 \times 10^{-19}$ C) moves at 3×10^5 m/s perpendicular to a 0.5 T field. The force has magnitude

$$F = (1.6 \times 10^{-19})(3 \times 10^5)(0.5) = 2.4 \times 10^{-14} \text{ N}$$

3.5 Circular Motion

If a charge moves perpendicular to a uniform field, the magnetic force always points toward the center of a circle, and the charge moves in a circle of radius

$$r = \frac{mv}{|q|B}$$

An electron ($m = 9.11 \times 10^{-31}$ kg) moving at 2×10^6 m/s in a 0.01 T field has a radius of about 1.1×10^{-3} m. Its frequency of revolution is $f = \frac{|q|B}{2\pi m} \approx 2.8 \times 10^8$ Hz, which does not depend on its speed.

Where does the radius formula come from? From the circular motion you know: the magnetic force $|q|vB$ supplies the centripetal force $\frac{mv^2}{r}$. Setting them equal and solving for r gives $r = \frac{mv}{|q|B}$. This is how the auroras form: charged particles from the Sun spiral along Earth's magnetic field lines down toward the poles, where they crash into the air and make it glow.

3.6 Force on a Current-Carrying Wire

A wire of length L carrying current I in a field B feels a force

$$F = ILB \sin \theta$$

where θ is the angle between the wire and the field. This follows from the force on each moving charge in the wire, since the current is the total charge per second moving through it. A 5 A current in a 0.2 m wire in a 0.3 T field, perpendicular to the field, feels a force of 0.3 N.

3.7 Solenoids

Wrap a wire into a coil of n turns per meter, and a current I gives a nearly uniform field inside the coil:

$$B = \mu_0 n I$$

For $n = 1000$ turns/m and $I = 2$ A, $B \approx 2.5 \times 10^{-3}$ T. A solenoid is an *electromagnet*: a magnet you can switch on and off. Electromagnets lift scrap cars in junkyards, lock doors, and ring doorbells. The giant superconducting solenoid in an MRI machine makes a uniform field of 1.5 to 3 T, about 50,000 times stronger than Earth's.

3.8 What Comes Next

A changing magnetic field induces a current in a nearby conductor. That effect, and its direction (Lenz's law), is covered in the chapter on electromagnetic induction.

Exercise 9 Field of a wire

A wire carries $I = 3$ A. Find the magnetic field at $r = 0.02$ m.

Working Space

Answer on Page 54

Exercise 10 Force on a charge

A charge $q = 2 \times 10^{-6}$ C moves at 4 m/s perpendicular to a 0.25 T field. Find the magnitude of the magnetic force.

Working Space

Answer on Page 54

Exercise 11 Force on a wire

A wire of length 0.5 m carries 2 A and sits at 30 degrees to a 0.4 T field. Find the force on the wire.

Working Space

Answer on Page 55

Exercise 12 Does the field do work?

A charged particle moves in a circle in a uniform magnetic field. Does the magnetic force change its kinetic energy? Explain.

*Working Space**Answer on Page 55***Exercise 13** Proton in a circle

A proton ($m = 1.67 \times 10^{-27}$ kg) moves at 3×10^5 m/s perpendicular to a 0.5 T field. What is the radius of its circle? How does that compare with an electron moving at the same speed?

*Working Space**Answer on Page 55*

Alternating Current

Plug in a phone charger, and something strange is going on inside the wall. The electrons in the wire are not flowing from the power plant to your house at all. They are sloshing back and forth, 60 times a second, going nowhere. Yet they deliver all the energy you use. This chapter explains how, and why the whole world's power grid works this way.

A battery pushes electrons in one direction at a constant voltage; this is *direct current*, or DC. Most batteries provide between 1.5 and 12 volts.

The electrical power that comes into your home on wires is different. If you plotted the voltage over time, it would look like this:

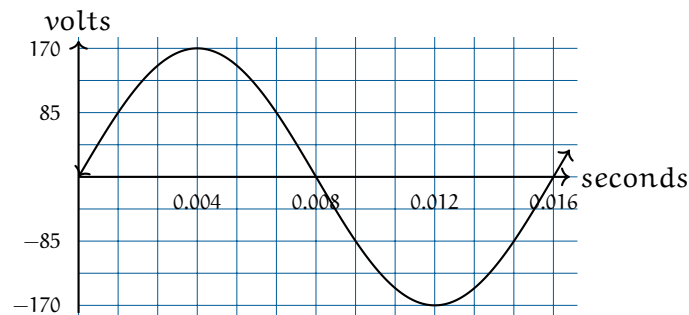


Figure 4.1: A graph of AC voltage over time.

The x axis here represents 0 volts. When you insert a two-prong plug into an outlet, one prong is “hot” and the other is “neutral.” The neutral wire is connected to the earth at your electrical panel, so it stays at about 0 volts, the same as the dirt under the building. (The round third hole on many outlets is a separate safety *ground*.)

The voltage is a sine wave at 60Hz. Your voltage fluctuates between -170v and 170v. Think for a second what that means: the power company pushes electrons with 170 volts, then pulls them back with 170 volts, alternating 60 times per second. This is *alternating current*, or AC. (In Europe and much of the world, it is 230 volts at 50 Hz.)

4.1 Power of AC

Let us say you turn on your toaster, which has a resistance of 14.4 ohms. How much power (in watts) does it turn into heat? We know that $I = V/R$ and that watts of power

are IV. So, given a voltage of V , the toaster is consuming V^2/R watts.

However, V is fluctuating. Let us plot the power the toaster is consuming:

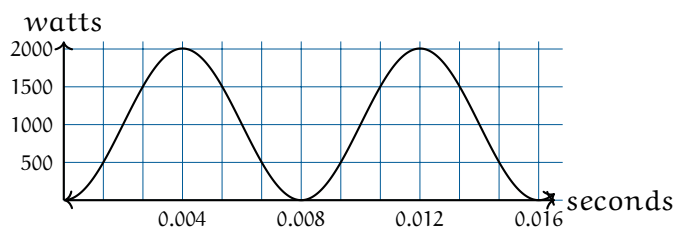


Figure 4.2: A plot of AC power over time.

Another wave! It never goes negative, because V^2 is never negative. Its shape comes from a power-reducing identity from the trigonometric identities chapter: $\sin^2(x) = \frac{1}{2} - \frac{1}{2} \cos(2x)$.

