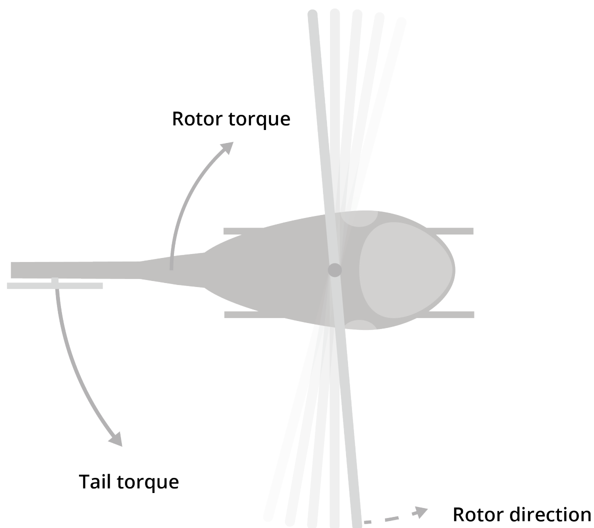
Contents

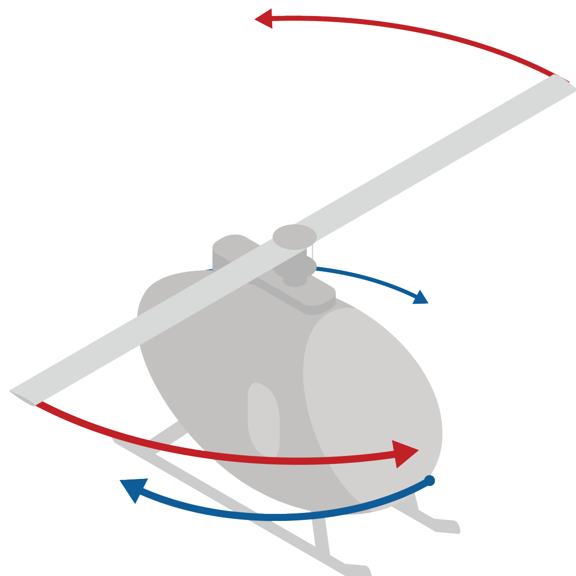
Chapter 1

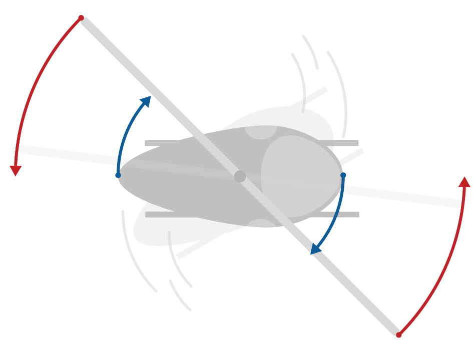
Helicopters

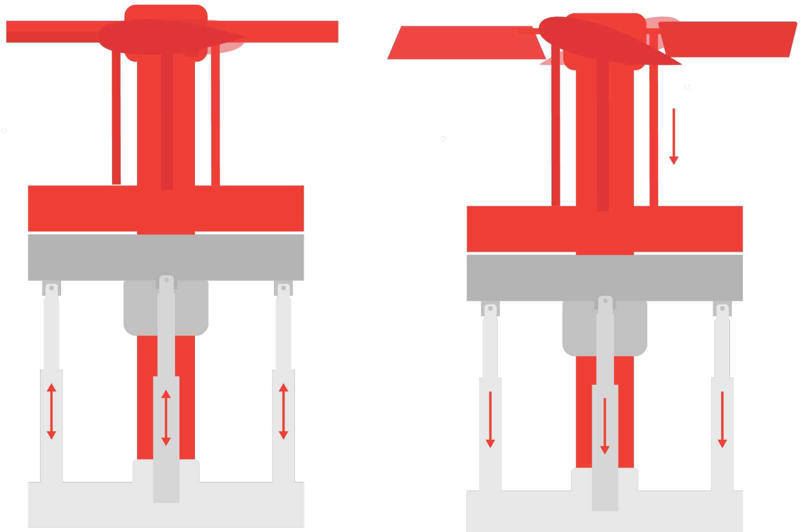
Instead of a fixed wing, the ”wing” rotates around to produce lift.



Sequence: Rotor/Tail, Show how the tail rotor works, then show how the control works for the main rotor







Still hard to see
what's going on in
the red hubs

Collective

Chapter 2

Free Body Diagrams

Now that you've mastered modeling *how things move*, you can learn to model *why things move*. Recall Newton's Second Law:

$$F = ma$$

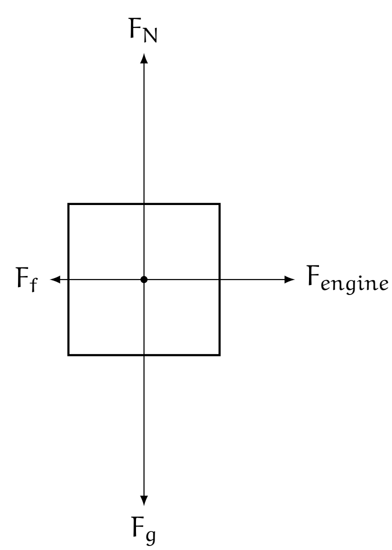
This is a simplification: what Newton's Second Law says in total is "the acceleration of a body is directly proportional to the *net force* acting on the body and inversely proportional to the mass of the body." Mathematically, that would be:

$$a = \frac{\Sigma F}{m}$$

Where ΣF is the vector sum of all the individual forces acting on the body. This sum is also called the *net force*. To visualize the magnitude and direction of the net force, it can be very helpful to draw a free body diagram.

