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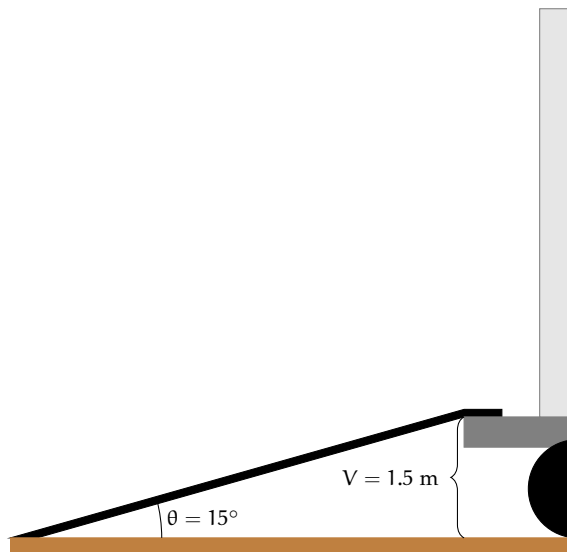
Inclined Planes

Why does a sled speed up on a steep hill but sit still on a gentle one? Why do mountain roads zigzag instead of heading straight up? Gravity always pulls straight down, but on a slope only part of that pull points along the ground. Finding that part takes the trigonometry from the Right-Triangle Trigonometry chapter and the vector components from the Vectors chapter.

This chapter also uses free body diagrams and the normal force from the Forces and Motion chapter, static and kinetic friction from the Friction chapter, and the ramps from the Simple Machines chapter. Here is the one new idea in a sentence: tilt your axes to match the ramp, and split gravity into a piece along the ramp, $mg \sin \theta$, and a piece into the ramp, $mg \cos \theta$.

1.1 Pushing a Load Up a Ramp

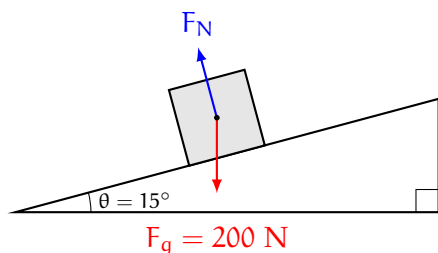
In the Simple Machines chapter, a family on moving day pushed a 200 N hand trolley up a ramp into a truck bed 1.5 m high. Lifting it straight up takes 300 J of work. There, we knew the ramp's length. Real ramps are usually described by their angle, so suppose this one makes a 15° angle with the ground:



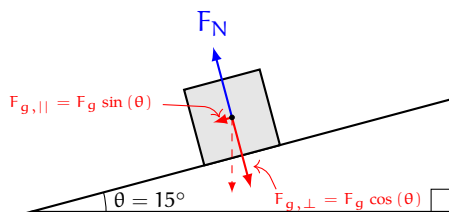
The ramp is the hypotenuse of a right triangle whose height is the rise, so $\sin \theta = \frac{V}{L}$ and

$L = \frac{V}{\sin \theta} = \frac{1.5}{\sin 15^\circ} \approx \frac{1.5}{0.259} \approx 5.8$ meters. How much force does it take to move the load of boxes up the ramp? Intuitively, we know it is less force. A free body diagram shows us the minimum force needed to push the load up the ramp.

Before you push it, there are two forces acting on the loaded hand trolley: its weight (F_g) and the normal force between the trolley and the ramp (F_N):

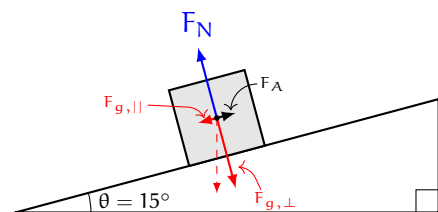


Notice that the normal force is perpendicular to the ramp! We want to know how much force it takes to push the load up the ramp, so we will “split” the weight force vector into two parts: one part parallel to the ramp ($F_{g,\parallel}$) and one part perpendicular ($F_{g,\perp}$):



We did this by treating the weight vector as the hypotenuse of a right triangle with legs perpendicular and parallel to the ramp. That right triangle has the same angle θ as the ramp, so by SOH-CAH-TOA, the part of the weight perpendicular to the ramp is $F_g \cos \theta$ and the part parallel to the ramp is $F_g \sin \theta$.

What force do you need to overcome to push the hand trolley up the ramp? Just the part of the weight that is parallel to the ramp! You will need to apply an equal force in the opposite direction (up the ramp) to move the hand trolley:



So, we know that you are pushing with an applied force of $F_A = F_{g,\parallel} = F_g \sin(\theta)$. Therefore, the work you would do pushing the hand trolley up the ramp is:

$$F_A \cdot L = F_g \sin(\theta) \cdot \left(\frac{V}{\sin(\theta)} \right) = F_g \cdot V = 300 \text{ J}$$

That is the same 300 J as lifting the load straight up, just as the Simple Machines chapter found: the trigonometry changes how we describe the ramp, not the physics.

So what makes it “easier” to use a ramp to lift the hand trolley? The fact that you need to apply less *force* to move the hand trolley ($F_A < F_g$). Now, let us look at the mechanical advantage of the ramp. In this case, the mechanical advantage is given by:

$$MA = \frac{F_g}{F_A}$$

Substituting for F_A , we see that:

$$MA = \frac{F_g}{F_g \sin(\theta)} = \frac{1}{\sin \theta} = \frac{L}{V}$$

This matches the mechanical advantage $\frac{L}{V}$ from the Simple Machines chapter, and it gives a new way to write the force: $F_A = F_g \sin \theta$. Here, $F_A = 200 \sin 15^\circ \approx 51.8 \text{ N}$, about a quarter of the load’s weight.

Ramps and Angles

On a frictionless ramp at angle θ , the force needed to push an object of weight W up the ramp is $W \sin \theta$, and the mechanical advantage is $\frac{1}{\sin \theta} = \frac{L}{V}$.

1.2 A Box on a Ramp

Here is a box resting on an inclined plane, held in place by static friction and gravity.