This is actually a cosine wave flipped upside down, scaled down by half the peak power, and translated up so that it is never negative. Note that it is also twice the frequency of the voltage sine wave.

If we say the peak voltage is V_p and the resistance of the toaster is R , the power is given by

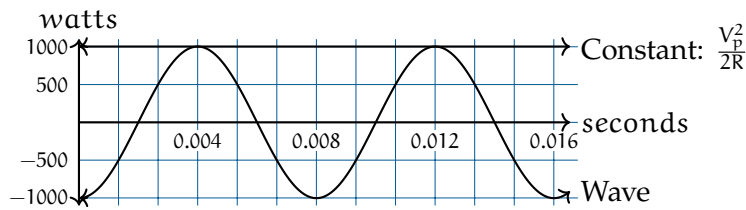
$$\frac{V_p^2}{2R} - \frac{V_p^2}{2R} \cos(2\pi \cdot 120 t)$$

(The power wave repeats 120 times per second, twice the frequency of the voltage, because it peaks at both the top and the bottom of the voltage wave.)

As a toaster user and as someone who pays a power bill, you are mostly interested in the average power. To get the average power, you take the area under the power graph and divide it by the amount of time.

Split the area under the curve into two pieces:

- A constant function of $y = \frac{V_p^2}{2R}$
- A wave $y = -\frac{V_p^2}{2R} \cos(2\pi \cdot 120 t)$



The area under the constant line over a time t is a rectangle: $\frac{V_p^2}{2R} \cdot t$.

The area under the wave over a complete cycle is...zero! The part above the axis exactly cancels the part below.

So the average power is $\frac{V_p^2}{2R}$ watts: exactly half the peak power. For the toaster, $\frac{170^2}{2(14.4)} \approx 1000$ watts.

Someone at some point said, “I am used to power being V^2/R . Can we define a voltage measure for AC power such that this is always true?”

So we use the *RMS voltage* $V_{\text{rms}} = \frac{V_p}{\sqrt{2}}$. Then the average power is $\frac{V_{\text{rms}}^2}{R}$, just like DC. If you look on the back of anything that plugs into a standard US power outlet, it will say something like “For 120v”. What they mean is, “For 120v RMS, we expect the voltage to fluctuate back and forth from 170v to -170v.”

RMS stands for root-mean-square: square the voltage, take the mean (average), then take the square root. For any sine wave, the RMS value is $\frac{1}{\sqrt{2}} \approx 0.707$ times the peak.

Worked example. A household outlet has an RMS voltage of 120 V. What is its peak voltage?

Solution. $V_{\text{peak}} = \sqrt{2} V_{\text{rms}} = 120\sqrt{2} \approx 170$ V.

For current, we do the same thing. If the current is AC, the power consumed by a resistor is $I_{\text{RMS}}^2 R$, where I_{RMS} is the peak current divided by $\sqrt{2}$.

4.2 Power Line Losses

A wire has some resistance. Thinner wires tend to have more resistance than thicker ones. Aluminum wires tend to have more resistance than copper wires.

Let us say that the power that comes to your house has to travel 20 km from the generator in a cable that has about 1Ω of resistance per km. Let us also say that your home is consuming 12 kilowatts of power. If that power traveled the whole way at 120 V RMS, how much would be lost heating the power line?

The current would be $I = \frac{P}{V} = \frac{12,000}{120} = 100$ amps RMS. In the $20\ \Omega$ line, $I^2R = (100^2)(20) = 200,000$ watts would be lost to heat. That is more than 16 times the power your home uses! The line would glow, and the plan is hopeless.

What if the power company moved the power at 120,000 volts RMS? Now only $\frac{12,000}{120,000} = 0.1$ amps flow in the line, and the loss is $I^2R = (0.1^2)(20) = 0.2$ watts. A thousand times the voltage means a thousandth of the current, and a millionth of the loss.

This is much, much more efficient. The only problem is that 120,000 volts would be incredibly dangerous. So the power company moves power long distances at very high voltages, like 765 kV. Before the power is brought into your home, it is converted into a lower voltage using a *transformer*.

4.3 Transformers

A transformer is a device that converts electrical power from one voltage to another. A good transformer is more than 95% efficient. The details of magnetic fields, flux, and inductance are beyond the scope of this chapter, so we are going to give a relatively simple (and admittedly incomplete) explanation for now.

A transformer is a ring with two sets of coils wrapped around it.

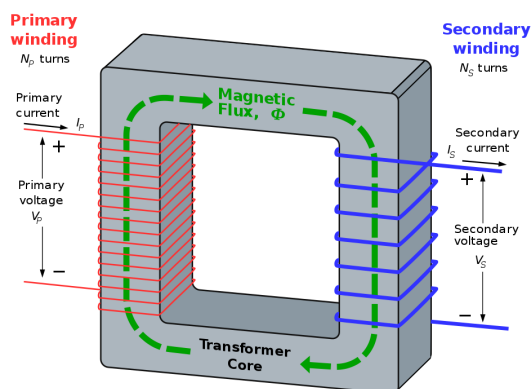


Figure 4.3: A diagram of a transformer from Wikipedia.

When alternating current is run through the primary winding, it creates magnetic flux in the ring. The magnetic flux induces current in the secondary winding. (This is called *induction*.)

If V_p is the voltage across the primary winding and V_s is the voltage across the secondary winding, they are related by the following equation:

$$\frac{V_P}{V_S} = \frac{N_P}{N_S}$$

where N_P and N_S are the number of turns in the primary and secondary windings.

There are usually at least two transformers between you and the very high voltage lines. There are transformers at the substation that make the voltage low enough to travel on regular utility poles. On the utility poles, you will see gray cans that contain smaller transformers. Those step the voltage down to the 120 V RMS that your home uses.

This is the big reason the world uses AC: transformers only work with alternating current, because they depend on a *changing* magnetic field.

Exercise 14 Turns ratio

A pole transformer takes 7,200 V RMS and delivers 240 V RMS to a neighborhood. The secondary coil has 50 turns. How many turns does the primary coil have?

Working Space

Answer on Page 55

4.4 Phase and 3-phase power

If two waves are “in sync”, we say they have the same *phase*.

