Contents

Chapter 1

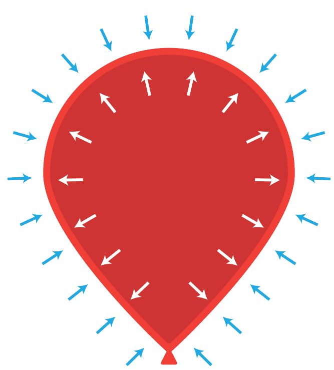
Atmospheric Pressure

The air you breathe is a blend of gases:

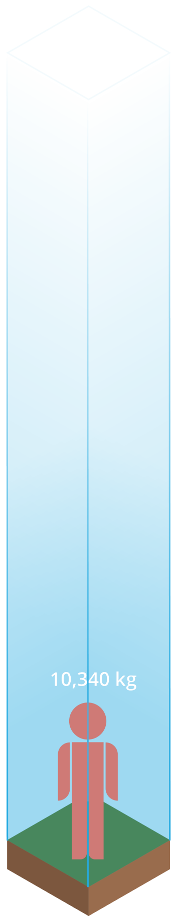
1. 78% nitrogen in the form of N_2
2. 21% oxygen in the form O_2
3. 1% other gases (mostly argon)

If you fill a balloon with helium (He), the helium will push against the interior of the balloon with a certain amount of pressure. The pressure is the same at every point in the interior of the balloon. Pressure, then, is force spread over some area. Force is commonly measured in newtons. Pressure is measured in *pascals*. A pascal is 1 newton per square meter.

We don't usually think about it, but the air outside the balloon is also pushing against the exterior of the balloon. We call this *barometric pressure* or *atmospheric pressure*, and it is caused by gravity pulling on the gas molecules above the balloon.



Imagine a square meter on the ground at sea level. Now imagine the column of air above it — reaching all the way to the top of the atmosphere.



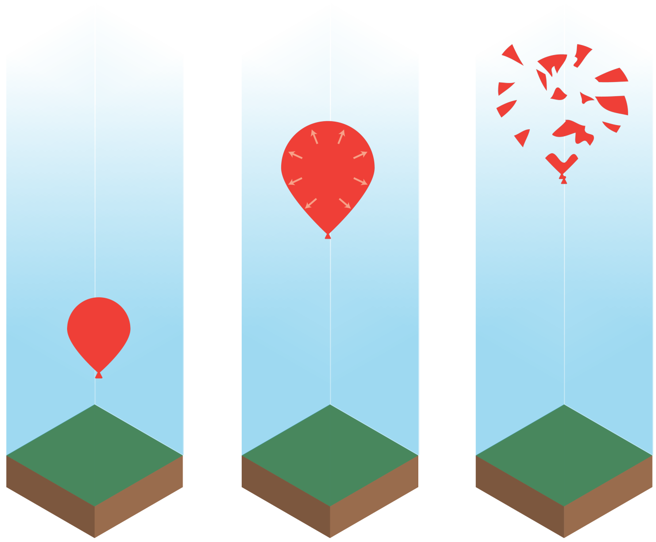
The air inside that column has a mass of about 10,340 kg. One kilogram on the earth experiences a gravitational force of 9.8 N. So the atmospheric pressure all around you is about 101,332 pascals. When dealing with such large numbers, we often use kilopascals. We'd say the barometric pressure at sea level is about 101.3 kPa.

That's a lot of pressure! Why doesn't your ribcage collapse crushing your lungs? The air *inside* your lungs is the same pressure as the air push on the outside of your rib cage.

This is why we go about our lives relatively oblivious to this huge force that is all around us — but you can see it sometimes. For example, if you suck the air out of a plastic bottle, the bottle will be crushed by the barometric pressure.

1.1 Altitude and Atmospheric Pressure

If you let go of the balloon, as it rises through this column there will be less and less air mass above it, and thus less and less atmospheric pressure on the outside of the balloon.



What would be the atmospheric pressure at h meters above sea level? Here is a handy formula for that:

$$p = 101,332 \times \left(1 - \left(2.25577 \times 10^{-5} \times h\right)\right)^{5.25588}$$

where p is the atmospheric pressure in pascals.

Exercise 1 Atmospheric Pressure

Working Space

You are thinking about riding your bicycle to the top of Mount Everest. You are worried when the atmospheric pressure outside the tire drops, the tire will fail. (I have had a tire fail before; It is very, very loud.)

Calculate the atmospheric pressure at the top of Mount Everest (9,144 meters above sea level).

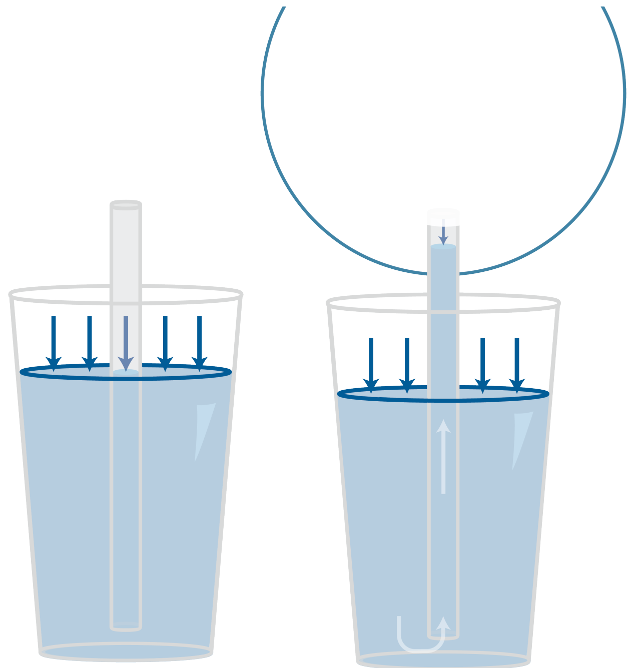
Answer on Page 35

1.2 How a Drinking Straw Works

When you suck on a drinking straw, why does the beverage rise? It is actually pushed by atmospheric pressure.

Before you put your mouth on the straw, the atmospheric pressure is pressing on the entire surface of the liquid (even inside the straw) evenly. Gravity pulls on the liquid making the surface level.

When you suck some air out of the straw, the pressure on the surface inside the straw drops. The atmospheric pressure on the surface outside the straw pushes into the straw and the beverage rises.