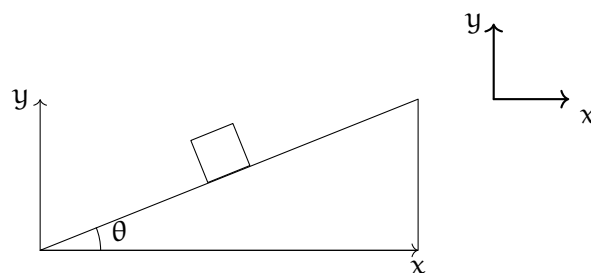


Figure 1.1: A box on an inclined plane with $a = 0$ due to static friction and gravity.

For a box on a ramp, it pays to tilt your axes. Nothing forces the x -axis to be horizontal. If you tip the axes so that x runs along the ramp and y points straight out of it, the normal force lies along y , friction lies along x , and only gravity needs to be split into pieces.

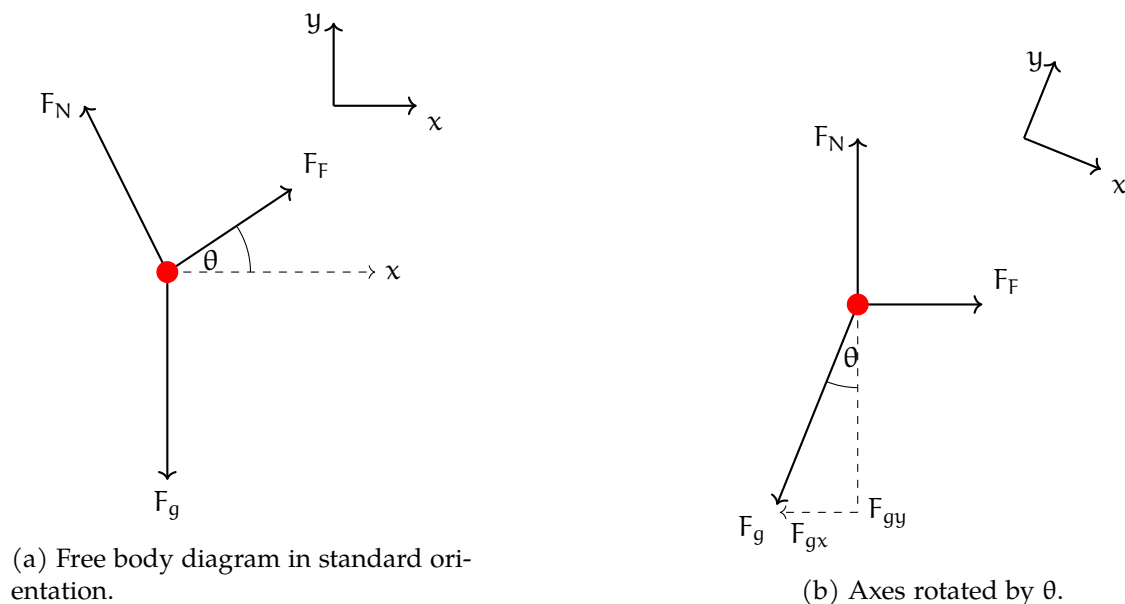


Figure 1.2: Free body diagrams on an incline.

From this new perspective, we can see that the gravitational force F_g can be broken down into two components: one parallel to the incline (F_{gx})¹ and one perpendicular to the incline (F_{gy}). The normal force F_N acts perpendicular to the surface of the incline, while the frictional force F_F acts parallel to the surface, opposing motion. The coordinate planes in the corner are included to show the original coordinate system relative to our rotated FBD.²

¹Note that some professors will use down the incline as positive. This book will consistently use up the incline as positive as rotating coordinate plane aligns well with this.

²Note also that the dashed line components are not actual forces, they are drawn in for illustration of how a force at an angle splits into pieces.

We can set up the following equations based on Newton's second law:

$$\sum F_x = F_{gx} - F_F = ma_x = 0, \quad \sum F_y = F_N - F_{gy} = ma_y = 0$$

Using the trigonometric components, we can express the forces in terms of the angle θ :

$$\sum F_x = mg \sin(\theta) - F_F = ma_x = 0, \quad \sum F_y = F_N - mg \cos(\theta) = ma_y = 0$$

Solving for the normal force and frictional force, we find:

$$F_N = mg \cos(\theta), \quad F_F = mg \sin(\theta)$$

When the box is sliding down, friction is $F_F = \mu F_N = \mu mg \cos \theta$, pointing up the ramp. The box speeds up down the ramp at

$$a = g(\sin \theta - \mu \cos \theta)$$

where μ is the coefficient of kinetic friction between the box and the ramp. (With up the ramp as positive, $a_x = -g(\sin \theta - \mu \cos \theta)$.)

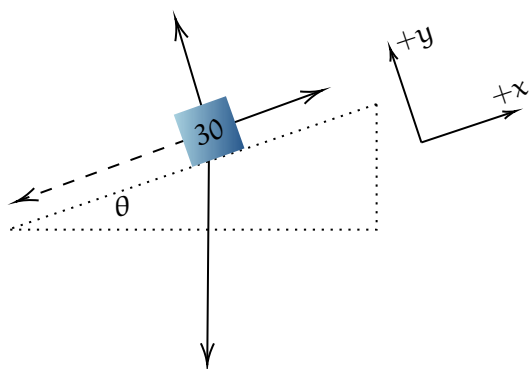
Let us apply all of this to an example problem:

Exercise 1 Box sliding down an incline

A 30 kg block is sliding down an incline plane that makes a 30° with the horizontal. Given that the coefficient of kinetic friction is 0.3, find the following. Note that up the inclined plane is considered the positive x-direction.

- The normal force acting on the block.
- The frictional force acting on the block.
- The acceleration of the block down the incline.

Working Space



Answer on Page 95

Exercise 2 Solving for angle θ

A 40 kg box is sitting on a rough inclined plane, held in place by static friction. The coefficient of static friction μ_s between the box and the incline is found to be 0.464. What is the maximum angle θ that the incline can be tilted to before the box starts to slide down?