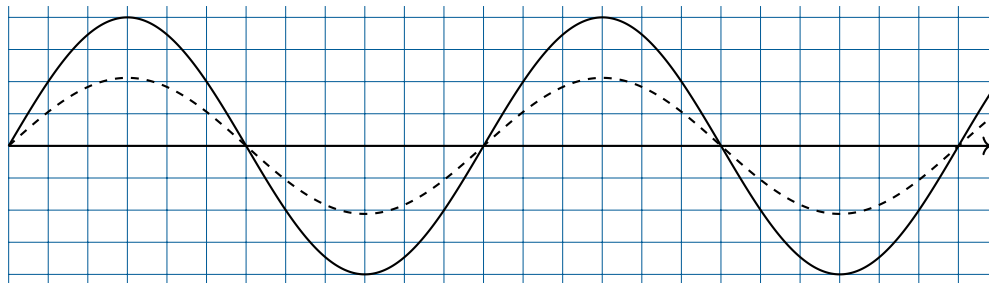


Figure 4.4: A diagram of two AC signals in sync.

If they are the same frequency, but are not in sync, we can talk about the difference in

their phase.

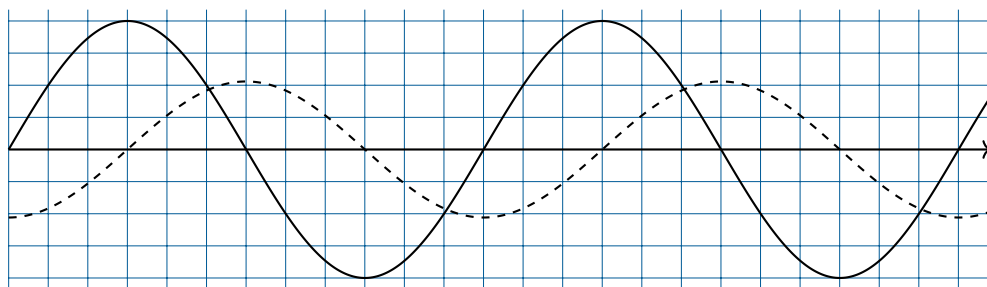


Figure 4.5: A diagram of two AC signals out of phase.

Here, we see that the smaller wave is lagging by $\pi/2$ or 90° .

In most power grids, there are usually three wires carrying the power. The voltage on each is $2\pi/3$ out of phase with the other two:

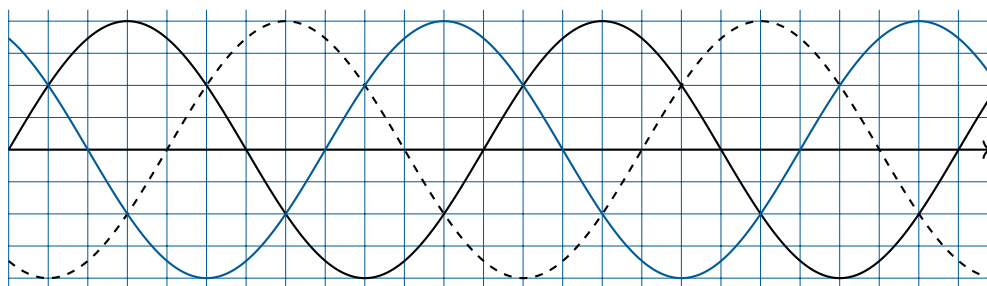


Figure 4.6: A diagram of three AC signals, each $2\pi/3$ radians out of phase.

This is nice in two ways:

- While the power in each wire is fluctuating, the total power is not fluctuating at all.
- At every instant, the three currents add up to zero, so no return wire is needed: three wires do the work of three separate circuits.

(Both these assume that each wire is attached to a load with the same constant resistance.)

In big industrial factories, you will see all three wires enter the building. Large amounts of smooth power delivery means a great deal to an industrial user.

In residential settings, each home gets its power from one of the three wires. However, two wires typically carry power into the home. Each one carries 120V RMS, but they are out of phase by 180 degrees. Lights and small appliances are connected to one of the wires and ground, so they get 120V RMS. Large appliances, like air conditioners, clothes

dryers, and electric ovens, are connected across the two wires, so they get 240 V RMS.

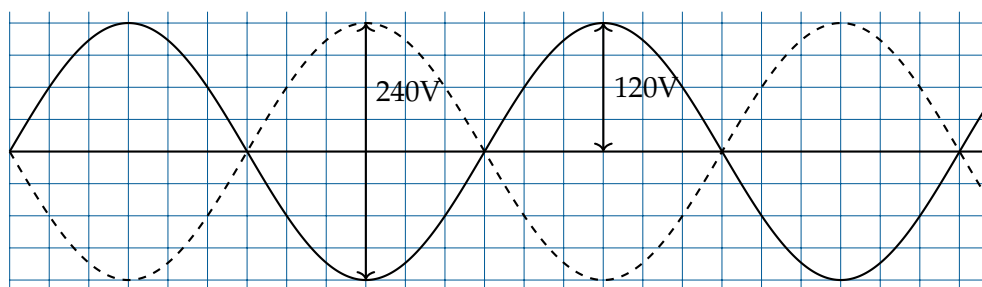


Figure 4.7: A diagram of AC out of phase in homes.

How do you get two circuits, 180 degrees out of phase, from one? With a *center-tap transformer*. Its secondary coil has an extra wire attached to its exact middle, which becomes the neutral. Whenever one end of the coil is 170 V above the middle, the other end is 170 V below it. So the two ends are perfectly out of phase, each 120 V RMS from the middle and 240 V RMS from each other.

Exercise 15 RMS from Peak

A signal has a peak voltage of 340 V.
What is its RMS voltage?

Working Space

Answer on Page 55

Electromagnetic Induction

Drop a strong magnet down a copper pipe, and something eerie happens: it drifts down slowly, as if falling through honey, even though copper is not magnetic at all. Lay your phone on a wireless charger, and it charges with no wires touching. Pluck an electric guitar string, and a signal flows to the amplifier. All three are the same effect: a *changing* magnetic field makes electricity. Michael Faraday discovered it in 1831, and it is how nearly all of the world's electricity is generated.

5.1 Magnetic Flux

In the charge chapter, electric flux measured how much electric field passes through a surface. *Magnetic flux* Φ_B does the same for a magnetic field. For a uniform field B passing through a flat loop of area A ,

$$\Phi_B = BA \cos \theta$$

where θ is the angle between the field and the line perpendicular to the loop. Flux is measured in *webers* (Wb): $1 \text{ Wb} = 1 \text{ T} \cdot \text{m}^2$.