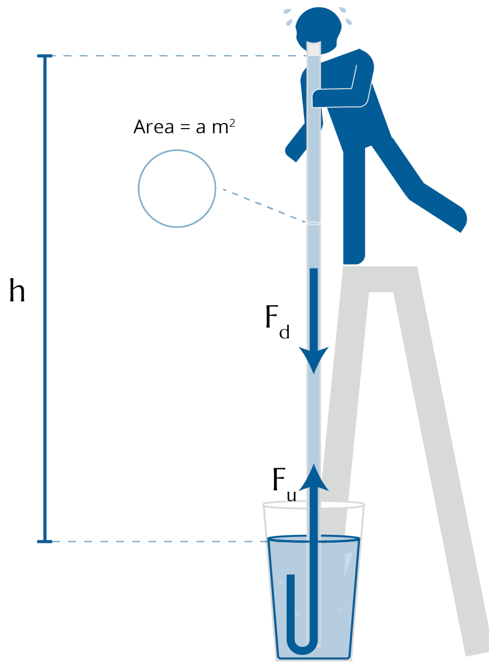


Of course, gravity is still trying to pull the liquid inside the straw back down. And for every inch that you lift the liquid in the straw, the force of gravity gets greater, demanding more suction.

1.2.1 The Longest Usable Straw

Assuming you are drinking water in a place with 100 kPa of atmospheric pressure, how high could you suck water with a perfect vacuum? That is, given a very, very long and very, very stiff drinking straw, if you created a pressure of 0 Pa inside, how far above the surface of the glass could you get the water?

Let's say a cross-section of the straw has an area of a square meters, and the very top of the column of water is h meters above the surface in the glass.



With how many newtons of force is the atmosphere pushing the water upward? 100 kPa = 100,000 newtons per square meter. So:

$$F_u = (100,000)a$$

With how many newtons of force is gravity pulling the water in the straw downward? The volume of the water is ah. A cubic meter of liquid water weights 1000 kg. The force of gravity is 9.8 Newtons per kg.

$$F_d = (ah)(1000)(9.8)$$

The water will stop rising when $F_u = F_d$. So, to find h, we substitute in:

$(100,000)a = (ah)(1000)(9.8)$

Notice that we can divide both sides by a getting:

$$h = \frac{100,000}{9,800} = 10.2 \text{ meters}$$

A perfect vacuum would only be able to drag the water up 10.2 meters.

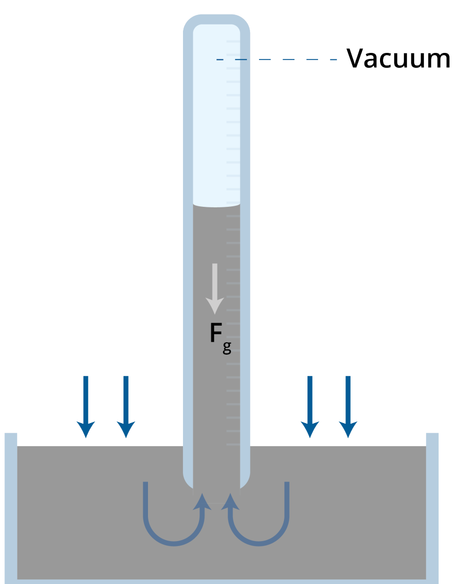
1.2.2 Millimeters Mercury

The density of mercury is 13,500 kg per cubic meter. How far up a straw would a perfect vacuum pull mercury?

$$h = \frac{100,000}{(9.8)(13,500)} \approx 0.756 \text{ meters}$$

When the atmospheric pressure is 100kPa, the mercury will rise 756 mm into a vacuum.

This is actually how scientists measure atmospheric pressure. They have a long glass tube filled with mercury. One end is closed off and pointed into the sky (exactly opposite the direction of gravity). The other end is placed into a dish of mercury. There are millimeter marks on the glass tube.



We use fluctuations in the atmospheric pressure to help us predict the weather. You might hear a weather nerd with a barometer in their house say, "Wow, the barometer has gone from 752 to 761 millimeters mercury in the last hour. A high-pressure system is moving in.”

1.3 How Siphon Works

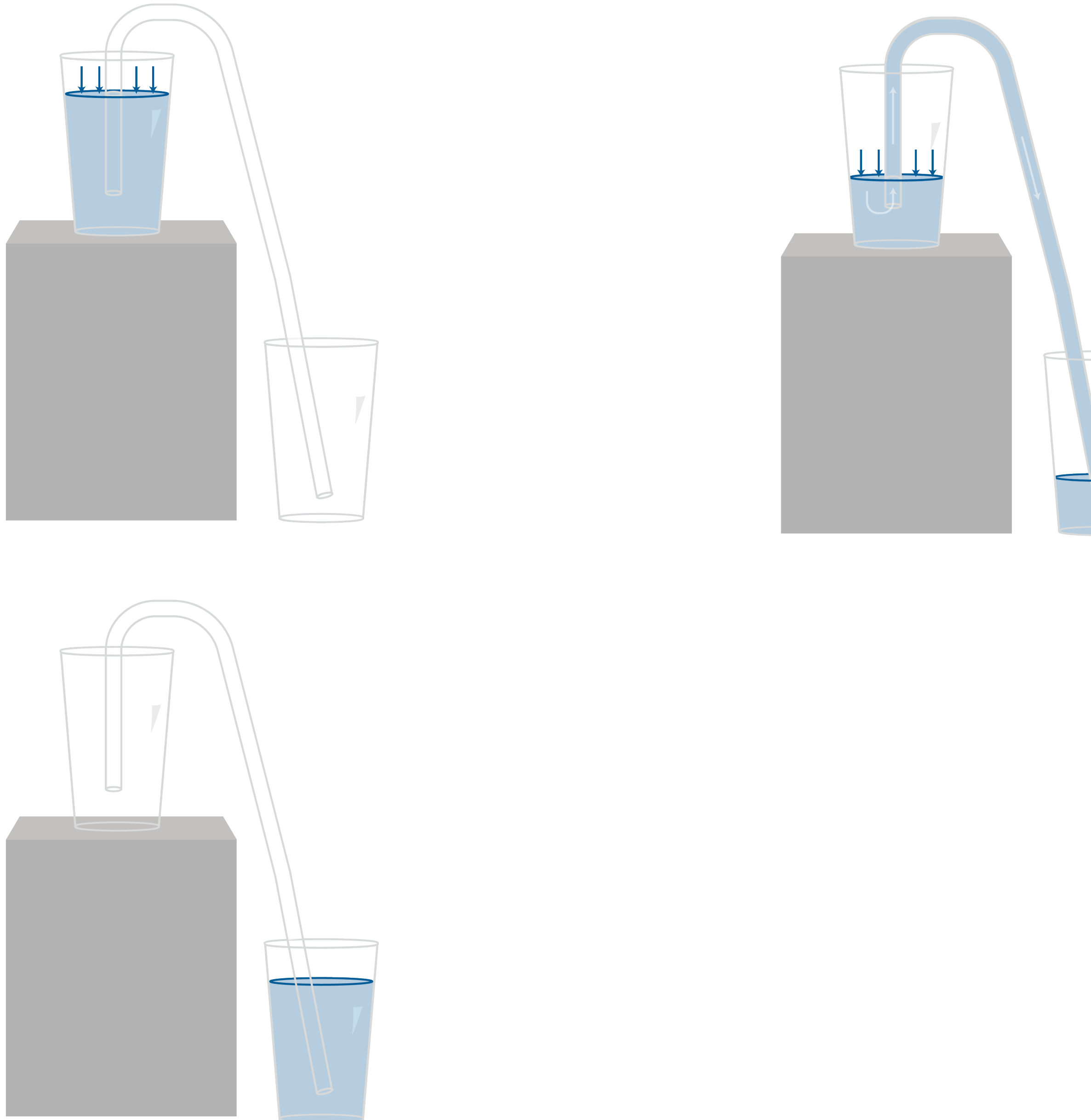
Let’s say you had two cups on a table; one is filled with water, one is empty. And you connected them with an empty U -shaped straw. Water will not crawl up the empty straw — the pressure on each end of the straw is the same, and crawling the straw (against gravity) would require energy.