Working Space

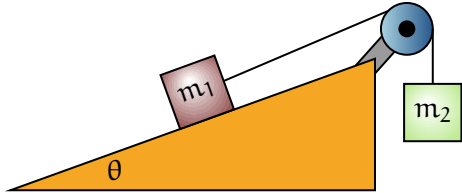
Answer on Page 95

Exercise 3 Pulleys and Inclines

A block at rest of mass m_1 is resting on a rough inclined plane making an angle θ with the horizontal. The coefficient of kinetic friction between the block and the incline is μ_k . The block is attached by a light string over a frictionless pulley to a hanging block of mass m_2 . Once they are connected by a string, the hanging block begins to descend and the block of m_1 moves upwards along the incline. Draw the free

Working Space

body diagrams of each mass. Derive an equation for the acceleration of m_2 , only in terms of m_1 , m_2 , g , θ , and μ_k .



Answer on Page 96

Projectile Motion

A basketball arcing toward the hoop, a soccer ball off a goal kick, a diver leaving the high board: each is a *projectile*, an object that, once launched, moves under the influence of gravity alone. NASA uses the same physics to aim rovers at Mars.

In the kinematics chapter, you handled objects that fall straight down. Now we add sideways motion. The key idea is surprisingly simple: the sideways motion and the up-and-down motion ignore each other. Once you see that, you can predict where a ball will land, how high it will go, and what angle sends it farthest. At the end, you will write a simulation that handles any projectile, even one slowed by air.

2.1 Comparing Projectiles

Hold one coin on your fingertip and another on the edge of a table. Flick the table coin off sideways at the same moment you drop the other one. Listen: they hit the floor at the same time. This video shows the same experiment with balls: <https://www.youtube.com/watch?v=zMF4CD7i3hg>.

Adding sideways speed does not change how fast an object falls. This is called *independence of motion*.

2.2 Independence of Motion

For a projectile, the horizontal and vertical parts of the motion are independent. Gravity pulls only downward, so it changes only the vertical velocity. Nothing pushes sideways (if we ignore air resistance), so the horizontal velocity never changes.

Vertically, the projectile behaves exactly like the falling objects in the kinematics chapter: constant acceleration of $9.8 \frac{\text{m}}{\text{s}^2}$ downward. (We often round to $10 \frac{\text{m}}{\text{s}^2}$ to make the arithmetic easier.)

Horizontally, it coasts at constant speed.

The two motions are tied together by just one thing: time. The ball lands when its vertical motion says it hits the ground, and that same moment ends its horizontal travel. So we use separate equations for each direction (figure 2.1) and connect them through t . Note that the velocity in the horizontal direction is *constant*, while the velocity in the vertical

Horizontal Motion	Vertical Motion
$\Delta x = v_{0x}t$	$\Delta y = v_{0y}t + \frac{1}{2}(-g)t^2$
$v_x = v_{0x}$ (constant!)	$v_y = v_{0y} + (-g)t$
$a_x = 0$	$a_y = -g$

Figure 2.1: Projectile motion equations from kinematics equations.

direction is *not constant* due to the acceleration of gravity ($9.8 \frac{\text{m}}{\text{s}^2}$ downward).

Figures 2.2 and 2.3 sketch the shapes of the graphs. (The scales are only illustrative.)