2.1 Interpreting Free Body Diagrams

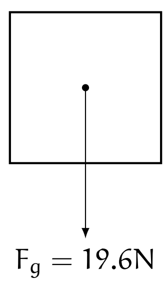
Before you learn to draw your own free body diagrams (FBDs), let's examine and analyze a few. Here is a FBD for an accelerating car:



We can consider the x and y axes separately. On the y-axis, we see that the weight (F_g) is the same magnitude as the normal force (F_N). This means in the y direction, the forces are balanced, and thus we do not expect to see acceleration in the y direction. This makes intuitive sense: if you are accelerating your car on a flat road, your car does not go up into the air. How about the x direction? Here we see friction (F_f) pulling the car backwards while the engine is pushing the car forwards. In this case, the forces are *not* balanced. See how the F_{engine} arrow is longer than the F_f arrow? This means the forward force from the engine is *greater in magnitude* than the backward force of F_f. Therefore, the *net force* in the x direction is to the right, and the car will accelerate to the right.

2.2 Drawing Free Body Diagrams

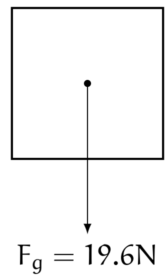
A free body diagram consists of the object, usually represented as a simple square, with various arrows representing the forces acting on the object. The direction and relative lengths of the arrows show the direction and relative strength of the forces. Here is a free body diagram for a 2-kg hammer falling through the air on Earth:



Both methods are useful; choose the one that you prefer.

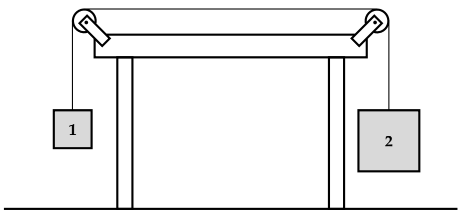
2.2.1 Objects At Rest

If an object is at rest, then its acceleration must be $0\frac{m}{s^2}$. By Newton's Second Law, the *net force* acting on that object must also be zero. Does this mean there are no forces acting on the object? NO! It means all the forces are *balanced*: for every upwards force, there is an equal downwards force, and so on. Consider the same 2-kg hammer, but this time it is sitting on a table. We know gravity must be acting on the hammer, so let's begin with that:


$F_g = 19.6N$

Exercise 1 Blocks on a Table

[This exercise was originally presented as a free-response question on the 2015 AP Physics 1 exam.] Two blocks are connected with a string and two pulleys over a table, as shown in the figure. Block 1 has a mass of 3 kg and Block 2 has a mass of 9 kg. The blocks are released from rest. Treat the string as massless and the pulleys as massless and frictionless when answering the questions below.

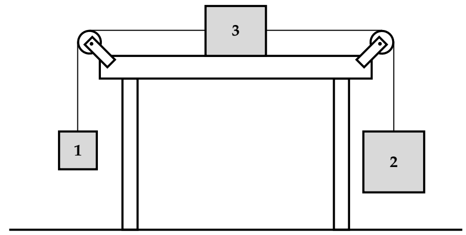


1. Draw free body diagrams for both blocks. Correctly show the relative magnitude of the forces using the relative lengths of the vectors.
2. Before doing any calculations, describe each block's acceleration as (a) positive or negative, (b) having a magnitude greater or less than g. Take the upward direction as positive.
3. Calculate the acceleration of each block. Take the upward direction as positive and express your answer in terms of g.

Working Space

Answer on Page 31

Now here's an interesting question: what happens if we add another mass to the system? Imagine that a third block with a mass of 15 kg is laid on the table from the previous exercise and the string is attached to each side, as shown below.



We could redraw free body diagrams for all three blocks, noting that the tensions on either side of Block 3 will be different. Then, we would have another system of equations to solve and we could find the acceleration. Or, we could think about the entire system. Originally, the net force on the two-block system was 58.8 Newtons (the weight of the larger block minus the weight of the smaller block). And the mass of the entire two-block system was 12 kilograms. We can apply Newton's Second Law:

$$a = \frac{58.8\text{N}}{12\text{kg}} = 4.9\frac{m}{s^2} = \frac{1}{2}g$$

(Notice, this is the same answer we found in the previous exercise). What changes when we add a third, 15-kg block? The net force on the system doesn't change: the table pushes back up on the third block with a force equal to the block's weight. However, the total mass of the system has changed: it's increased. Now the total mass is 27 kg, and we can again apply Newton's Second Law:

$$a = \frac{58.8\text{N}}{27\text{kg}} \approx 2.18\frac{m}{s^2}$$

This answer makes sense: the mass of the system has increased while the net force acting on the system has stayed the same, resulting in a slower acceleration.

Chapter 3

Introduction to Spreadsheets

Spreadsheets are the perfect tool for many real world problems. In this chapter, you will be introduced to how to use a spreadsheet. There are numerous spreadsheet programs, such as Google Sheets, Microsoft Excel, Apple Numbers, and OpenOffice Calc. All of them are relatively similar. This instruction will use Google Sheets, but if you are using one of the others, you should be able to follow along.

The first spreadsheet program (VisiCalc) was introduced in 1979 as a tool for finance people to play "what if" games. For example, a company might make a spreadsheet that told them how much more profit they would make if they changed from using an expensive metal to using a cheaper alloy.