However, what if the straw were filled with water? Then the force of gravity is pulling the water on each side of the hump in different directions. However, the side going into the empty glass pulls a little harder. That is sufficient to create enough suction to pull the water in the other side up and over the hump.

In turn, this will pull more water. Water will continue to flow from the full glass to the empty one until their surfaces are at the same level. At this point, the pull of gravity is the same on each side of the tube.

This is known as a *siphon*. Notice that atmospheric pressure makes the siphon possible. When the water on one side is pulled down by gravity, the atmospheric pressure pushes the other side up. If you were on a planet with plenty of gravity, but no atmosphere, a siphon wouldn’t work.

A siphon is especially useful when you want to get liquid out of a container that is too big to pour. For example, if you wanted to take the gasoline out of a car, you could use any flexible tubing to make a siphon. You would put the hose in the gas tank, suck enough gas up into the hose to get the siphon going, then put the hose into your jug. (If you ever do this, be extra careful not to suck any of the gasoline into your mouth; ingesting even a little bit of gasoline can make you incredibly sick, or even kill you.)



There are two rules to siphons:

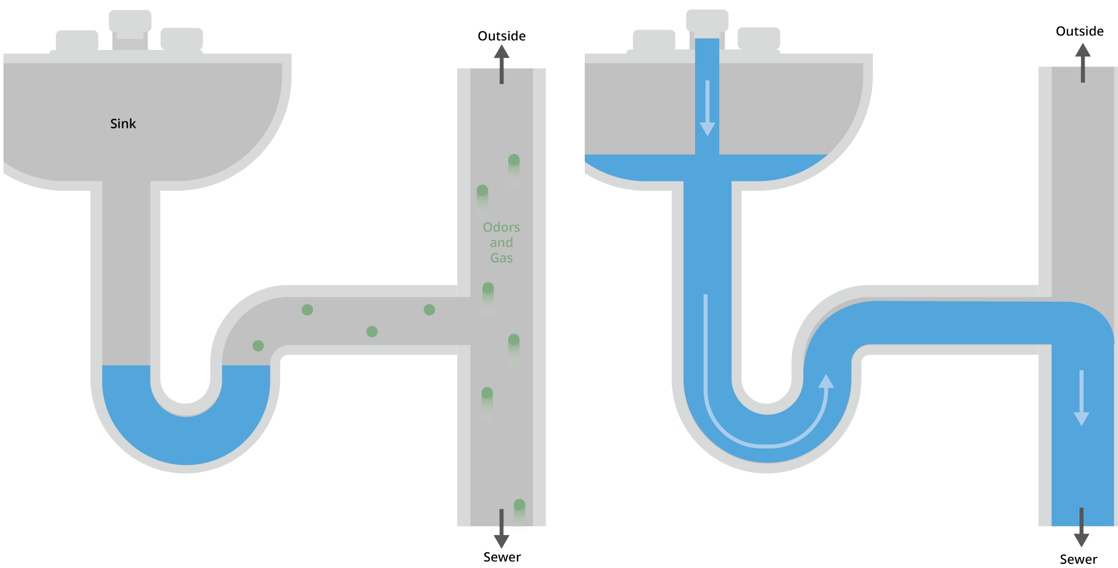
- The peak of the siphon needs to be low enough for atmospheric pressure to push the liquid that high. For water at sea-level, for example, the peak of the siphon can't be more than 10.2 meters above the surface of the source liquid.
- The tube has to carry the liquid to a lower level than the surface of the source liquid. If the destination end of the siphon is submerged, the surface of the liquid it is submerged in must be lower than the surface of the source liquid. If the destination

end of the siphon is not submerged, its opening must be lower than the surface of the source liquid.

As long as you follow these two rules, your siphons can be very creative. For example, every toilet has a siphon in it.

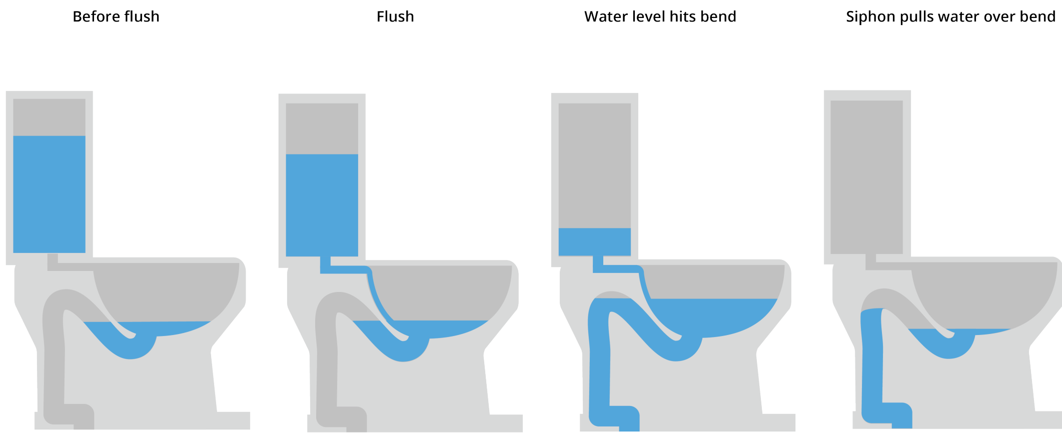
1.4 How a Toilet Works

Before we talk about toilets, you should know about P-traps. The drain from every sink, shower, and toilet in your house curves up and then down. This is known as a *P-trap*. The P-trap should always have some water in it. That keeps stinky (and flammable!) gases in the sewer from coming up and into your house.



If one of your fixtures (especially one that hasn't been used in a while) smells like raw sewage, run some water to ensure that the P-trap is full.

Now, back to the toilet! The drain in the bottom of the toilet is connected to a siphon into the sewer. The siphon is filled with air most of the time. However, when you flush the toilet, water rushes from the tank into the bowl, which also fills the siphon. Once the siphon is filled, it pulls water out of the toilet until air starts to enter the siphon. At that point, the water stops flowing.



The toilet tank is pretty simple: it has a float and valve that opens with the float is too low. So, anytime the water-level is too low, the value is open and slowly filling the tank. This means the tank is nearly always filled with a precise amount of water.