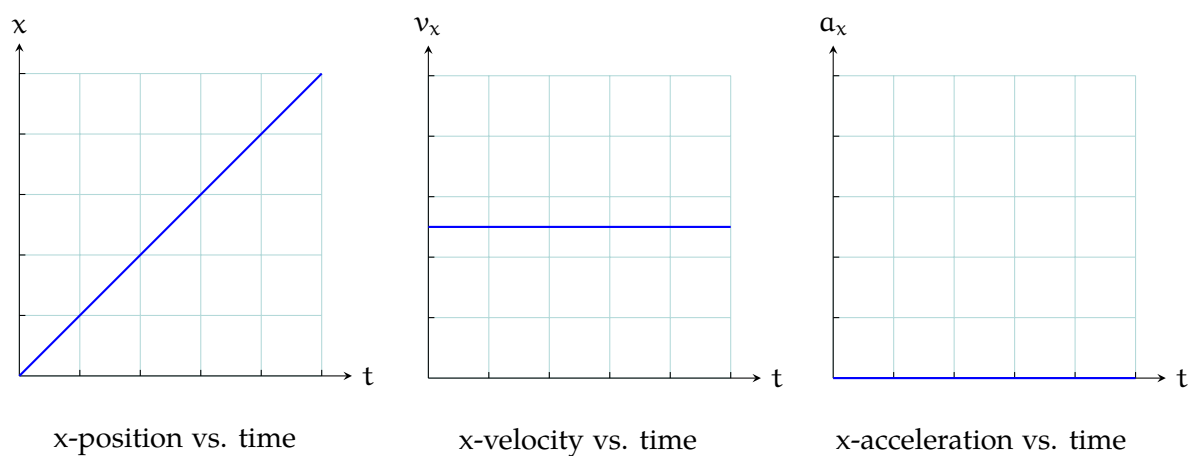


Figure 2.2: Horizontal motion graphs

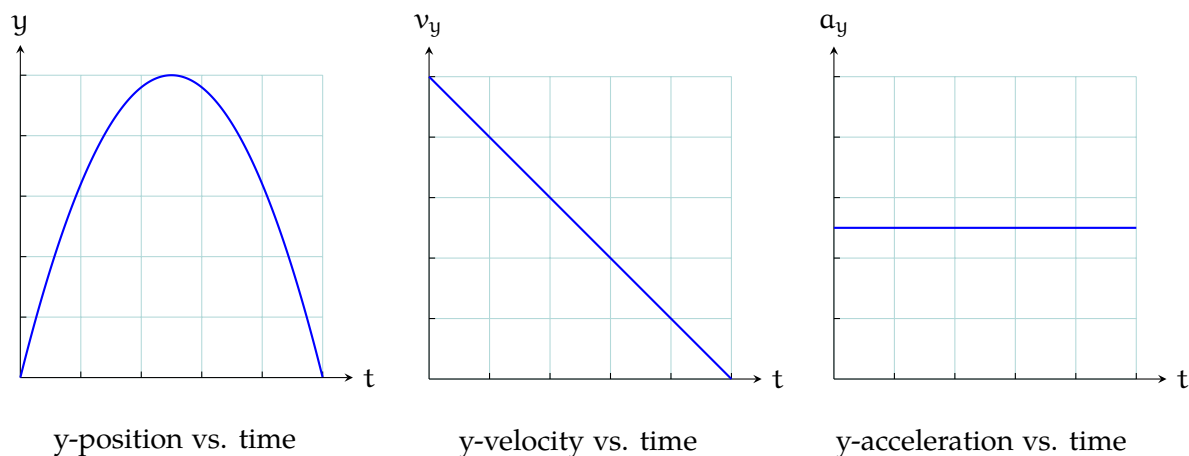


Figure 2.3: Vertical motion graphs

2.3 Horizontally-launched Projectiles

Imagine a ball is thrown horizontally off of a cliff. The ball has an initial horizontal velocity of 20 m/s, but no initial vertical velocity. The ball will continue to move horizontally at a constant velocity, while simultaneously accelerating downward due to gravity. The ball falls for 5 seconds before hitting the ground. We will use $-10 \frac{\text{m}}{\text{s}^2}$ for the acceleration due to gravity to make calculations easier. See figure 2.4.

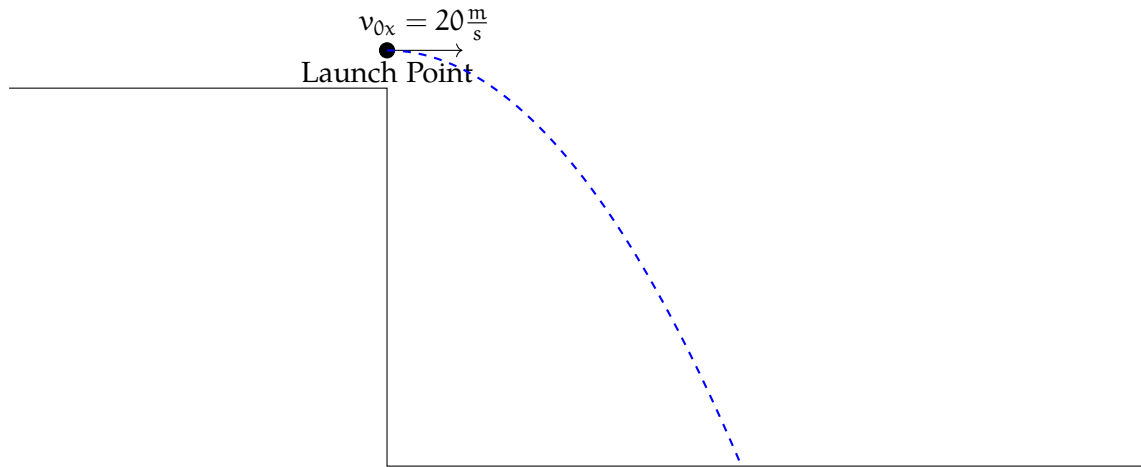


Figure 2.4: Diagram of a ball being thrown horizontally off of a cliff. Note: figure is not to scale.

Let us calculate the change in y .

The vertical motion can be described by the equation:

$$\Delta y = v_{0y}t + \frac{1}{2}(-g)t^2$$

Since the initial vertical velocity $v_{0y} = 0$, this simplifies to:

$$\Delta y = \frac{1}{2}(-g)t^2$$

Substituting in the values for g and t :

$$\Delta y = \frac{1}{2}(-10 \frac{\text{m}}{\text{s}^2})(5\text{s})^2$$

Calculating this gives:

$$\Delta y = -125\text{m}$$

So the ball falls 125 m before hitting the ground. The cliff is 125 m tall.

Now let us calculate the change in x . The horizontal motion can be described by the equation:

$$\Delta x = v_{0x}t$$

Substituting in the values for v_{0x} and t :

$$\Delta x = (20\frac{\text{m}}{\text{s}})(5\text{s}) = 100\text{m}$$

Exercise 4 **Horizontally-launched Projectile**

A rock is thrown horizontally off of the edge of a cliff with an initial velocity of 8 m/s. If the rock falls for 3 seconds before hitting the ground, (a) how far from the base of the cliff does the rock land? (b) How high is the cliff?

Use $-10\frac{\text{m}}{\text{s}^2}$ for the acceleration due to gravity.

Working Space

Answer on Page 97

2.3.1 Newton's Cannon and Escape Velocity

Isaac Newton imagined a cannon on top of a mountain so tall that it pokes above the atmosphere. This thought experiment is called Newton's cannon. Fire the cannonball horizontally, and one of three things happens:

- The cannonball would not have enough horizontal velocity, and fall back to the Earth.
- The cannonball would get shot at enough velocity to be in continuous orbit around the

Earth. (This is how satellites stay in orbit around the Earth!).

- The cannonball would get shot at such a high velocity that it would escape the Earth's gravitational pull, and fly off into space. This is known as *escape velocity*.

Take a look at this simulation of Newton's cannon: <https://physics.weber.edu/schroeder/software/NewtonsCannon.html>. Then, watch this video: <https://www.youtube.com/watch?v=ALRdYPMpqQs>.

Here is the beautiful part. The cannonball is always falling. But Earth is round, so the ground curves away beneath it. Fire it fast enough (about 7,900 m/s near Earth's surface), and the ground curves away exactly as fast as the ball falls. The ball falls forever without landing. That is an orbit. The International Space Station is doing this right now. For Earth, the escape velocity is about 11,200 m/s.

For any given planet, the escape velocity can be calculated using the formula:

$$v_e = \sqrt{\frac{2GM}{d}}$$

where G is the gravitational constant, M is the mass of the planet, and d is the distance from the center of the planet to the object. We will cover this in more detail in the orbits chapter.