In honor of this history, let's start by studying a business question: You have a friend who dreams of quitting her job to become a cooper. (A cooper makes barrels that are used for aging wine and whiskey.) According to her:

- It costs $45 dollars in materials to build one barrel.
- A barrel sells for $100 dollars.
- The workshop/warehouse she wants to rent costs $2000 per month.
- Taxes take 20% of her profits.
- She needs to make $4000 monthly after taxes.

She has asked you, "How many barrels do I need to make each month?"

3.1 Solving It Symbolically

Many problems can be solved two ways: symbolically or numerically. To solve this problem symbolically, you would write out the facts as equations or inequalities, then do symbol manipulations until you ended up with an answer. In this case, you would let b be the number of barrels and create the following inequality:

$$(1.0 - 0.2)\,(b(100 - 45) - 2000) \geq 4000$$

You would simplify it:

$$(0.8)(55b - 2000) \geq 4000$$

And simplify it more:

$$44b - 1600 \geq 4000$$

If that is true, then:

$$44b \geq 5600$$

And if that is true, then:

$$b \geq \frac{1400}{11}$$

1400/11 is about 127.27, so she needs to make and sell 128 barrels each month.

That is a perfect answer, and we didn't need a spreadsheet at all. However:

- As problems get larger and more realistic, it gets much more difficult to solve them symbolically.
- As soon as you say "Yes, you need to make and sell 128 barrels each month." Your friend will ask "What if I make and sell 200 barrels? How much money will I make then?"

So, we use a spreadsheets to solve the problem numerically.

3.2 Solving It Numerically (with a spreadsheet)

Let's get back to our example. Put labels in the A column:

- Barrels produced (per month)
- Materials cost (per barrel)
- Sale price (per barrel)
- Pre-tax earnings (per month)

- Taxes (per month)
- Take home pay (per month)

Format them any way you like. It should look something like this:

	A
1	**Barrels Produced** (per month)
2	**Materials cost** (per barrel)
3	**Sale price** (per barrel)
4	**Rent** (per month)
5	**Pretax Earnings** (per month)
6	**Taxes** (per month)
7	**Take home pay** (per month)
8	

In the B column, the first four cells are values (not formulas):

- 115 formatted as a number with no decimal point
- 45 formatted as currency
- 100 formatted as currency
- 2000 formatted as currency

It should look something like this:

	A	B
1	**Barrels Produced** (per month)	115
2	**Materials cost** (per barrel)	$45.00
3	**Sale price** (per barrel)	$100.00
4	**Rent** (per month)	$2,000.00

The next three cells in the B column will have formulas:

- B1 * (B3 - B2) - B4
- 0.2 * B5
- B5 - B6

It should look something like this:

	A	B
1	**Barrels Produced** (per month)	115
2	**Materials cost** (per barrel)	$45.00
3	**Sale price** (per barrel)	$100.00
4	**Rent** (per month)	$2,000.00
5	**Pretax Earnings** (per month)	$4,325.00
6	**Taxes** (per month)	$865.00
7	**Take home pay** (per month)	$3,460.00
8		

Now you can share this spreadsheet with your friend, and she can put different values into the cells for what-if games, such as "If I can get my materials cost down to $42 per barrel, what happens to my take home pay?"

Sometimes it is nice to show a range of values for a variable or two. In this case, it might be nice to show your friend what the numbers look like if she produces 115, 120, 125, 130, 135, or 140 barrels per month.

We have one column, and now we need six. How do we duplicate cells?

1. Click B1 to select it, then shift-click on B7 to select all seven cells.
2. Copy them. (Depending on what program you are using, there is likely a menu item for this.)
3. Click C1 to select it
4. Paste them.

	A	B	C
1	**Barrels Produced** (per month)	115	115
2	**Materials cost** (per barrel)	$45.00	$45.00
3	**Sale price** (per barrel)	$100.00	$100.00
4	**Rent** (per month)	$2,000.00	$2,000.00
5	**Pretax Earnings** (per month)	$4,325.00	$4,325.00
6	**Taxes** (per month)	$865.00	$865.00
7	**Take home pay** (per month)	$3,460.00	$3,460.00

We want the first cell in the new column to be 120. You could just type in 120, but let's do something more clever. Put a formula into that cell: $= B1 + 5$. Now, the cell should show 120.

Why did we put in a formula? When we duplicate this column, this cell will always have 5 more barrels than the cell to its left.

Next, let's duplicate the second column a few times. The easy way to do this is to select the cells as you did before, then drag the lower-right corner to the right until column G is in the selection. When you end the drag, the copies will appear:

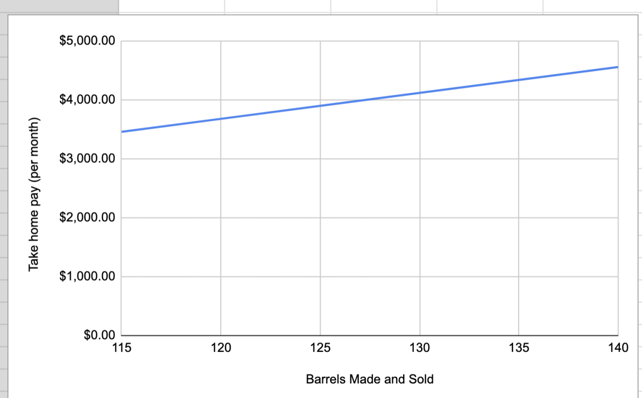
	A	B	C	D	E	F	G
1	**Barrels Produced** (per month)	115	120	125	130	135	140
2	**Materials cost** (per barrel)	$45.00	$45.00	$45.00	$45.00	$45.00	$45.00
3	**Sale price** (per barrel)	$100.00	$100.00	$100.00	$100.00	$100.00	$100.00
4	**Rent** (per month)	$2,000.00	$2,000.00	$2,000.00	$2,000.00	$2,000.00	$2,000.00
5	**Pretax Earnings** (per month)	$4,325.00	$4,600.00	$4,875.00	$5,150.00	$5,425.00	$5,700.00
6	**Taxes** (per month)	$865.00	$920.00	$975.00	$1,030.00	$1,085.00	$1,140.00
7	**Take home pay** (per month)	$3,460.00	$3,680.00	$3,900.00	$4,120.00	$4,340.00	$4,560.00