When you flush, a small door in the bottom of the tank is opened and the water rushes into the bowl. When the water is out, the door closes again so the tank can refill.

Chapter 2

Exponents

Let's take quick look at exponents. Ancient scientists started coming up with a lot of formulas that involved multiplying the same number several times. For example, if they knew that a sphere was r centimeters in radius, its volume in milliliters was

$$V = \frac{4}{3} \times \pi \times r \times r \times r$$

They did two things to make the notation less messy. First, they decided that if two numbers were written next to each other, the reader would assume that meant "multiply them". Second, they came up with the exponent, a little number that was lifted off the baseline of the text, that meant "multiply it by itself". For example 5^3 was the same as $5 \times 5 \times 5$.

Now, the formula for the volume of a sphere is written

$$V = \frac{4}{3}\pi r^3$$

Tidy, right? In an exponent expression like this, we say that r is *the base* and 3 is *the exponent*.

2.1 Identities for Exponents

What about exponents of exponents? What is $\left(5^3\right)^2$?

$$\left(5^3\right)^2 = (5 \times 5 \times 5)^2 = (5 \times 5 \times 5)(5 \times 5 \times 5) = 5^6$$

In general, for any a, b, and c:

$$\left(a^b\right)^c = a^{(bc)}$$

If you have $\left(5^3\right)\left(5^4\right)$, that is just $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$ or 5^7

The general rule is that for any a, b, and c

$$\left(a^b\right)\left(a^c\right) = a^{(b+c)}$$

Mathematicians *love* this rule, so we keep extending the idea of exponents to keep this rule true. For example, at some point, someone asked "What about 5^0?" According to the rule, 5^2 must equal $5^{(2+0)}$ which must equal $\left(5^2\right)\left(5^0\right)$. Thus, 5^2 must be 1. So mathematicians declared "Anything to the power of 0 is 1".

We don't typically assume that $0^0 = 1$. It is just too weird. So, we say that for any a not equal to zero,

$$a^0 = 1$$

What about $5^{(-2)}$? By our beloved rule, we know that $\left(5^{-2}\right)\left(5^5\right)$ must be equal to 5^3, right? So 5^{-2} must be equal to $\frac{1}{5^2}$.

We say that for any a not equal to zero and any b,

$$a^{-b} = \frac{1}{a^b}$$

This makes dividing one exponential expression by another (with the same base) easy:

$$\frac{a^b}{a^c} = a^{(b-c)}$$

We often say "cancel out" for this. Here, we can "cancel out" x^2:

$$\frac{x^5}{x^2} = x^3$$

What about $5^{\frac{1}{3}}$? By the beloved rule, we know that $5^{\frac{1}{3}}5^{\frac{1}{3}}5^{\frac{1}{3}}$ must equal 5^1. Thus, $5^{\frac{1}{3}} = \sqrt[3]{5}$.

We say that for any a and b not equal to zero and any c greater than zero,

$$a^{\frac{b}{c}} = a^b\sqrt[c]{a}$$

Before you go on to the exercises, note that the beloved rule demands a common base.

- We can combine these: $(5^2)(5^4) = 5^6$
- We cannot combine: $(5^2)(3^5)$

With that said, we note for any a,b, and c:

$$(ab)^c = (a^c)(b^c)$$

So, for example, if Iwe were asked to simplify $(3^4)(6^2)$, we would note that $6 = 2 \times 3$, so

$$\left(3^4\right)\left(6^2\right) = \left(3^4\right)\left(3^2\right)\left(2^2\right) = \left(3^6\right)\left(2^2\right)$$

If these ideas are new to you (or maybe you are just having trouble remembering them), watch the Khan Academy's **Intro to rational exponents** video at `https://youtu.be/lZfXc4nHooo`.

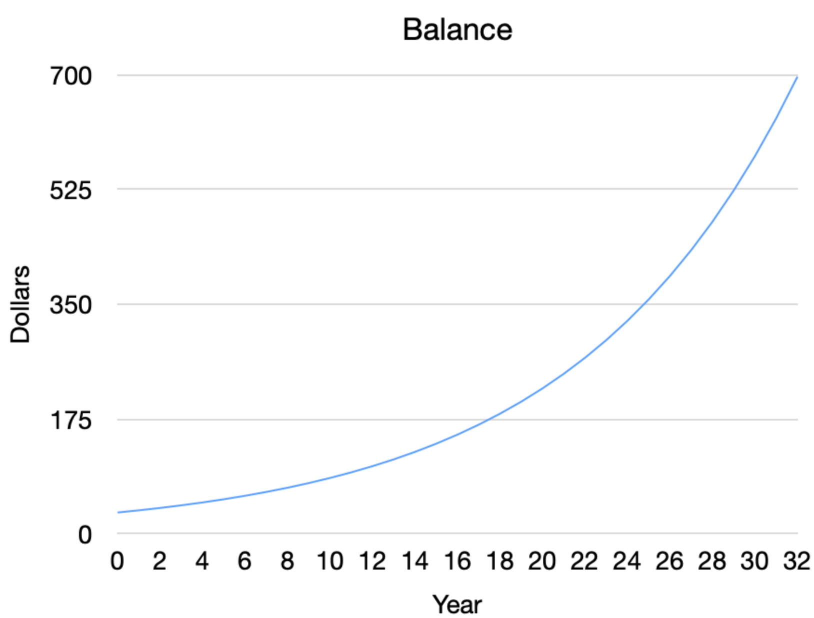
Chapter 3

Exponential Decay

In a previous chapter, we saw that an investment of P getting compound interest with an annual interest rate of r, grows exponentially. At the end of year t, your balance would be

$$P(1+r)^t$$

Because r is positive, this number grows as time passes. You get a nice exponential growth curve that looks something like this:



This is \$30 invested with a 10% annual interest rate. So, the formula for the balance after t years would be

$$(30)(1.1)^t$$

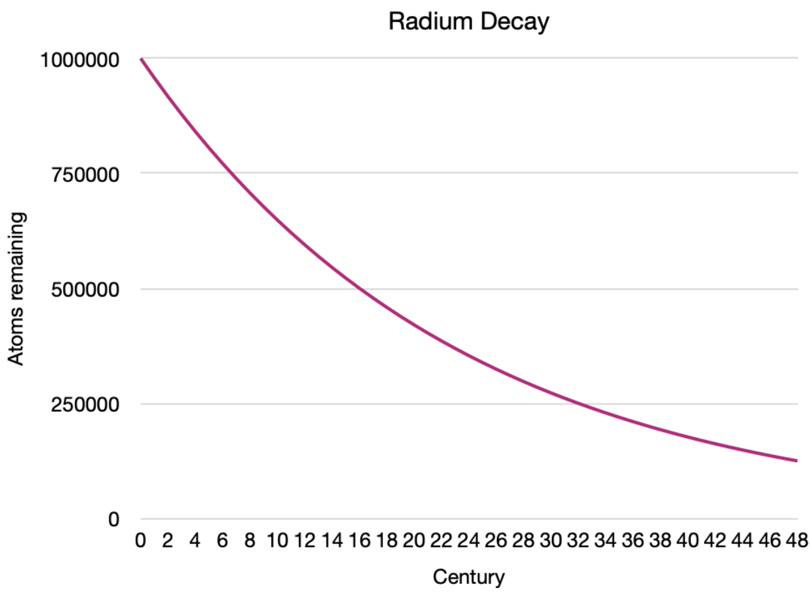
What if r were negative? This would be *exponential decay*.