You can change the flux through a loop in three ways:

- Change the field B (move a magnet closer, or switch on a current).
- Change the area A (stretch or squeeze the loop).
- Change the angle θ (spin the loop).

5.2 Faraday's Law

Faraday found that whenever the flux through a loop of wire changes, a voltage appears around the loop. This induced voltage is called the *electromotive force*, or emf, written $\mathcal{E}$ and measured in volts. For a coil of N turns,

Faraday's Law

$$\mathcal{E} = -N \frac{\Delta \Phi_B}{\Delta t}$$

The faster the flux changes, the bigger the emf. A steady flux, no matter how large, induces nothing. Hold a magnet still inside a coil, and no current flows; move it, and current flows. Each turn of the coil adds its own emf, so more turns mean more voltage. (In calculus notation, the rate of change is written $\frac{d\Phi_B}{dt}$.)

Worked example. A coil with 100 turns has its magnetic flux change by 0.02 Wb in 0.5 s. What emf is induced?

Solution. $\mathcal{E} = N \frac{\Delta\Phi_B}{\Delta t} = (100) \frac{0.02}{0.5} = 4 \text{ V}.$

If the field changes steadily through a coil of fixed area A held perpendicular to it, then $\Delta\Phi_B = A \Delta B$ and

$$\mathcal{E} = -N \frac{A \Delta B}{\Delta t}$$

Exercise 16 Induced EMF in a Coil

A coil with 100 turns and area 0.2 m^2 experiences a change in magnetic field from 0 to 5 T over 0.1 s. Calculate the induced emf.

Working Space

Answer on Page 55

Exercise 17 Twice the Turns

A coil with 200 turns has a flux change of 0.01 Wb in 0.1 s. What is the induced emf?

Working Space

Answer on Page 56

5.3 Lenz's Law

Which way does the induced current flow? *Lenz's law* answers: *the induced current always flows so that its own magnetic field opposes the change in flux that caused it.* That is what the

minus sign in Faraday's law means. Nature resists changes in flux.

Push the north pole of a magnet toward a coil. The flux through the coil grows, so the induced current makes the near end of the coil a north pole, pushing back on your magnet. Pull the magnet away, and the current reverses to make a south pole that tugs the magnet back.

That explains the magnet in the copper pipe. As the magnet falls, the flux through each ring of copper below it grows and the flux above it shrinks. Currents swirl around the pipe (called *eddy currents*) and their fields push up on the magnet from below and pull up on it from above. The faster it falls, the stronger the push, so it settles at a slow, steady speed.

Lenz's law is really conservation of energy in disguise. If the induced current *helped* the change instead of opposing it, the magnet would speed up on its own, and you would get energy from nothing.

5.4 Moving Wires

A wire moving through a magnetic field also gets an emf, even if the field never changes. Slide a straight rod of length L at speed v along two rails, perpendicular to a field B . The loop formed by the rod and rails sweeps out an area Lv every second, so the flux changes at a rate BLv , and

$$\mathcal{E} = BLv$$

You can also see this from the magnetic force. Each electron in the rod moves with the rod, so it feels a force qvB along the rod, which pushes electrons toward one end.

Exercise 18 A sliding rod

A 0.5 m metal rod slides at 4 m/s along rails, perpendicular to a 0.3 T field. What emf is induced? If the circuit has a resistance of 2 Ω , what current flows?

Working Space

Answer on Page 56

5.5 Applications of Electromagnetic Induction

- **Electric generators** spin coils in a magnetic field, so the angle θ changes constantly and the flux rises and falls. The result is an alternating emf. Almost every power plant, whether it burns coal, splits atoms, or catches wind, ends with a spinning generator.
- **Transformers** use the changing current in one coil to make a changing flux through a second coil. This is why they only work with AC.
- **Induction cooktops** drive a rapidly changing field under the pan. Eddy currents in the iron pan heat it directly, while the glass top stays cool.
- **Wireless chargers** are transformers with an air gap: a coil in the pad induces a current in a coil inside your phone.
- **Guitar pickups** are tiny coils wrapped around magnets. The vibrating steel string changes the flux through the coil.
- **Regenerative brakes** in electric cars run the motor as a generator. Lenz's law slows the car, and the induced current recharges the battery.

Exercise 19 Which way?

You pull the south pole of a magnet away from the end of a coil. Does the induced current make that end of the coil a north pole or a south pole? Does the coil attract or repel the magnet?

Working Space

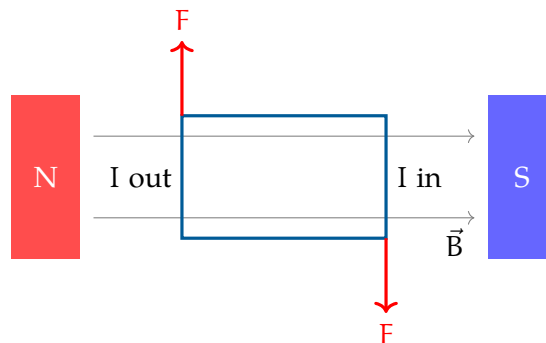
Answer on Page 56

Electric Motors and Generators

Count the electric motors in your home: the refrigerator compressor, the fan in your laptop, the washing machine, the vibrator in your phone, the spinning disc in a microwave. There are probably more than fifty. Now follow the electricity that runs them back to the power plant, and you will find a generator, which is a motor running backward. In this chapter, you will see that motors and generators are the same machine, built on the two ideas from the last chapters: currents feel forces in magnetic fields, and changing magnetic fields create currents.

6.1 How a Motor Turns

In the magnetism chapter, you saw that a wire carrying current I across a magnetic field B feels a force $F = ILB$. Now bend the wire into a rectangular loop and set it between the poles of a magnet. Current flows one way along one side of the loop and the opposite way along the other side. So the two sides are pushed in *opposite* directions: one up, one down. That pair of forces twists the loop. A twisting effect is called a *torque*.



Seen from the end, the current flows out of the page on one side of the loop and into the page on the other. The forces push one side up and the other down, so the loop turns.

For a coil of N turns and area A , the torque is largest when the plane of the coil lines up with the field:

$$\tau_{\max} = NBIA$$

There is a catch. Once the loop has turned a half turn, the side that was being pushed up is now on the other side, and it gets pushed back. The loop would just rock back and

forth. The fix is a *commutator*: a split ring on the shaft. Carbon *brushes* press against it to carry current into the coil, and every half turn the gap in the ring swaps the connections, reversing the current in the coil. The push always points the same way around, and the motor keeps spinning.