2.4 Projectiles Launched at an Angle

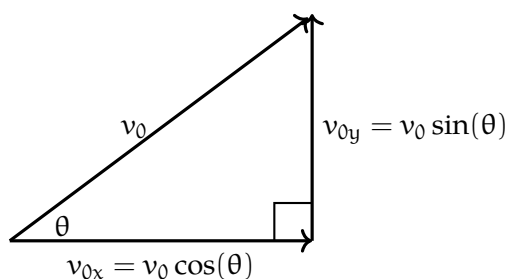
2.4.1 From the Ground

Now, let us imagine a ball is launched from the ground at an angle. The angle of launch influences both horizontal and vertical components of the initial velocity. We can use trigonometry to separate the initial velocity into its horizontal and vertical components.

$$v_{0x} = v_0 \cos(\theta) \qquad v_{0y} = v_0 \sin(\theta)$$

A horizontal throw is the special case $\theta = 0^\circ$: then $v_{0x} = v_0$ and $v_{0y} = 0$. At any other angle, the launch speed is shared between the two directions, so each component is smaller than v_0 .

The horizontal velocity stays v_{0x} the whole flight. The vertical velocity v_y starts at v_{0y} and drops by $9.8 \frac{\text{m}}{\text{s}}$ every second. At the peak, $v_y = 0$: the ball has stopped rising and has not started falling.

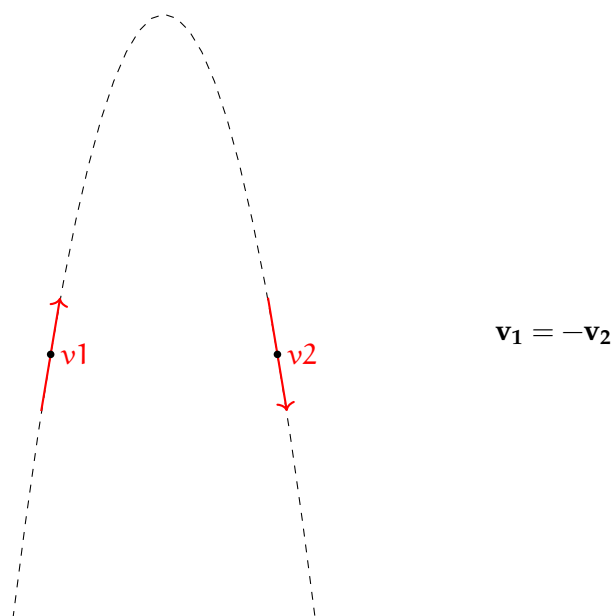


For example, take a watermelon launched at $20 \frac{\text{m}}{\text{s}}$ at an angle of 30° above the horizontal. What would the horizontal and vertical components of the initial velocity be?

Well, we can use the equations above to find out:

$$v_{0x} = (20) \cos(30^\circ) \approx 17.32 \text{ m/s}, \quad v_{0y} = (20) \sin(30^\circ) = 10 \text{ m/s}$$

Projectile paths are symmetric. At any given height, the ball is rising on the way up exactly as fast as it is falling on the way down. The way down is a mirror image of the way up.



When the ball lands at the same height it was launched from (level ground), its horizontal distance, or *range*, has a neat formula. You will derive it in Exercise 6.

Range Formula

$$R = \frac{v_0^2 \sin(2\theta)}{g} \quad (2.1)$$

where the object starts and lands at the same height.

Worked example. A ball is launched at 20 m/s at 30° above horizontal on level ground. Take $g = 9.8 \text{ m/s}^2$. How high does it go, and how far?

Solution. $v_{0y} = 20 \sin 30^\circ = 10 \text{ m/s}$. At the peak, $v_y = 0$, so the rise time is $\frac{10}{9.8} \approx 1.02 \text{ s}$ and the height is $\frac{(10)^2}{2(9.8)} \approx 5.1 \text{ m}$. The range is $R = \frac{20^2 \sin 60^\circ}{9.8} = \frac{400 \cdot 0.866}{9.8} \approx 35.3 \text{ m}$.

Exercise 5 Projectile Motion at an angle

A pumpkin is launched at 25 m/s at an angle of 40° above the horizontal. Find (a) the peak height of the pumpkin, and (b) the horizontal distance the pumpkin travels before hitting the ground. Use $-10 \frac{\text{m}}{\text{s}^2}$ for the acceleration due to gravity. You may assume the ground is level the entire time.

Working Space

Answer on Page 98

Exercise 6 Launch Angle for Maximum Range

(a) At what angle should you launch for an object to go the furthest given a maximum launch velocity? (b) How can you get the same horizontal distance with two different launch angles?

Working Space

Answer on Page 98

Exercise 7 **Projectile Motion at an angle**

A target is 100 meters away. Your bow launches arrows at 14 meters per second. At what angle should you aim? If you cannot reach the target, what is the slowest launch speed that could? Use $-10 \frac{\text{m}}{\text{s}^2}$ for the acceleration due to gravity. You may assume the ground is level the entire time.

Working Space

Answer on Page 99

2.4.2 From a Height Above the Ground

Now launch at an angle from a height, like throwing a ball from the top of the bleachers.

Worked example. A ball is launched from a height of 50 m at 30° above the horizontal at 20 m/s. Use $g = 10 \frac{\text{m}}{\text{s}^2}$. Find its peak height, how long it flies, and where it lands.

Solution. The components of the initial velocity are

$$v_{0x} = (20) \cos(30^\circ) \approx 17.32 \text{ m/s}, \quad v_{0y} = (20) \sin(30^\circ) = 10 \text{ m/s}.$$

Measure height from the ground, so the ball starts at $y_0 = 50$ m:

$$y = y_0 + v_{0y}t + \frac{1}{2}(-g)t^2 = 50 + 10t - 5t^2$$

Horizontally:

$$x = v_{0x}t = 17.32t$$

To find the peak of the projectile, we can set $v_y = 0$ and solve for t :

$$\begin{aligned} v_y &= v_{0y} + (-g)t \\ 0 &= 10 - 10t \\ t_{\text{peak}} &= 1 \text{ s} \end{aligned}$$

Substituting this time into the vertical and horizontal equations, we can find the peak height and horizontal distance at the peak:

$$y = 50 + 10(1) - 5(1)^2 = 55 \text{ m}$$

$$x = 17.32(1) = 17.32 \text{ m}$$

The ball lands when $y = 0$:

$$0 = 50 + 10t - 5t^2$$

$$0 = -5t^2 + 10t + 50$$

$$0 = t^2 - 2t - 10$$

Using the quadratic formula from the Solving Quadratics chapter, we find:

$$\begin{aligned} t &= \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-10)}}{2(1)} \\ &= \frac{2 \pm \sqrt{4 + 40}}{2} \\ &= \frac{2 \pm \sqrt{44}}{2} \\ &= 1 \pm \sqrt{11} \approx 4.32 \text{ s (taking the positive root)} \end{aligned}$$

(The negative root, about -2.32 s, is when a ball on this same path would have left the ground before the launch. It is not part of our flight.)