3.1 Radioactive Decay

Until around 1970, there were companies making watches whose faces and hands were coated with radioactive paint. The paint usually contained radium. When a radium atom decays, it gives off some energy, loses two protons and two neutrons, and becomes becomes a different element (radon). Some of the energy given off is visible light. Thus, these watches glow in the dark.

How many of the radium atoms in the paint decay each century? About 4.24%.

Notice the quantity of atoms lost is proportional to the number of atoms you have. This is exponential decay. If we assume that we start with a million radium atoms, the number of atoms decreases over time like this:

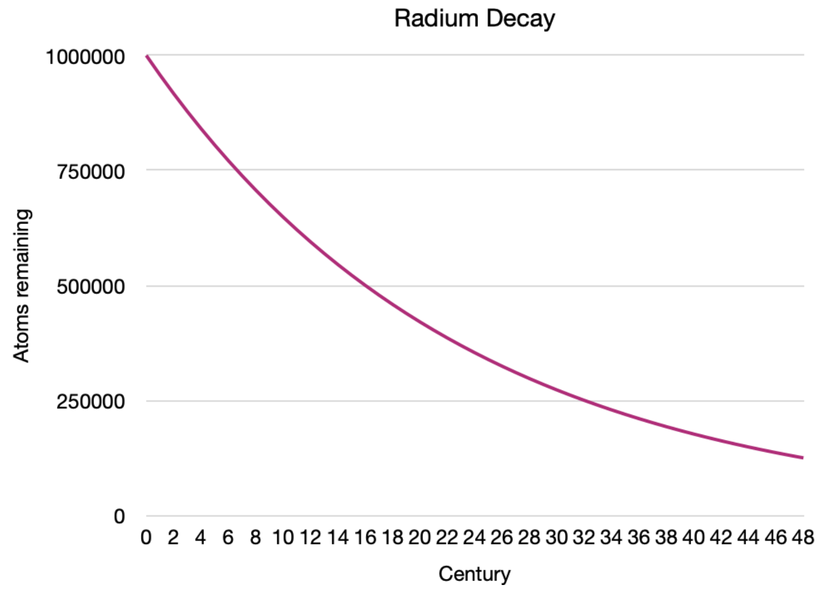


- We start with 1,000,000 atoms.
- At 16 centuries, we have only 500,000 (half as many) left.
- 16 centuries after that, we have only 250,000 (half again) left.
- 16 centuries after that, we have only 125,000 (half again) left.

A nuclear chemist would say that radium has a *half-life* of 1,600 years. Note that this means that if you bought a watch with glowing hands in 1960, it will be glowing half as brightly in the year 3560.

How do we calculate the amount of radium left at the end of century t? If you start with P atoms, at the end of the t-th century, you will have

$$P\,(1 - 0.0424)^{t}$$



This is exponential decay.

3.2 Model Exponential Decay

Let's say you get hired to run a company with 480,000 employees. Each year, 1/8 of your employees leave the company for one reason or another (retirement, quitting, etc.). For some reason, you never hire any new employees.

Make a spreadsheet that indicates how many of the original 480,000 employees will still be around at the end of each year for the next 12. Next, make a bar graph from that data.

Chapter 4

Logarithms

After the world created exponents, it needed the opposite. We could talk about the quantity $? = 2^3$, that is, "What is the product of 2 multiplied by itself three times?" We needed some way to talk about $2^? = 8$, that is, "2 to the what is 8?" This is why we developed the logarithm.

Here is an example:

$$\log_2 8 = 3$$

In English, you would say, "The logarithm base 2 of 8 is 3."

The base (2, in this case) can be any positive number. The argument (8, in this case) can also be any positive number.

Try this one: What is the logarithm base 2 of 1/16?

You know that $2^{-4} = \frac{1}{16}$, so $\log_2 \frac{1}{16} = -4$.

4.1 Logarithms in Python

Most calculators have pretty limited logarithm capabilities, but python has a nice `log` function that lets you specify both the argument and the base. Start python, import the math module, and try taking a few logarithms:

```
>>> import math
>>> math.log(8,2)
3.0
>>> math.log(1/16, 2)
-4.0
```

Let's say that a friend offers you 5% interest per year on your investment for as long as you want. You wonder, "How many years before my investment is 100 times as large?" You can solve this problem with logarithms:

```
>>> math.log(100, 1.05)
```

```
94.3872656381287
```

If you leave your investment with your friend for 94.4 years, the investment will be worth 100 times what you put in.

4.2 Logarithm Identities

The logarithm is defined this way:

$$\log_b a = c \iff b^c = a$$

Notice that the logarithm of 1 is always zero, and $\log_b b = 1$.

The logarithm of a product:

$$\log_b ac = \log_b a + \log_b c$$

This follows from the fact that $b^{a+c} = b^a b^c$. What about a quotient?

$$\log_b \frac{a}{c} = \log_b a - \log_b c$$

Exponents?

$$\log_b (a^c) = c \log_b a$$

Notice that because logs and exponents are the opposite of each other, they can cancel each other out:

$$b^{\log_b a} = a$$

and

$$\log_b (b^a) = a$$

4.3 Changing Bases

We mentioned that most calculators have pretty limited logarithm capabilities. Most calculators don't allow you to specify what base you want to work with. All scientific calculators have a button for "log base 10". So, you need to know how to use that button to get logarithms for other bases. Here is the change-of-base identity:

$$\log_b a = \frac{\log_c a}{\log_c b}$$

For example, if you wanted to find $\log_2 8$, you would ask the calculator for $\log_{10} 8$, then divide that by $\log_{10} 2$. You should get 3.

4.4 Natural Logarithm

When you learn about circles, you are told that the circumference of a circle is about 3.141592653589793 times its diameter. Because we use this unwieldy number a lot, we give it a name: We say "The circumference of a circle is π times its diameter."

There is a second unwieldy number that we will eventually use a great deal in solving problems. It is about 2.718281828459045 (but the digits actually go on forever, just like π). We call this number e. (We are not going to talk about why e is special quite yet, but we will soon.)

Most calculators have a button labeled "ln". That is the *natural logarithm* button. It takes the log in base e.