This design, a coil (the *armature*) spinning between magnets with a commutator and brushes, is a *brushed DC motor*. It powers toys, drills, and car window lifts.

Worked example. A motor coil has 50 turns and an area of 0.01 m^2 , carries 2 A, and sits in a 0.2 T field. What is the largest torque?

Solution. $\tau_{\max} = NBIA = (50)(0.2)(2)(0.01) = 0.2 \text{ N}\cdot\text{m}$. That is about the twist you would make pushing with 2 newtons (the weight of a cup of coffee) on the end of a 10 cm wrench.

6.1.1 Other Kinds of Motors

Brushes wear out and make sparks. A *brushless motor* flips the design around: the magnets spin and the coils stay still, and a small computer switches the current in the coils at just the right moments. Drones, computer fans, and electric cars use them. A *stepper motor* turns in small, precise steps, perfect for 3D printers. An *AC induction motor*, invented by Nikola Tesla, uses alternating current to make a rotating magnetic field, which induces currents in the rotor and drags it around. It needs no brushes or magnets, so it is cheap and nearly indestructible; it runs fans, pumps, and factory machines.

6.2 Power and Efficiency

A motor turns electrical power into mechanical power. The electrical power going in is $P = VI$.

Worked example. A motor draws 2 A from a 120 V supply. What electrical power does it use?

Solution. $P = VI = (120)(2) = 240 \text{ W}$.

No motor is perfect. Some energy is lost to friction and to heat in the wires. The *efficiency* is the mechanical power out divided by the electrical power in. Large industrial motors can be more than 95% efficient. Compare that with a car engine, which turns only about 30% of the energy in its fuel into motion.

Exercise 20 **Power at Higher Current**

The same motor now draws 5 A from a 120 V supply. What is its power?

Working Space

Answer on Page 56

Exercise 21 **Lifting with a motor**

A winch motor draws 240 W and lifts a 10 kg box straight up at a steady 0.2 m/s. What mechanical power does it deliver? What is its efficiency?

Working Space

Answer on Page 56

6.3 Generators

Now run the motor backward. Instead of feeding it current, turn its shaft by hand. As the coil spins between the magnets, the magnetic flux through it rises and falls. By Faraday's law, the changing flux induces an emf, and current flows through anything connected to it. A generator turns mechanical energy into electrical energy.

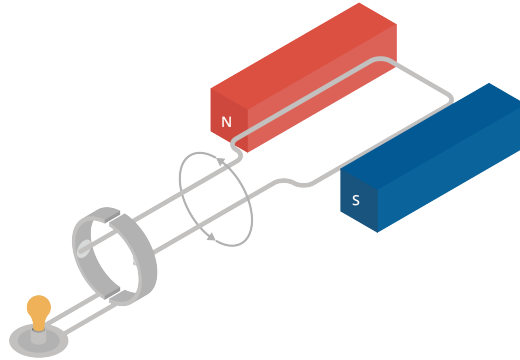


Figure 6.1: A basic generator: a loop spins between the poles of a magnet. Brushes on the rotating rings carry the induced current to a light bulb.

Every small DC motor is also a generator. Connect a motor to a light bulb and turn the shaft with a crank, and the bulb lights.

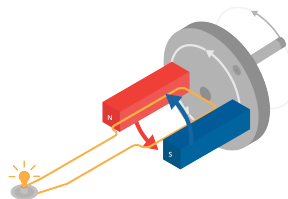


Figure 6.2: Turning the generator with a hand crank.

You will notice something as you crank: the brighter the bulb, the harder the crank is to turn. That is Lenz's law. The current in the coil feels a force from the magnets that opposes the turning. The energy in the light comes from the work your arm does.

When a coil of N turns and area A spins at a steady rate in a field B , the angle of the coil

changes steadily, so the flux follows a cosine and the emf follows a sine wave:

$$\mathcal{E} = NBA\omega \sin(\omega t)$$

where ω is the rotation rate in radians per second. That is alternating current! A generator naturally makes AC. Spin it 60 times a second, and you get the 60 Hz power in your outlet.

Worked example. A generator coil has 100 turns, an area of 0.01 m^2 , and spins 60 times per second in a 0.5 T field. What is its peak emf?

Solution. 60 turns per second is $\omega = 2\pi(60) \approx 377 \text{ rad/s}$. The peak emf is $NBA\omega = (100)(0.5)(0.01)(377) \approx 188 \text{ V}$.

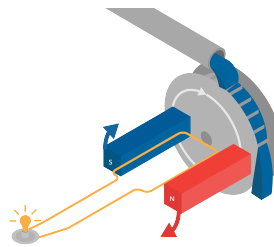


Figure 6.3: Any spinning wheel can drive a generator.

Almost all of the world's electricity comes from spinning generators. In a hydroelectric dam, falling water spins the turbines. In a wind farm, the wind does. In coal, natural gas, and nuclear plants, heat boils water, and the high-pressure steam spins turbines. (Solar panels are the big exception: they turn light directly into electricity with no moving parts.)

Exercise 22 Spin it faster

The generator in the worked example is spun twice as fast. What happens to its peak emf and to the frequency of the AC it produces?

Working Space

Answer on Page 56

Drag

One of the first electronic computers, ENIAC, was built during World War II to calculate how artillery shells fly. Why did the army need a computer for that? You already know the projectile formulas. The trouble is that a shell flies hundreds of meters per second, and at that speed the air pushes back hard. With air resistance, the neat formulas stop working, and the only way to get answers is to simulate the flight one tiny step at a time. That is exactly what you will do in this chapter.