Nice, right? Now your friend can easily see how many barrels correspond to how much take-home pay. But do you know what would be even more helpful? A graph.

3.3 Graphing

Graphing is a little different on every different platform. Here is what you want the graph to look like:

	A	B	C	D	E	F	G
1	**Barrels Produced** (per month)	115	120	125	130	135	140
2	**Materials cost** (per barrel)	$45.00	$45.00	$45.00	$45.00	$45.00	$45.00
3	**Sale price** (per barrel)	$100.00	$100.00	$100.00	$100.00	$100.00	$100.00
4	**Rent** (per month)	$2,000.00	$2,000.00	$2,000.00	$2,000.00	$2,000.00	$2,000.00
5	**Pretax Earnings** (per month)	$4,325.00	$4,600.00	$4,875.00	$5,150.00	$5,425.00	$5,700.00
6	**Taxes** (per month)	$865.00	$920.00	$975.00	$1,030.00	$1,085.00	$1,140.00
7	**Take home pay** (per month)	$3,460.00	$3,680.00	$3,900.00	$4,120.00	$4,340.00	$4,560.00

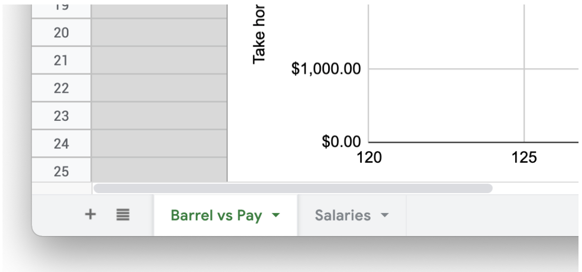


On Google Sheets:

1. Select cells B7 through G7.
2. Choose the menu item Insert -> Chart.
3. Choose the chart type (Line)
4. Add the X-axis to be B1 through G1.
5. Under the Customize tab, Set the label for the X-axis to be “Barrels Made and Sold”.
6. Delete the chart title (which is the same as the Y-axis label).

3.4 Other Things You Should Know About Spreadsheets

Your spreadsheet document can have several "Sheets". Each has its own grid of cells. The sheet has a name; usually, you call it something like "Salaries". When you need to use a value from the "Salaries" sheet in another sheet, you can specify "Salaries!A2" — that is, cell A2 on sheet "Salaries". To flip between the sheets, there is usually a tab for each at the bottom of the document.



By default, the cell references are relative. In other words, when you write a formula in cell H5 that references the value in cell G4, the cell remembers "The cell that is one up and one to the left of me." Thus, if you copy that formula into B9, now that formula reads the value from A8.

If you want an absolute reference, you use $. If H5 references G4, G4 will be used no matter where on the sheet the formula is copied to.

You can use the $ on the row or column. In $A4, the column is absolute and the row is relative. In A$4, the row is absolute and the column is relative.

3.5 Challenge: Make a spreadsheet

You have a company that bids on painting jobs. Make a spreadsheet to help you do bids. Here are the parameters:

- The client will tell you how many square meters of wall needs to be painted.
- Paint costs $0.02 per square meter of wall
- On average, a square meter of wall takes 0.02 hours to paint.
- You can hire painters at $15 per hour.
- You add 20% to your estimated costs for a margin of error and profit.

Make a spreadsheet such that when you type in the square meters to be painted, the spreadsheet tells you how much you will spend on paint and labor. It should also tell you what your bid should be.

Chapter 4

Compound Interest

When you loan money to someone, you typically charge them some sort of interest. The most common loan of this sort is what the bank calls a "savings account". Any money you put in the account is loaned to the bank. The bank then lends it to someone else, who pays interest to the bank. The bank gives some of that interest to you. However, what if you leave the interest in your account? And you start making *interest on the interest*? This is known as *compound interest*.

4.1 An example with annual interest payments

Let's say that you put $1000 in a savings account that pays 6% interest every year. How much money would you have after 12 years? Let's make a spreadsheet.

	A	B	C
1	**Interest Rate**	6.00%	
2			
3	**After year:**	**Interest**	**Balance**
4	0	$0.00	1000
5	1	$60.00	$1,060.00

Create a new spreadsheet and edit the cells to look like this. All the cells in rows 1 - 4 are just values: just type in what you see.

The fifth row is all formulas:

After year	Interest	Balance
$= A4 + 1$	$= B\$1 * C4$	$= C4 + B5$

The interest rate field should be formatted as a percentage. One thing to know when dealing with percentages in the spreadsheet: If the field says "600%", its value is 6.

The cells in the Interest and Balance column should be formatted as currency.

You are about to make several copies of the cells in the fifth row, so make sure they look right.