Similarly, in python, if you don't specify a base, the logarithm is done in base e:

```
>>> math.log(10)
2.302585092994046
>>> math.log(math.e)
1.0
```

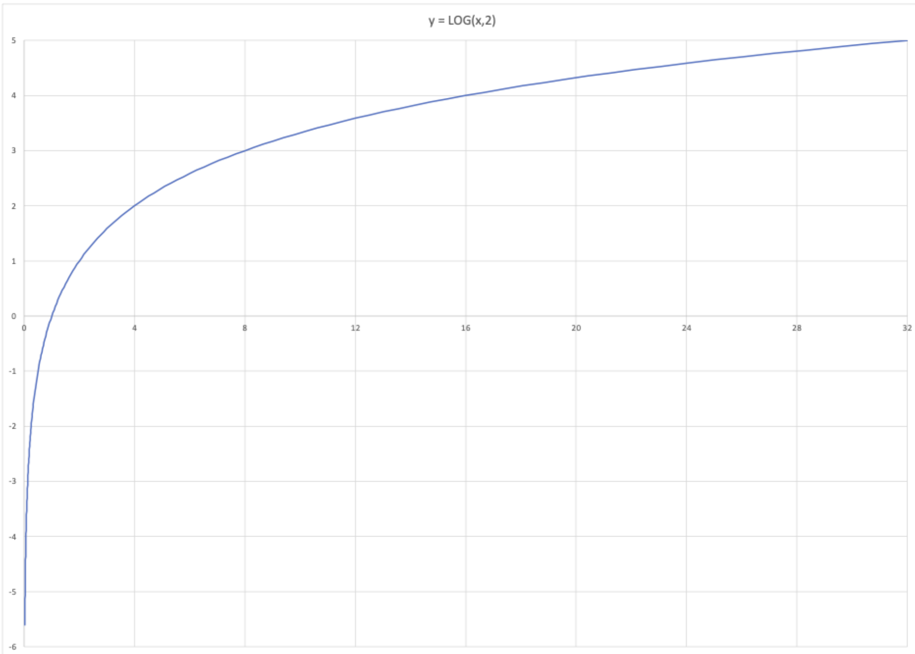
4.5 Logarithms in Spreadsheets

Spreadsheets have three log functions:

- `LOG` takes both the argument and the base. `LOG(8,2)` returns 3.
- `LOG10` takes just the argument and uses 10 as the base.

- `LN` takes just the argument and uses e as the base.

Here is a plot from a spreadsheet of a graph of $y = \mathrm{LOG}(x, 2)$.



Spreadsheets also have the function `EXP(x)`, which returns e^x. For example, `EXP(2)` returns 7.38905609893065.

Chapter 5

Trigometric Functions

As mentioned in an earlier chapter, in a right triangle where one angle is θ, the sine of θ is the length of the side opposite θ divided by the length of the hypotenuse.

The sine function is defined for any real number. We treat that real number θ as an angle, we draw a ray from the origin out to the unit circle. The y value of that point is the sine. For example, the $\sin(\frac{4\pi}{3})$ is $-\sqrt{3}/2$ (see figure 5.1).

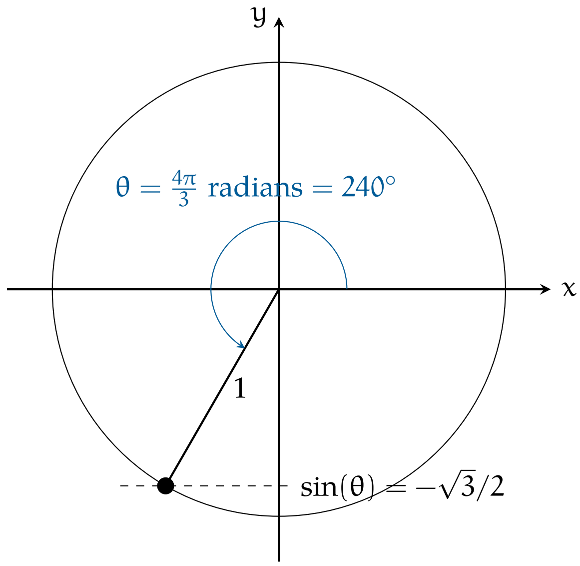


Figure 5.1: $\sin\frac{4\pi}{3} = \frac{-\sqrt{3}}{2}$

(Note that in this section, we will be using radians instead of degrees unless otherwise noted. While degrees are more familiar to most people, engineers and mathematicians nearly always use radians when solving problems. Your calculator should have a radians mode and a degrees mode; you want to be in radians mode.)

Similarly, we define cosine using the unit circle. To find the cosine of θ, we draw a ray from the origin at the angle θ. The x component of the point where the ray intersects the unit circle is the cosine of θ (shown in figure 5.2).

From this description, it is easy to see why $\sin(\theta)^2 + \cos(\theta)^2 = 1$. They are the legs of a right triangle with a hypotenuse of length 1.

It should also be easy to see why $\sin(\theta) = \sin(\theta + 2\pi)$: Each time you go around the circle, you come back to where you started.

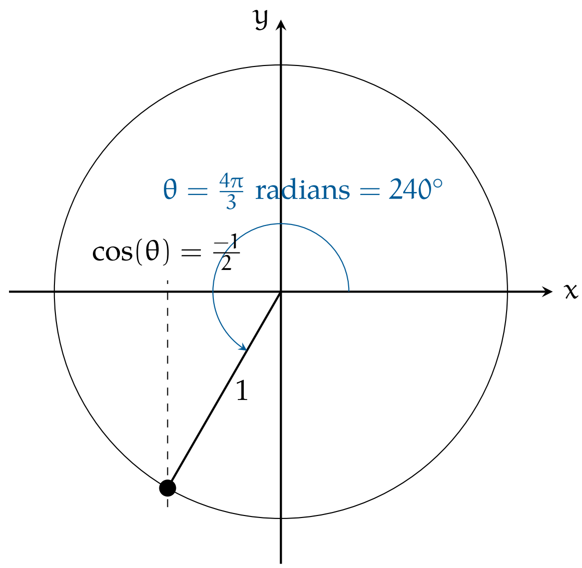
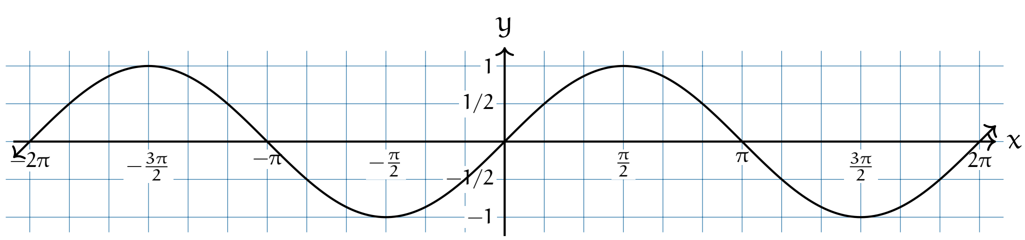


Figure 5.2: $\cos\frac{4\pi}{3} = \frac{-1}{2}$

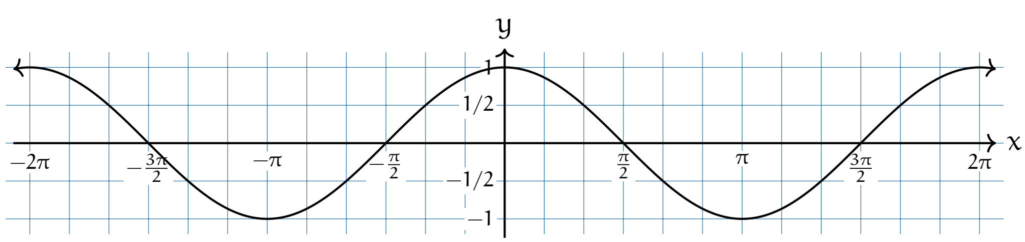
Can you see why $\cos(\theta) = \sin(\theta + \pi/2)$? Turn the picture sideways.