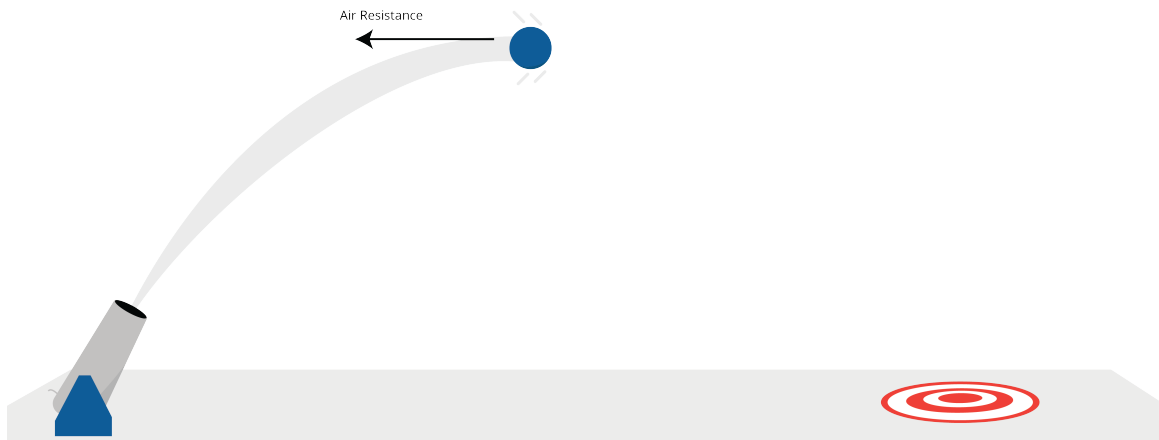


Figure 7.1: Without air, a cannonball would follow a parabola. With air resistance, the path is shorter and steeper on the way down.

7.1 Wind resistance

Engineers start by putting a shell in a wind tunnel and measuring the force when air blows past it at 1 m/s. Say it is 0.1 newtons.

One of the interesting things about the drag from the air (often called *wind resistance*) is that it increases with the *square* of the speed. Thus, if the wind pushing on the shell is 3 m/s, instead of 1 m/s, the resistance is $3^2 \times 0.1 = 0.9$ newtons.

(Why? Intuitively, three times as many air molecules are hitting the shell and each

molecule is hitting it three times harder.)

Drag always points exactly opposite the velocity. If μ is the drag force at 1 m/s, then at speed $|v|$ the drag has magnitude $\mu|v|^2$. As a vector, that is

$$\vec{F}_{\text{drag}} = -\mu|v|\vec{v}$$

(The vector $\vec{v}$ points along the motion and has length $|v|$, so $\mu|v|\vec{v}$ has length $\mu|v|^2$; the minus sign flips it backward.)

7.2 Initial velocity and acceleration due to gravity

Let us say a shell is shot out of a tube at s m/s, and the tube is tilted θ radians above level. The initial velocity will be given by the vector $[s \cos(\theta), s \sin(\theta)]$

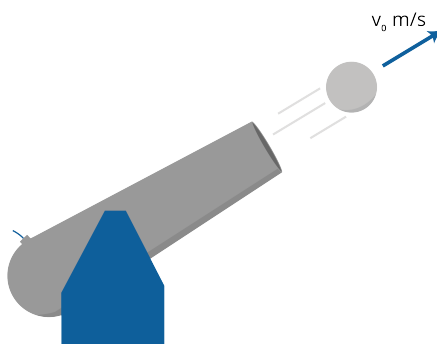


Figure 7.2: The initial velocity vector of the shell.

(The velocity of the shell is actually a 3-dimensional vector, but we are only going to worry about height and horizontal distance; we are assuming that the operator pointed it in the right direction.)

To figure out the path of the shell, we need to compute its acceleration. We remember that

$$F = ma$$

(Note that F and a are vectors.) Dividing both sides by m , we get:

$$a = \frac{F}{m}$$

Let us figure out the net force on the shell, so that we can calculate the acceleration vector.

If the shell has a mass of m , the force of gravity points down with a magnitude of $9.8m$ newtons.

To get the net force, we will need to add the force due to gravity with the force due to wind resistance.

7.3 Simulating artillery in Python

Create a file called `artillery.py`.

```
1 import numpy as np
2 import matplotlib.pyplot as plt
3
4 # Constants
5 mass = 45 # kg
6 start_speed = 300.0 # m/s
7 theta = np.pi/5 # radians (36 degrees above level)
8 time_step = 0.01 # s
9 wind_resistance = 0.05 # newtons in 1 m/s wind
10 force_of_gravity = np.array([0.0, -9.8 * mass]) # newtons
11
12 # Initial state
13 position = np.array([0.0, 0.0]) # [distance, height] in meters
14 velocity = np.array([start_speed * np.cos(theta), start_speed * np.sin(theta)])
15 time = 0.0 # seconds
16
17 # Lists to gather data
18 distances = []
19 heights = []
20 times = []
21
22 # While shell is aloft
23 while position[1] >= 0:
24     # Record data
25     distances.append(position[0])
26     heights.append(position[1])
27     times.append(time)
28
29     # Calculate the next state
30     time += time_step
31     position += time_step * velocity
32
33     # Drag points opposite the velocity, with size wind_resistance * speed**2
```

```

34     speed = np.linalg.norm(velocity)
35     force = force_of_gravity - wind_resistance * speed * velocity
36
37     # Calculate the current acceleration vector
38     acceleration = force / mass
39
40     # Update the velocity vector
41     velocity += time_step * acceleration
42
43     print(f"Hit the ground {position[0]:.2f} meters away at {time:.2f} seconds.")
44
45     # Plot the data
46     fig, ax = plt.subplots()
47     ax.plot(distances, heights)
48     ax.set_title("Distance vs. Height")
49     ax.set_xlabel("Distance (m)")
50     ax.set_ylabel("Height (m)")
51     plt.show()

```

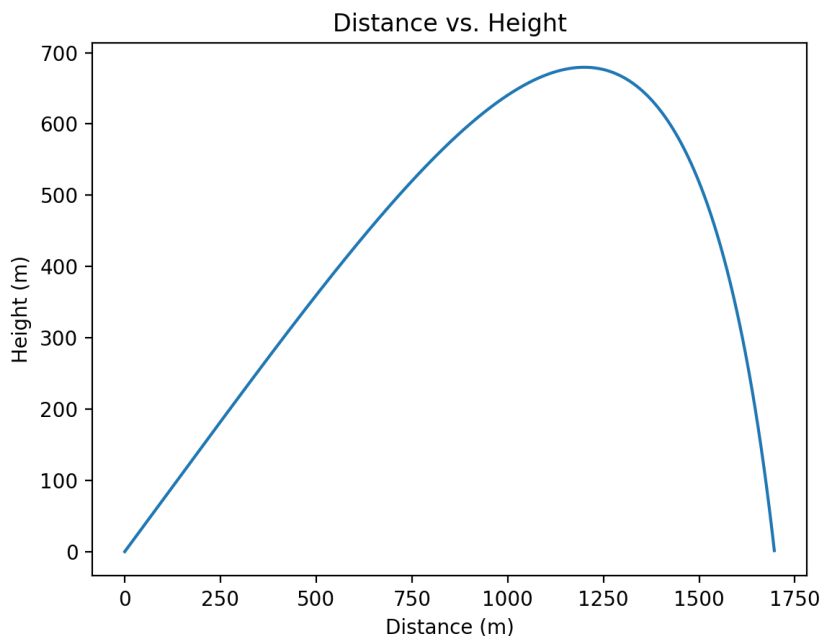
When you run it, you should get a message like:

```

1 Hit the ground 1519.46 meters away at 19.34 seconds.