Click on A5 and shift-click on C5 to select all three cells. Drag the lower-right corner down to fill the rows 6 - 15.

A5:C16 *fx* = A4 + 1

	A	B	C
1	**Interest Rate**	6.00%	
2			
3	**After year**	**Interest**	**Balance**
4	0	$0.00	1000
5	1	$60.00	$1,060.00
6	2	$63.60	$1,123.60
7	3	$67.42	$1,191.02
8	4	$71.46	$1,262.48
9	5	$75.75	$1,338.23
10	6	$80.29	$1,418.52
11	7	$85.11	$1,503.63
12	8	$90.22	$1,593.85
13	9	$95.63	$1,689.48
14	10	$101.37	$1,790.85
15	11	$107.45	$1,898.30
16	12	$113.90	$2,012.20
17			

Look at the numbers. The first interest payment is $60, but the last is $113.90. Your balance has more than doubled!

4.2 Exponential Growth

We figured this out numerically by repeatedly multiplying the balance by the interest rate. What if you wanted to know what the balance would be n years after investing P_0 dollars with an annual interest rate of r? (Note that r in our example would be 0.06, not 6.0.)

Each year, the balance is multiplied by $1+r$, so after one year, P_0 would become $P_0 \times (1+r)$. The next year you would multiply this number by $(1+r)$ again: $P_0 \times (1+r) \times (1+r)$. The next year? $P_0 \times (1+r) \times (1+r) \times (1+r)$ See the pattern? P_n is this balance after n years, then

$$P_n = P_0(1+r)^n$$

Because n is an exponent, we call this *exponential growth*. There are few things as terrifying to a scientist as the phrase "The population is undergoing exponential growth".

4.3 Sensitivity to interest rate

For most people, the first surprising thing about compound interest is how quickly your money grows after a few years. The second thing that is surprising is how much difference a small change in the percentage rate makes.

Let's add another set of columns that shows what happens to your money if you convince the bank to pay you 8% instead of 6%.

Copy everything from columns B and C:

	A	B	C	D	E
1	Interest Rate	6.00%		6.00%	
2					
3	After year	Interest	Balance	Interest	Balance
4	0	$0.00	1000	$0.00	1000
5	1	$60.00	$1,060.00	$60.00	$1,060.00
6	2	$63.60	$1,123.60	$63.60	$1,123.60
7	3	$67.42	$1,191.02	$67.42	$1,191.02
8	4	$71.46	$1,262.48	$71.46	$1,262.48
9	5	$75.75	$1,338.23	$75.75	$1,338.23
10	6	$80.29	$1,418.52	$80.29	$1,418.52
11	7	$85.11	$1,503.63	$85.11	$1,503.63
12	8	$90.22	$1,593.85	$90.22	$1,593.85
13	9	$95.63	$1,689.48	$95.63	$1,689.48
14	10	$101.37	$1,790.85	$101.37	$1,790.85
15	11	$107.45	$1,898.30	$107.45	$1,898.30
16	12	$113.90	$2,012.20	$113.90	$2,012.20
17					

Now edit the second interest rate to be 8%:

	A	B	C	D	E
1	Interest Rate	6.00%		8.00%	
2					
3	After year	Interest	Balance	Interest	Balance
4	0	$0.00	1000	$0.00	1000
5	1	$60.00	$1,060.00	$80.00	$1,080.00
6	2	$63.60	$1,123.60	$86.40	$1,166.40
7	3	$67.42	$1,191.02	$93.31	$1,259.71
8	4	$71.46	$1,262.48	$100.78	$1,360.49
9	5	$75.75	$1,338.23	$108.84	$1,469.33
10	6	$80.29	$1,418.52	$117.55	$1,586.87
11	7	$85.11	$1,503.63	$126.95	$1,713.82
12	8	$90.22	$1,593.85	$137.11	$1,850.93
13	9	$95.63	$1,689.48	$148.07	$1,999.00
14	10	$101.37	$1,790.85	$159.92	$2,158.92
15	11	$107.45	$1,898.30	$172.71	$2,331.64
16	12	$113.90	$2,012.20	$186.53	$2,518.17
17					

Chapter 5

Introduction to Data Visualization

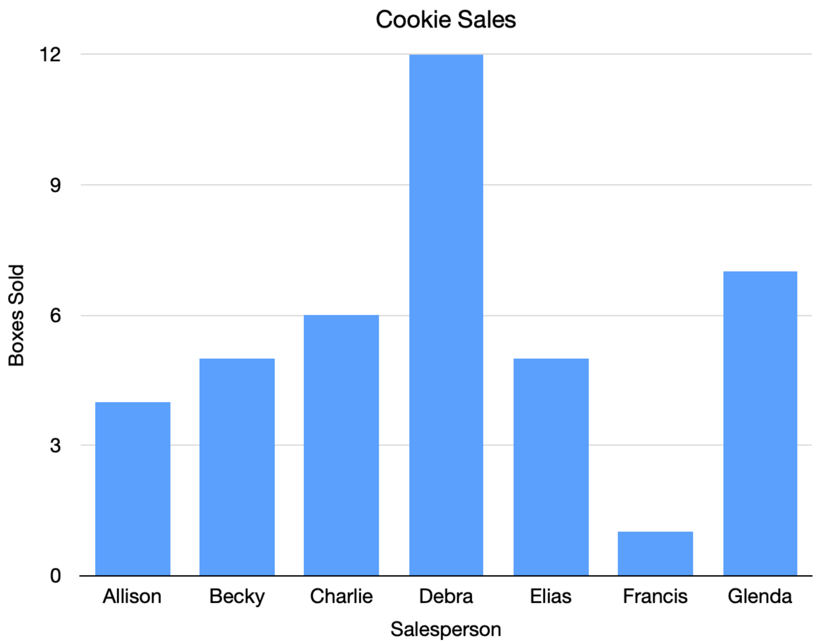
It is difficult for the human mind to look at a list of numbers and identify the patterns in them, so we often use these numbers to make a picture. These pictures are called *graphs*, *charts*, or *plots*. Often, the right picture can make the meaning in the data obvious. *Data visualization* is the process of making pictures from numbers.