In 4.32 s, the ball travels $x = 17.32(4.32) \approx 74.8$ m sideways.

2.5 Simulating Projectile Motion

Formulas are great for clean problems. Real projectiles feel air resistance, and then the formulas stop working. A simulation does not care. It chops time into tiny steps and, in each one, nudges the position by the velocity and the velocity by the acceleration.

Create `projectile.py`:

```

1 import math
2 import matplotlib.pyplot as plt
3
4 g = 9.8          # m/s^2
5 dt = 0.001      # s, the size of each time step
6 speed = 20.0    # m/s

```

```

7  angle = math.radians(45)
8
9  x, y = 0.0, 0.0
10 vx = speed * math.cos(angle)
11 vy = speed * math.sin(angle)
12 xs, ys = [x], [y]
13
14 while y >= 0:
15     x += vx * dt      # move
16     y += vy * dt
17     vy -= g * dt      # gravity changes only the vertical velocity
18     xs.append(x)
19     ys.append(y)
20
21 print(f"Range: {x:.1f} m")
22 plt.plot(xs, ys)
23 plt.xlabel("x (m)")
24 plt.ylabel("y (m)")
25 plt.gca().set_aspect("equal")
26 plt.show()

```

Run it. It prints Range: 40.8 m and draws the arc. The range formula agrees: $\frac{20^2 \sin 90^\circ}{9.8} \approx 40.8$ m. Notice that v_x never changes inside the loop. That is independence of motion, written in Python.

Exercise 8 Adding air

Air resistance pushes against the velocity, and it grows with the square of the speed. Replace the line $vy -= g * dt$ with

```

1  k = 0.01                                # drag
   ↳ strength, per meter
2  s = math.hypot(vx, vy)                  #
   ↳ current speed
3  vx -= k * s * vx * dt
4  vy -= (g + k * s * vy) * dt

```

What is the range now? Try several angles between 30° and 50° . Is 45° still the best?

Working Space

Answer on Page 100

Exercise 9 Range at 45 Degrees

A ball is launched at 20 m/s at 45° . Take $g = 9.8 \text{ m/s}^2$. What is its range?

Working Space

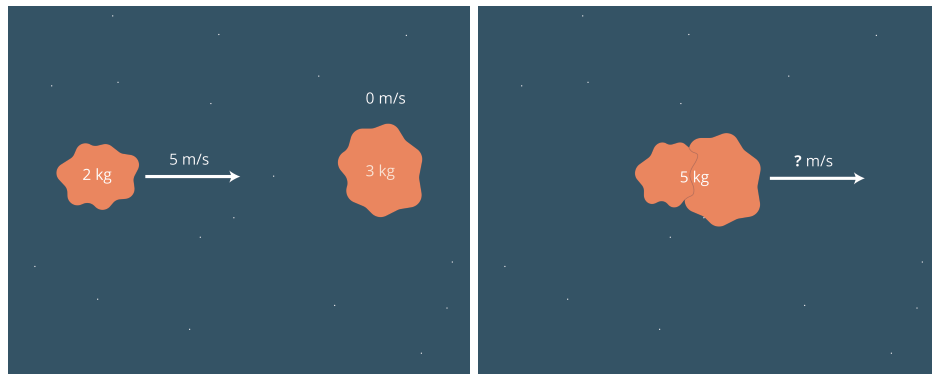
Answer on Page 100

CHAPTER 3

Momentum

Why does a bowling ball scatter pins while a beach ball bounces off them? Why do airbags save lives? Why does a rifle kick back into your shoulder? All three answers come from one idea: momentum.

Start with a puzzle. A 2 kg block of putty flies through space at 5 meters per second and smacks into a 3 kg block of putty that is sitting still. The blocks squish and stick together. How fast does the combined blob move?



Formula for Momentum

Every object has *momentum*. The momentum is a vector quantity — it points in the direction that the object is moving and has a magnitude equal to its mass times its speed.

$$p = mv \quad (3.1)$$

Add up the momentum vectors of a group of objects, and you get their total momentum. When the objects only push on each other, that total never changes.

$$p = p_1 + p_2 + \cdots + p_n = m_1 v_1 + m_2 v_2 + \cdots + m_n v_n \quad (3.2)$$

In our puzzle, the moving block has momentum $(2)(5) = 10 \text{ kg m/s}$ and the resting block has 0. After they merge, the 5 kg blob must still carry 10 kg m/s, so it moves at 2 m/s in the original direction.

Conservation of Momentum

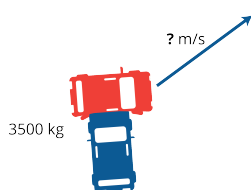
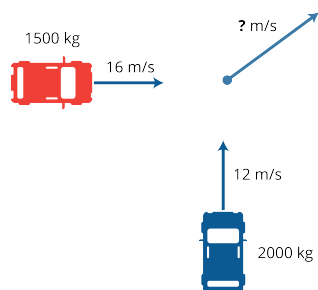
The total momentum of a system remains constant as long as no external forces act on it. In a collision or interaction, the momentum before the event must equal the momentum after the event:

$$p_{\text{initial}} = p_{\text{final}} \quad (3.3)$$

This applies whether the collision is elastic, inelastic, or perfectly inelastic. Even if kinetic energy is lost (as heat, sound, or deformation), momentum is still conserved because it depends only on mass and velocity, not on energy.