```

Without air, the same shell would travel $\frac{300^2 \sin 72^\circ}{9.8} \approx 8,700$ meters. Air resistance cuts the range by more than 80%! You should also see a plot of the shell's path. Notice that it is not a parabola: it comes down more steeply than it went up.



7.4 Terminal velocity

If you shot the shell very, very high in the sky, it would keep accelerating toward the ground until the force of gravity and the force of the wind resistance were equal. The speed at which this happens is called the *terminal velocity*. A skydiver falling belly-down reaches about 53 m/s (190 km/h). Opening the parachute increases the drag so much that the terminal velocity drops to about 5 m/s, a safe landing speed.

Worked example. A skydiver and gear have a mass of 80 kg and a drag of $\mu = 0.28$ N at 1 m/s. What is the terminal velocity?

Solution. At terminal velocity, drag equals weight: $0.28v^2 = (80)(9.8) = 784$, so $v = \sqrt{\frac{784}{0.28}} = \sqrt{2800} \approx 53$ m/s.

Note that kinematic equations do not apply to terminal velocity, because the acceleration is not constant. Instead, we can use the fact that at terminal velocity, the force of wind resistance equals the force of gravity.

Exercise 23 Terminal velocity

What is the terminal velocity of the shell described in our example?

Working Space

Answer on Page 56

Exercise 24 Parachute

When the skydiver from the worked example opens the parachute, the drag constant becomes $\mu = 30$ N at 1 m/s. What is the new terminal velocity?

Working Space

Answer on Page 57

Exercise 25 **Without air**

In `artillery.py`, set `wind_resistance = 0` and run it again. How far does the shell go now? Compare it with the range formula from the projectiles chapter.

Working Space

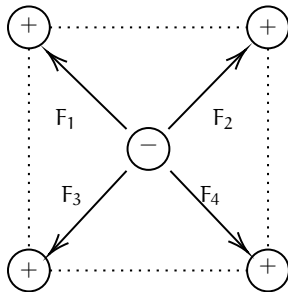
Answer on Page 57

Answers to Exercises

Answer to Exercise 1 (on page 4)

$$F = k \frac{|q_1 q_2|}{r^2} = (8.988 \times 10^9) \frac{(-5 \times 10^{-9})(-5 \times 10^{-9})}{0.12^2} = \frac{224.7 \times 10^{-9}}{0.0144} = 15.6 \times 10^{-6} \text{ N or } 15.6 \text{ micronewtons}$$

Answer to Exercise 2 (on page 6)



The attractive forces of F_1 and F_3 cancel each other out. Similarly, the attractive forces of F_2 and F_4 also cancel each other out. The net force on the negative point charge is zero.

Answer to Exercise 3 (on page 8)

We noted above that the electric field caused by a point charge is defined as $E = \frac{kq}{r^2}$. We need to find the electric fields for each charge and then subtract their strengths.

Inputting known variables as constants for X from point P:

$$E = \frac{kQ}{d^2}$$

And doing the same for Y:

$$E = \frac{2kQ}{(2d)^2} = \frac{kQ}{2d^2}$$

Both charges are positive, so each field points away from its charge: X's field at P points right, and Y's points left. Because the fields are in opposite directions, we subtract the smaller magnitude from the larger one. Let us define *to the right* as the positive direction:

$$E_{\text{net}} = E_X - E_Y \implies \frac{kQ}{d^2} - \frac{kQ}{2d^2}$$

To subtract, we find a common denominator:

$$E_{\text{net}} = \frac{2kQ}{2d^2} - \frac{kQ}{2d^2} = \frac{kQ}{2d^2}$$

The correct answer is (c). (Answer (a) is a trap: Y has twice the charge, but it is twice as far away, and distance counts squared.)

Answer to Exercise 4 (on page 12)

The amount of charge dQ on a small *surface* section of the belt is $dq = \sigma dA$. For a belt of width w , the area passing the wire in time dt is $dA = w(v dt)$. Substituting this in:

$$I = \frac{\sigma(w \cdot v \cdot dt)}{dt} = \sigma \cdot w \cdot v \quad (1.12)$$

Plugging in our values:

$$\begin{aligned} I &= (5.0 \times 10^{-3} \text{ C/m}^2) \cdot (0.60 \text{ m}) \cdot (2.5 \text{ m/s}) \\ I &= 7.5 \times 10^{-3} \text{ A} = 7.5 \text{ mA} \end{aligned}$$

Answer to Exercise 5 (on page 13)

The force falls by a factor of $2^2 = 4$, so $F \approx 0.22 \text{ N}$.

Answer to Exercise 6 (on page 15)

Since the field is uniform, the charge has a constant electric force felt on it: $\vec{F}_E = q\vec{E}$. Since q is positive, $\vec{F}_E$ points in the same direction as $\vec{E}$. Because the force and the displacement $\vec{r}$ are in the same direction, the work W_E done by the electric field is positive:

$$W_E = F_E \cdot r = qEr$$

Thus the change in electric potential energy is

$$\Delta U_E = -qEr$$

It is *negative* because when the field does positive work, the potential energy goes down.