5.1 Graphs of sine and cosine

Here is a graph of $y = \sin(x)$:



It looks like waves, right? It goes forever to the left and right. Remembering that $\cos(\theta) = \sin(\theta + \pi/2)$, we can guess what the graph of $y = \cos(x)$ looks like:

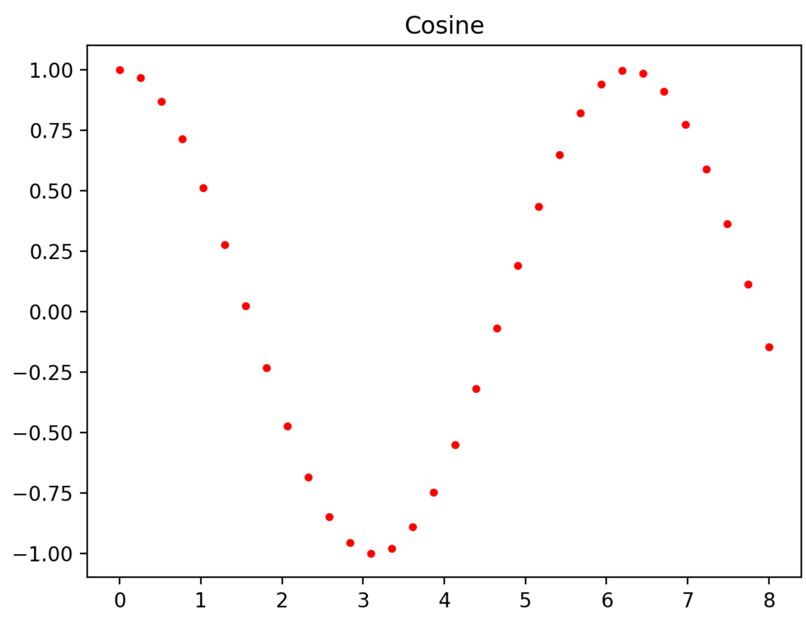


5.2 Plot cosine in Python

Create a file called `cos.py`:

```
import numpy as np
import matplotlib.pyplot as plt

until = 8.0

# Make a plot of cosine
thetas = np.linspace(0, until, 32)
cosines = []
for theta in thetas:
    cosines.append(np.cos(theta))

# Plot the data
fig, ax = plt.subplots()
ax.plot(thetas, cosines, 'r.', label="Cosine")
ax.set_title("Cosine")
plt.show()
```

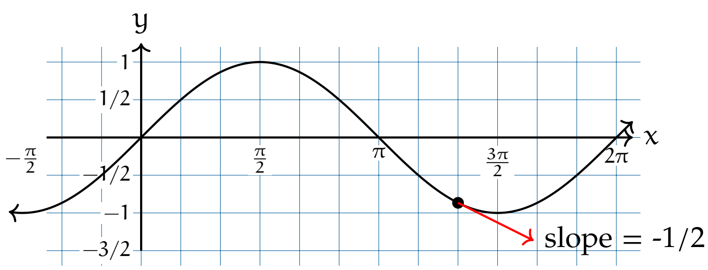
This will plot 32 points on the cosine wave between 0 and 8. When you run it, you should see something like this:



5.3 Derivatives of trigonometic functions

Here is a wonderful property of sine and cosine functions: At any point θ, the slope of the sine graph at θ equals $\cos(\theta)$.

For example, we know that $\sin(4\pi/3) = -(1/2)\sqrt{3}$ and $\cos(4\pi/3) = -1/2$. If we drew a line tangent to the sine curve at this point, it would have a slope of -1/2:



We say "The derivative of the sine function is the cosine function."

Can you guess the derivative of the cosine function? For any θ, the slope of the graph of the $\cos(\theta)$ is $-\sin(\theta)$.

Exercise 2 Derivatives of Trig Functions Practice 1

Use the limit definition of a derivative to show that $\frac{d}{dx}\cos x = -\sin x$

Working Space

Answer on Page 35

The derivatives of all the trigonometric functions are presented below:

$\frac{d}{dx}\sin x = \cos x$	$\frac{d}{dx}\csc x = -\csc x \cdot \cot x$
$\frac{d}{dx}\cos x = -\sin x$	$\frac{d}{dx}\sec x \sec x \cdot \tan x$
$\frac{d}{dx}\tan x = \sec^2 x$	$\frac{d}{dx}\cot x = -\csc^2 x$

Example: Find the derivative of $f(x)$ if $f(x) = x^2 \sin x$ **Solution**: Using the product rule, we find that:

$$\frac{d}{dx}f(x) = (x^2)\frac{d}{dx}(\sin x) + (\sin x)\frac{d}{dx}(x^2)$$

Taking the derivatives:

$$= x^2(\cos x) + 2x(\sin x)$$

Exercise 3 Derivatives of Trig Functions 2

Find the derivative of the following functions:

1. $f(x) = \frac{\sec x}{1+\tan x}$
2. $y = \sec t \tan t$
3. $f(\theta) = \frac{\theta}{4-\tan\theta}$
4. $f(t) = 2\sec t - \csc t$
5. $f(\theta) = \frac{\sin\theta}{1+\cos\theta}$
6. $f(x) = \sin x \cos x$

Working Space

Answer on Page 36

5.4 A weight on a spring

Let's say you fill a rollerskate with heavy rocks and attach it to the wall with a stiff spring. If you push the skate toward the wall and release it, it will roll back and forth. Engineers would say "The skate will oscillate."

Intuitively, you can probably guess:

- If the spring is stronger, the skate will oscillate more times per minute.
- If the rocks are lighter, the skate will oscillate more times per minute.