Exercise 10 Cars on Ice

A 1000 kg car is going north at 12 m/s. A 1500 kg car is going east at 16 m/s. They both hit a patch of frictionless ice and collide. Steel bends, and the two cars become one tangled object. What is its velocity (speed and direction) as it slides across the ice?



Working Space

Answer on Page 100

Note that kinetic energy ($\frac{1}{2}mv^2$) is *not* conserved here. Before the collision, the moving putty block has $(\frac{1}{2})(2)(5^2) = 25$ joules of kinetic energy. Afterward, the big block has $(\frac{1}{2})(5)(2^2) = 10$ joules of kinetic energy. What happened to the energy that was lost? It went into squishing the putty (and a little into heat and sound).

What if the blocks were marble instead of putty? Then there would be very little deforming, so kinetic energy *and* momentum would be conserved. The two blocks would end up having different velocity vectors.

Let us assume for a moment that they strike each other straight on, so there is motion in only one direction, both before and after the collision. Can we solve for the speeds of the first block (v_1) and the second block (v_2)?

We end up with two equations. Conservation of momentum says:

$$2v_1 + 3v_2 = 10$$

Conservation of kinetic energy says:

$$(1/2)(2)(v_1^2) + (1/2)(3)(v_2^2) = 25$$

Using the first equation, we can solve for v_1 in terms of v_2 :

$$v_1 = \frac{10 - 3v_2}{2}$$

Substituting this into the second equation, we get:

$$\left(\frac{10 - 3v_2}{2}\right)^2 + \frac{3v_2^2}{2} = 25$$

Simplifying, we get:

$$v_2^2 - 4v_2 + 0 = 0$$

This quadratic has two solutions: $v_2 = 0$ and $v_2 = 4$. $v_2 = 0$ represents the situation before the collision. Substituting in $v_2 = 4$:

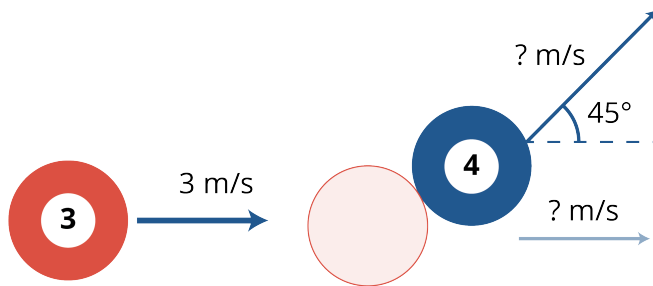
$$v_1 = \frac{10 - 3(4)}{2} = -1$$

Thus, if the blocks are hard enough that kinetic energy is conserved, after the collision, the smaller block will be heading in the opposite direction at 1 m/s. The larger block will be moving at 4 m/s in the direction of the original motion.

Exercise 11 Billiard Balls*Working Space*

A 0.4 kg billiard ball traveling at 3 m/s hits an identical billiard ball at rest. It strikes obliquely (neither perpendicular nor parallel), so that the ball at rest starts to move at a 45 degree angle from the path of the ball that hit it.

Assuming all kinetic energy is conserved, what is the velocity vector of each ball after the collision?

*Answer on Page 101***3.1 Impulse**

A change in momentum is called an *impulse*. Like momentum, impulse is a vector.

Formula for Impulse

The impulse $\mathbf{J}$ is the change in momentum:

$$\mathbf{J} = \Delta \mathbf{p} \quad (3.4)$$

It also equals the force times how long the force acts:

$$\mathbf{J} = \mathbf{F} \Delta t \quad (3.5)$$

If the force changes over time, the impulse is the area under the force-versus-time graph. (In calculus, you will write that area as an integral.) The fact that impulse equals change in momentum is called the **impulse-momentum theorem**.

So a big force for a short time *or* a small force for a long time can produce the same change in momentum.

3.1.1 Airbags and Egg Tosses

Have you ever played the egg toss? The trick to catching the egg is to let your hands move back with it. Here is why.

Worked example. A 0.06 kg egg arrives at 5 m/s. To stop it, you must change its momentum by $(0.06)(5) = 0.3$ kg m/s. That impulse is fixed. If your stiff hands stop it in 0.01 s, the force is

$$F = \frac{J}{\Delta t} = \frac{0.3}{0.01} = 30 \text{ N}$$

That is enough to crack the shell. If you let your hands ride back and take 0.2 s to stop it, the force is only $\frac{0.3}{0.2} = 1.5$ N. Same impulse, one twentieth of the force.

Airbags, bike helmets, crumple zones, and the bend in your knees when you land a jump all work the same way: they stretch out the stopping time so the force stays small.

Exercise 12 Landing a jump

A 50 kg student jumps off a wall and hits the ground at 4 m/s. What impulse does the ground deliver? What is the average force if the student lands stiff-legged and stops in 0.02 s? What if the student bends their knees and stops in 0.25 s?

Working Space

Answer on Page 102

3.2 Collisions

In every collision between isolated objects, momentum is conserved. Kinetic energy is another story. We sort collisions into three kinds by what happens to the kinetic energy.

3.2.1 Elastic Collision

In an *elastic collision*, both momentum and kinetic energy are conserved. The billiard balls in Exercise 11 collided elastically. When the hit is dead center, something striking happens: the moving ball stops cold, and the resting ball takes off with all of its velocity (figure 3.1).

No real collision is perfectly elastic. A little energy always goes into sound, heat, and tiny dents. But billiard balls come close. Listen: a billiard collision makes a quiet click, not the crunch of a car crash.