Answer to Exercise 7 (on page 17)

Using our equations, we can derive an equation for the potential difference V :

$$\Delta U_E = q\Delta V$$

$$W_E = \Delta U_E$$

$$W_E = qV$$

gain in kinetic energy = work done by field

$$\frac{1}{2}mv^2 = qV$$

$$V = \frac{1}{2} \frac{mv^2}{q}$$

Since our we are given the speed, and the charge and mass of the electron, we can plug in our values:

$$\begin{aligned} V &= \frac{\frac{1}{2}(9.11 \times 10^{-31})(9.4 \times 10^6)^2}{1.60 \times 10^{-19}} \\ &= \frac{\frac{1}{2}(9.11 \times 10^{-31})(8.836 \times 10^{13})}{1.60 \times 10^{-19}} \\ &= \frac{\frac{1}{2}(8.05 \times 10^{-17})}{1.60 \times 10^{-19}} \\ &= \frac{4.03 \times 10^{-17}}{1.60 \times 10^{-19}} \\ &= 252 \text{ V} \end{aligned}$$

A potential difference of about 252 volts does the job. (Since an electron is negative, it speeds up when it moves toward *higher* potential.)

Answer to Exercise 8 (on page 18)

First off, why do we know that the electron will hit a plate? Because the electric field

between the plate exerts a force on the electron. The electron is attracted to the **positively** charged upper plate, so it will accelerate upwards and eventually hit the upper plate. Let us first find the acceleration caused by the electric field. The electric field strength E between two parallel plates is given by:

$$E = \frac{V}{d} = \frac{30}{0.040} = 750 \text{ V m}^{-1} \quad (2.8)$$

Converting this to solve for the electron's acceleration:

$$a = \frac{F}{m} = \frac{qE}{m} = \frac{(1.6 \times 10^{-19})(750)}{9.11 \times 10^{-31}} \approx 1.32 \times 10^{14} \text{ m s}^{-2} \quad (2.9)$$

To find the horizontal distance travelled, we use the vertical motion to find time, and then use that time to find the horizontal distance travelled. The vertical motion is a simple kinematics problem with constant acceleration:

$$\begin{aligned} y &= v_{y0}t + \frac{1}{2}a_yt^2 \\ 0.02 &= 0 + \frac{1}{2}(1.32 \times 10^{14})t^2 \\ t^2 &= \frac{0.04}{1.32 \times 10^{14}} \\ t &\approx 1.74 \times 10^{-8} \text{ s} \end{aligned}$$

Then, putting this into a horizontal motion equation:

$$\begin{aligned} x &= v_{x0}t \\ &= (9.4 \times 10^6)(1.74 \times 10^{-8}) \\ &\approx 0.16 \text{ m} \end{aligned}$$

So the electron travels a horizontal distance of about **0.16 meters** before hitting the upper plate.

Answer to Exercise 9 (on page 24)

$$B = \frac{(4\pi \times 10^{-7})(3)}{2\pi(0.02)} = 3 \times 10^{-5} \text{ T.}$$

Answer to Exercise 10 (on page 24)

$$F = qvB \sin 90^\circ = (2 \times 10^{-6})(4)(0.25) = 2 \times 10^{-6} \text{ N.}$$

Answer to Exercise 11 (on page 24)

$$F = (2)(0.5)(0.4) \sin 30^\circ = 0.2 \text{ N.}$$

Answer to Exercise 12 (on page 25)

No. The magnetic force is always perpendicular to the velocity, so it does no work. The speed, and therefore the kinetic energy, stays constant.

Answer to Exercise 13 (on page 25)

$r = \frac{mv}{|q|B} = \frac{(1.67 \times 10^{-27})(3 \times 10^5)}{(1.6 \times 10^{-19})(0.5)} \approx 6.3 \times 10^{-3} \text{ m}$, about 6 mm. An electron has the same size charge but about $\frac{1}{1836}$ the mass, so its circle is about 1836 times smaller (about 3.4 micrometers), and it curves the opposite way.

Answer to Exercise 14 (on page 31)

$$\frac{N_P}{N_S} = \frac{V_P}{V_S} = \frac{7,200}{240} = 30, \text{ so } N_P = 30 \times 50 = 1,500 \text{ turns.}$$

Answer to Exercise 15 (on page 33)

$$V_{\text{rms}} = \frac{340}{\sqrt{2}} \approx 240 \text{ V.}$$

Answer to Exercise 16 (on page 36)

The change in magnetic flux is $\Delta\Phi_B = A \Delta B = (0.2 \text{ m}^2)(5 \text{ T}) = 1 \text{ Wb}$. The induced emf is

$$\mathcal{E} = -N \frac{\Delta\Phi_B}{\Delta t} = -100 \cdot \frac{1 \text{ Wb}}{0.1 \text{ s}} = -1000 \text{ V}$$

The magnitude of the induced emf is 1000 V.

Answer to Exercise 17 (on page 36)

$$\mathcal{E} = (200)\frac{0.01}{0.1} = 20 \text{ V.}$$

Answer to Exercise 18 (on page 37)

$$\mathcal{E} = BLv = (0.3)(0.5)(4) = 0.6 \text{ V. The current is } I = \frac{\mathcal{E}}{R} = \frac{0.6}{2} = 0.3 \text{ A.}$$

Answer to Exercise 19 (on page 38)

Pulling the magnet away decreases the flux, so the coil opposes the decrease by trying to hold on to the magnet: the near end becomes a north pole, which attracts the retreating south pole.

Answer to Exercise 20 (on page 41)

$$P = (120)(5) = 600 \text{ W.}$$

Answer to Exercise 21 (on page 41)

Lifting at steady speed takes a force equal to the weight, $(10)(9.8) = 98 \text{ N}$. Mechanical power is force times speed: $(98)(0.2) = 19.6 \text{ W}$. The efficiency is $\frac{19.6}{240} \approx 8\%$. This winch wastes most of its power as heat, probably in a gearbox.

Answer to Exercise 22 (on page 43)

Both double: the peak emf becomes about 377 V, and the frequency becomes 120 Hz. That is why power plants must spin their generators at a precise, steady speed.

Answer to Exercise 23 (on page 49)

The force of gravity is $9.8 \times 45 = 441 \text{ newtons}$.

At any speed s , the force of wind resistance is $0.05 \times s^2 = 0.05s^2 \text{ newtons}$.

At terminal velocity, $0.05s^2 = 441$.

Solving for s , we get $s = \sqrt{\frac{441}{0.05}}$

Thus, terminal velocity should be about 94 m/s.

Answer to Exercise 24 (on page 49)

$30v^2 = 784$, so $v = \sqrt{\frac{784}{30}} \approx 5.1$ m/s.

Answer to Exercise 25 (on page 50)

About 8,700 m (the simulation gives a value very close to $\frac{300^2 \sin 72^\circ}{9.8} \approx 8,734$ m). The simulation and the formula agree when there is no drag, which is a good check that the code is right.



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