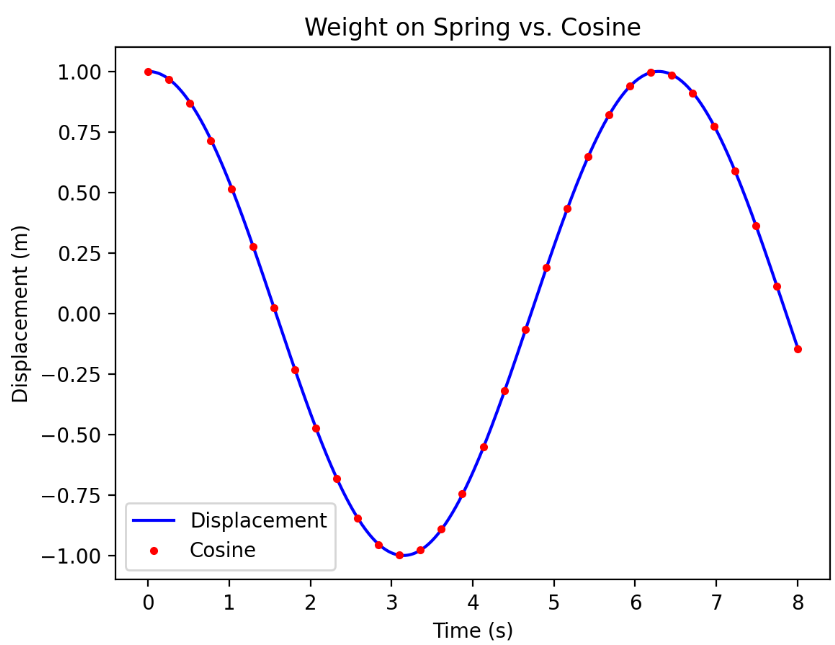
The force that the spring exerts on the skate is proportional to how far its length is from its relaxed length. When you buy a spring, the manufacturer advertises its "spring rate", which is in pounds per inch or newtons per meter. If a spring has a rate of 5 newtons per meter, that means that if you stretch or compress it 10 cm, it will push back with a force of 0.5 newtons. If you stretch or compress it 20 cm, it will push back with a force of 1 newton.

Let's write a simulation of the skate-on-a-spring. Duplicate `cos.py`, and name the new copy `spring.py`. Add code to implement the simulation:

```
import numpy as np
import matplotlib.pyplot as plt

until = 8.0

# Constants
mass = 100 # kg
spring_constant = -1 # newtons per meter displacement
time_step = 0.01 # s

# Initial state
displacement = 1.0 # height above equilibrium in meters
velocity = 0.0
time = 0.0 # seconds

# Lists to gather data
displacements = []
times = []

# Run it for a little while
while time <= until:
    # Record data
    displacements.append(displacement)
    times.append(time)

    # Calculate the next state
    time += time_step
    displacement += time_step * velocity
    force = spring_constant * displacement
    acceleration = force / mass
    velocity += acceleration

# Make a plot of cosine
thetas = np.linspace(0, until, 32)
cosines = []
for theta in thetas:
    cosines.append(np.cos(theta))

# Plot the data
fig, ax = plt.subplots()
ax.plot(times, displacements, 'b', label="Displacement")
ax.plot(thetas, cosines, 'r.', label="Cosine")
```

```
ax.set_title("Weight on Spring vs. Cosine")
ax.set_xlabel("Time (s)")
ax.set_ylabel("Displacement (m)")
ax.legend()
plt.show()
```

When you run it, you should get a plot of your spring and the cosine graph on the same plot.



The position of the skate is following a cosine curve. Why?

Because a sine or cosine waves happen whenever the acceleration of an object is proportional to -1 times its displacement. Or in symbols:

$$a \propto -p$$

where a is acceleration and p is the displacement from equilibrum.

Remember that if you take the derivative of the displacement, you get the velocity. And if you take the derivative of that, you get acceleration. So, the weight on the spring must follow a function f such that

$$f(t) \propto -f''(t)$$

Remember that the derivative of the $\sin(\theta)$ is $\cos(\theta)$.

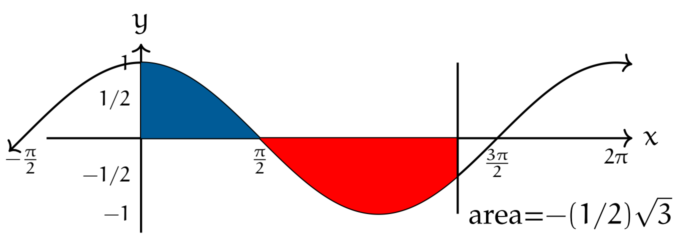
And the derivative of the $\cos(\theta)$ is $-\sin(\theta)$

These sorts of waves have an almost-magical power: Their acceleration is proportional to -1 times their displacement.

Thus, sine waves of various magnitudes and frequencies are ubiquitous in nature and technology.

5.5 Integral of sine and cosine

If we take the area between the graph and the x axis of the cosine function (and if the function is below the x axis, it counts as negative area), from 0 to $4\pi/3$, we find that it is equal to $-(1/2)\sqrt{3}$.



We say "The integral of the cosine function is the sine function."

5.5.1 Integrals of Trig Functions Practice

Exercise 4

Evaluate the following integrals:

1. $\int \sec x \tan x \, dx$

Working Space

Answer on Page 36

Appendix A

Answers to Exercises

Answer to Exercise 1 (on page 6)

$$p = 101,332 \times \left(1 - 2.25577 \times 10^{-5} \times h\right)^{5.25588}$$

and $h = 9,144$. Thus,

$$p \approx 30.1\text{kPa}$$

Answer to Exercise 2 (on page 30)

We start by writing out the limit:

$$\frac{d}{dx}\cos x = \lim_{h \to 0} \frac{\cos x + h - \cos x}{h}$$

Applying the sum formula for $\cos(x + h)$, we get:

$$= \lim_{h \to 0} \frac{\cos x \cos h - \sin x \sin h - \cos x}{h}$$

Rearranging to group the $\cos x$ and applying the Difference Rule:

$$= \lim_{h \to 0} \frac{\cos x \cos h - \cos x}{h} - \lim_{h \to 0} \frac{\sin x \sin h}{h}$$

Applying the Constant Multiple Rule:

$$= \cos x \lim_{h \to 0} \frac{\cos h - 1}{h} - \sin x \lim_{h \to 0} \frac{\sin h}{h}$$

Recalling that $\lim_{h\to 0} \frac{\sin h}{h} = 1$,

$$= \cos x \lim_{h\to a} \frac{\cos h - 1}{h} - \sin x \cdot 1$$

Recalling that $\lim_{h\to 0} \frac{\cos h - 1}{h} = 0$:

$$= \cos x \cdot 0 - \sin x = -\sin x$$

Therefore, $\frac{d}{dx} \cos x = -\sin x$

Answer to Exercise 3 (on page 31)

1. $\frac{\sec x(\tan x - 1)}{(1+\tan x)^2}$
2. $\sec t[\sec^2 t + \tan^2 t]$
3. $\frac{4-\tan\theta+\theta\sec^2\theta}{(4-\tan\theta)^2}$
4. $2\sec t \tan t + \csc t \cot t$
5. $\frac{2}{(1+\cos\theta)^2}$
6. $\cos^2 x - \sin^2 x$

Answer to Exercise 4 (on page 34)

1. $\sec x + C$

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