5.1 Common Types of Data Visualizations

Depending on the type of data and what you are trying to demonstrate about it, you will use different types of data visualizations. How many types of data visualizations are there? Hundreds, but we will concentrate on just four: The bar chart, the line graph, the pie chart, and the scatter plot.

5.1.1 Bar Chart

Here is an example of a bar chart.



Each bar represents the cookie sales of one person. For example, Charlie has sold 6 boxes of cookies, so the bar goes over Charlie's name and reaches to the number 6.

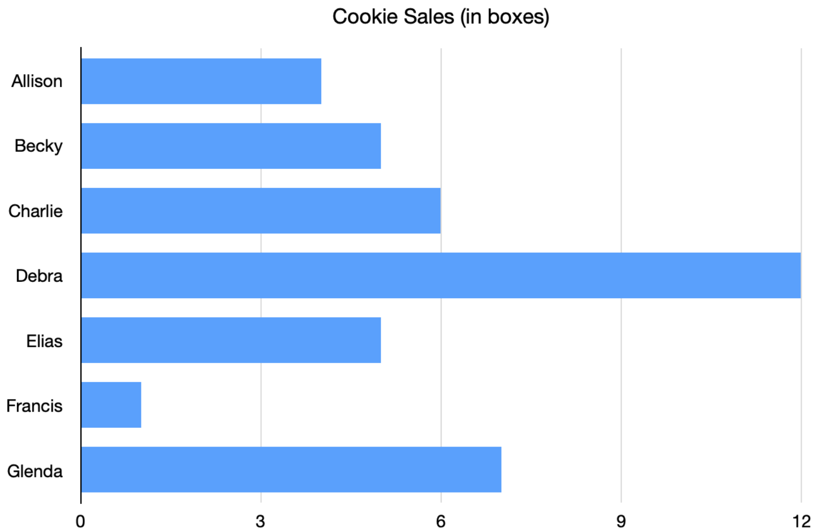
Looking at this chart, you probably think, "Wow, Debra has sold a lot more cookies than anyone else, and Francis has sold a lot fewer."

The same data could be in a table like this:

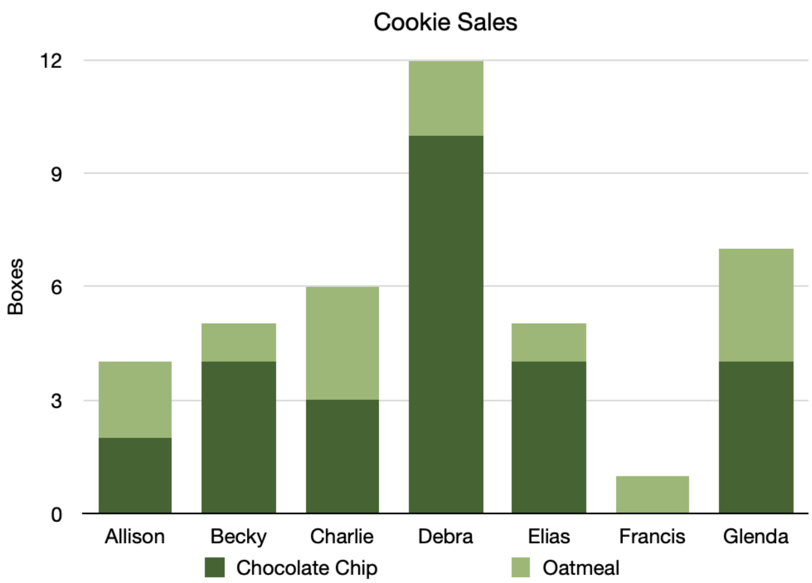
Salesperson	Boxes Sold
Allison	4
Becky	5
Charlie	6
Debra	12
Elias	5
Francis	1
Glenda	7

A table (especially a large table) is often just a bunch of numbers. A chart helps our brains understand what the numbers mean.

Bar charts can also go horizontally.



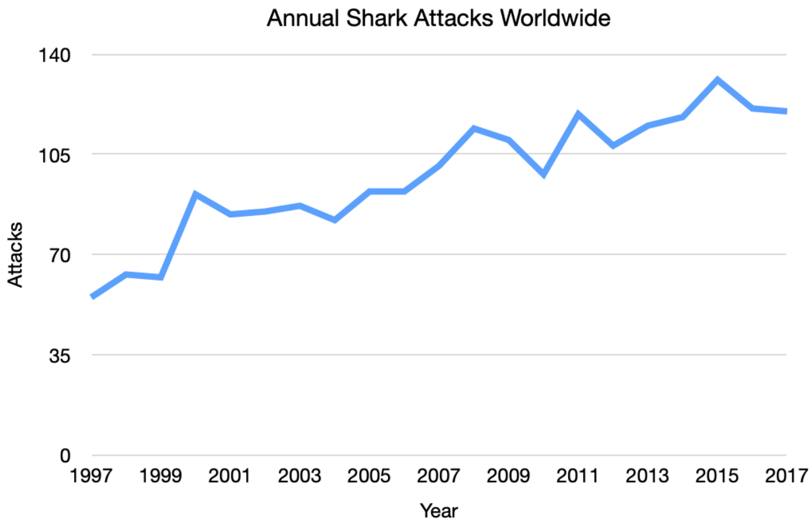
Sometimes we use colors to explain what contributed to the number.



This tells us that Becky sold more boxes of chocolate chip cookies than boxes of oatmeal cookies.

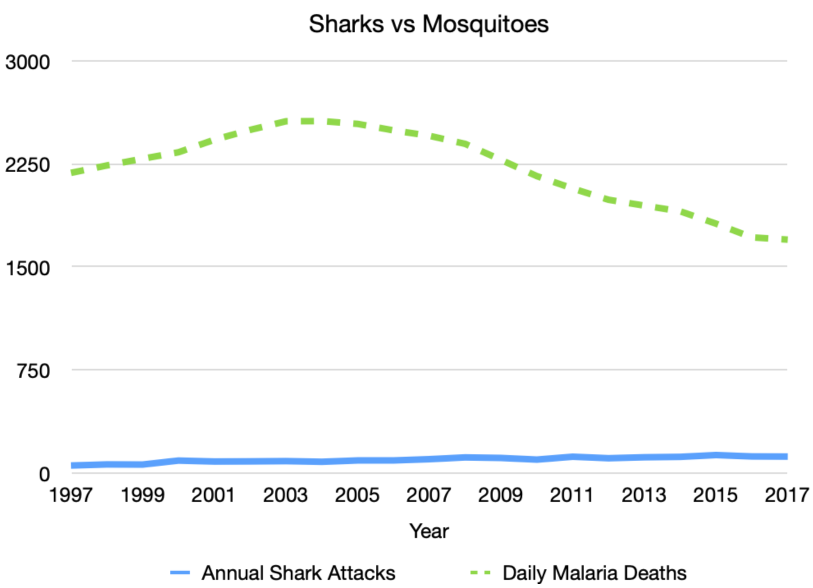
5.1.2 Line Graph

Here is a line graph:



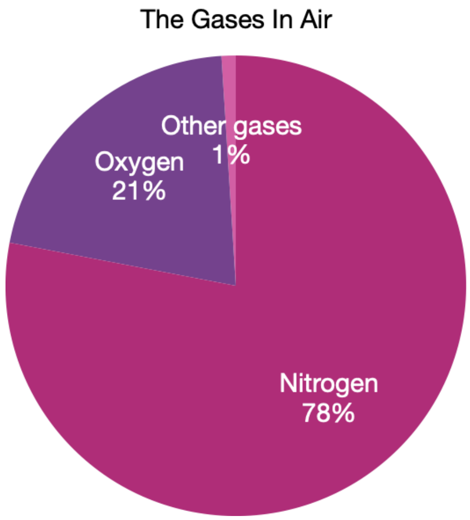
These are often used to show trends over time. Here, for example, you can see that the number of shark attacks has been increasing over time.

You can have more than one line on a graph.



5.1.3 Pie Chart

You use a pie chart when you are looking at the comparative size of numbers.



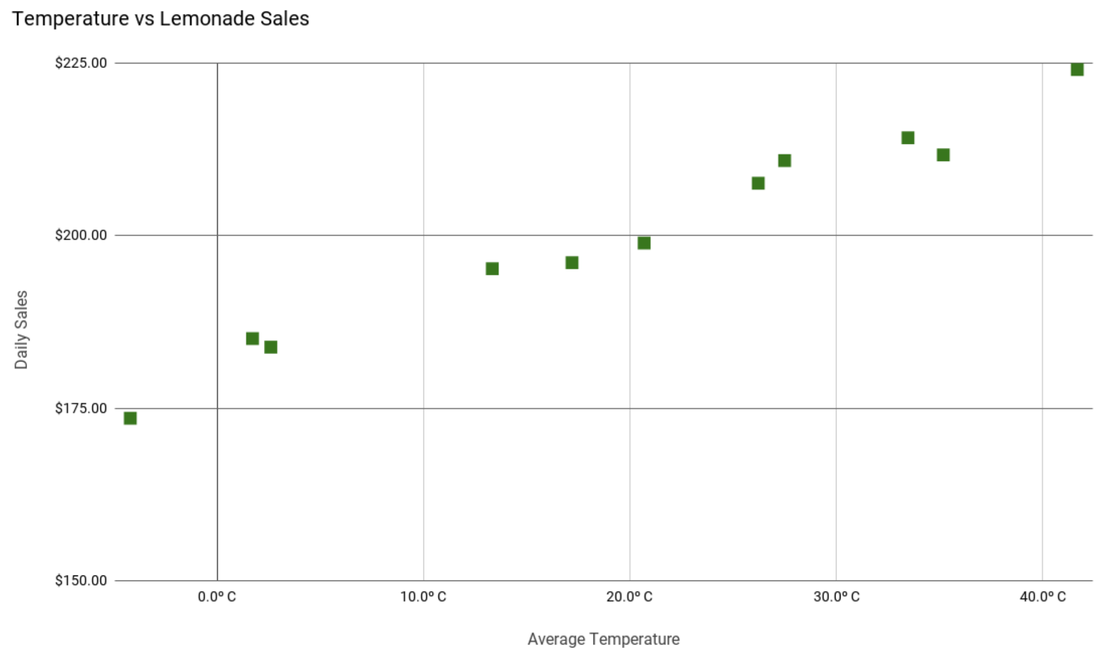
5.1.4 Scatter Plot

Sometimes, you have a large number of data points with two values, and you are looking for a relationship between them. For example, maybe you write down the average temperature and the total sales for your lemonade stand on the 15th of every month:

Date	Avg. Temp.	Total Sales
15 January 2022	2.6º C	$183.85
15 February 2022	-4.2º C	$173.56
15 March 2022	13.3º C	$195.22
15 April 2022	26.2º C	$207.61
15 May 2022	27.5º C	$210.88
15 June 2022	31.3º C	$214.18
15 July 2022	33.5º C	$215.23
15 Aug 2022	41.7º C	$224.07
15 September 2022	20.7º C	$198.94
15 October 2022	17.2º C	$196.10
15 November 2022	1.7º C	$185.10
15 December 2022	0.2º C	$188.70

You may wonder, ”Do I sell more lemonade on hotter days?”

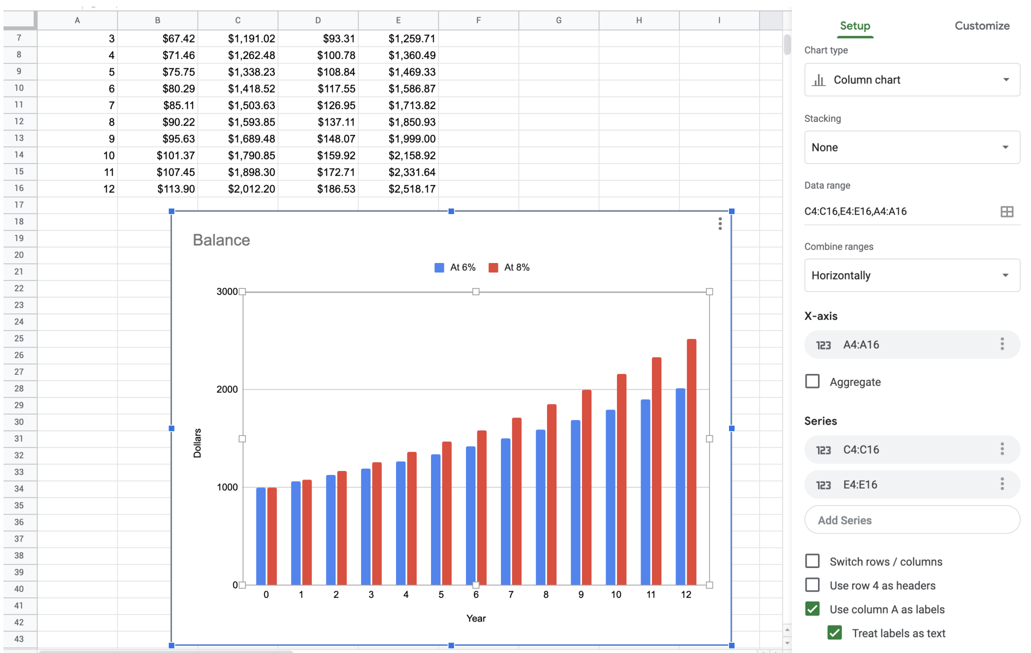
To figure this out, you might create a scatter plot. For each day, you put a mark that represents that temperature and the sales that day:



From this scatter plot, you can easily see that you do sell more lemonade as the temperature goes up.

5.2 Make Bar Graph

Go back to your compound interest spreadsheet and make a bar graph that shows both balances over time:



The year column should be used as the x-axis. There are two series of data that come from C4:C16 and E4:E16. Tidy up the titles and legend as much as you like.

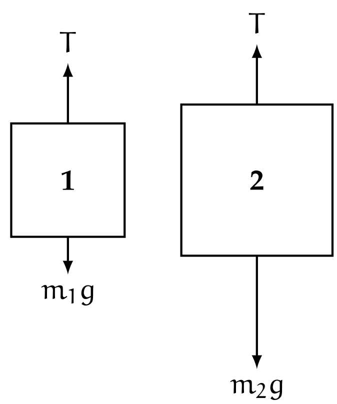
Looking at the graph, you can see the balances start the same, but balance of the account with the larger interest rate quickly pulls away from the account with the smaller interest rate.

Appendix A

Answers to Exercises

Answer to Exercise 1 (on page 10)

1. Each block is acted on by gravity and the tension of the string. The force of gravity on block 2 is 3 times that of block 1, and the tension vector should be between the lengths of the gravity vectors:



2. Block 1 will have a positive acceleration with a magnitude less than g. Block 2 will have a negative acceleration with a magnitude greater than g.

3. Since the blocks are connected, they will move as a system and therefore have the same magnitude acceleration. From this, the free body diagrams, and Newton's Second Law, we know that:

$$m_1 a = T - m_1 g$$

$$m_2(-a) = T - m_2 g$$

We know m_1 and m_2, so the two unknowns are T and a. One way to solve this system of equations would be to subtract equation 2 from equation 1:

$$m_1 a + m_2 a = m_2 g - m_1 g$$

$$a = \frac{m_2 - m_1}{m_1 + m_2} g$$

Substituting for the masses, we see that:

$$a = \frac{9-3}{9+3} g = \frac{6}{12} g = \frac{1}{2} g$$

Therefore, Block 1 is accelerating at $+\frac{1}{2}g$ and Block 2 is accelerating at $-\frac{1}{2}g$.

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