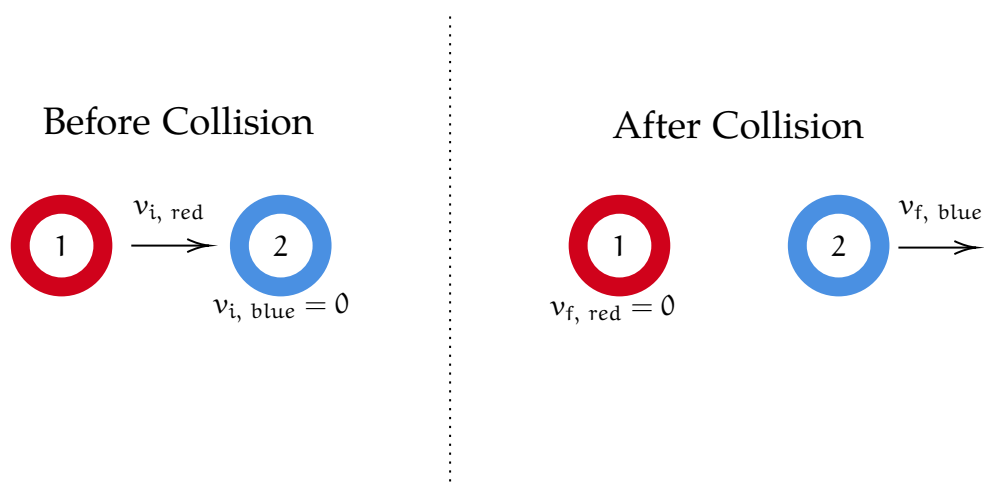


Figure 3.1: A head-on elastic collision between equal billiard balls.

Newton's cradle, the desk toy with a row of hanging steel balls, is a chain of nearly elastic collisions. Your digital resources include two videos of it in action.

In equations, for an elastic collision:

$$\text{total } \mathbf{p}_{\text{before collision}} = \text{total } \mathbf{p}_{\text{after collision}} \implies m_1 v_{i,1} + m_2 v_{i,2} + \cdots = m_1 v_{f,1} + m_2 v_{f,2} + \cdots$$

and

$$\text{total } \mathbf{KE}_{\text{before collision}} = \text{total } \mathbf{KE}_{\text{after collision}}$$

3.2.2 Inelastic Collisions

In an *inelastic collision*, some kinetic energy is lost, usually to heat, sound, or bending. Momentum is still conserved:

$$\text{total } \mathbf{p}_{\text{before collision}} = \text{total } \mathbf{p}_{\text{after collision}} \implies m_1 \mathbf{v}_{i,1} + m_2 \mathbf{v}_{i,2} + \cdots = m_1 \mathbf{v}_{f,1} + m_2 \mathbf{v}_{f,2} + \cdots$$

Drop a basketball, and it never bounces back to the height you dropped it from. Some kinetic energy went into the thump you heard and into squishing the ball. The missing height measures the lost energy (figure 3.2). (Where did the ball's momentum go? Into the Earth, which recoils by an amount far too small to notice.)

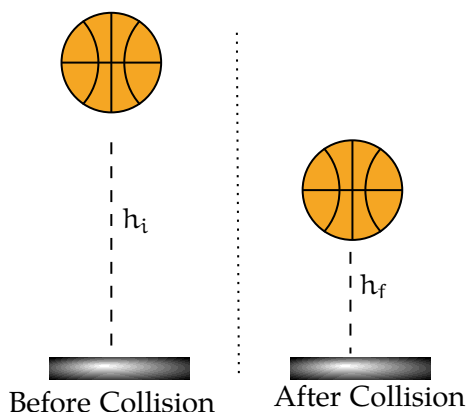


Figure 3.2: An example of a (partially) inelastic collision between the floor and a basketball.

3.2.3 Perfectly Inelastic

A collision is *perfectly inelastic* when the objects stick together and move off with the same velocity, like the putty and the cars on ice. This loses as much kinetic energy as momentum conservation allows.

Worked example. A 2 kg cart moving at 3 m/s hits a 1 kg cart at rest, and they stick together. Momentum is conserved: $(2)(3) + (1)(0) = (2 + 1)v$, so $v = 2$ m/s. The kinetic energy drops from $\frac{1}{2}(2)(3^2) = 9$ J to $\frac{1}{2}(3)(2^2) = 6$ J.

Exercise 13 Two Carts

Two carts on a frictionless track collide

Working Space

elastically.

Cart A has mass 2.0 kg and is moving to the right at 3.0 m/s.

Cart B has mass 1.0 kg and is initially at rest.

They collide head-on elastically.

Find the final velocities of both carts.
Hint: You may have to do a few equation substitutions

Answer on Page 102

Exercise 14 Car Crash

Sometimes, car companies will intentionally crash cars in order to test the safety of their prototypes.

Two cars collide head-on on a straight, frictionless test track, where east is considered positive.

Car A has mass 1000 kg and is traveling *east* at 20 m/s. Car B has mass 1500 kg and is traveling *west* at 10 m/s.

During the collision, the cars do not stick together. After the collision, Car A is observed to move east at 5 m/s.

Find the final velocity of Car B. How much kinetic energy is lost? What kind of collision is this?

Working Space

Answer on Page 104

Exercise 15 **Ready, Aim, Fire!**

A $m_{\text{bullet}} = 10 \text{ g}$ bullet is fired horizontally at 500 m/s into a wooden block of mass $m_{\text{block}} = 2.00 \text{ kg}$ resting on a frictionless horizontal surface. The bullet embeds itself in the block, such that a loud bang was heard and the *block-bullet system* slides together after the collision.

Working Space

- (a) What is the velocity of the *block-bullet system* after the collision?
- (b) How much kinetic energy was lost in the collision?
- (c) What percent of the bullet's kinetic energy was lost? What kind of collision is this?

Answer on Page 104

3.3 Momentum, Center of Mass, and Explosions

Picture a firework shell bursting at the top of its arc. Pieces fly in every direction. Yet the center of mass of all those pieces keeps sailing along the exact parabola the unexploded shell would have followed.

Why? The explosion pushes the fragments apart, but every push on one fragment comes with an equal and opposite push on another (Newton's third law). These internal forces cancel, so they cannot change the total momentum. And the total momentum is the total mass times the velocity of the center of mass:

$$M\vec{v}_{\text{COM}} = m_1\vec{v}_1 + m_2\vec{v}_2 + \dots$$

Since the explosion cannot change the left side, $\vec{v}_{\text{COM}}$ right after the explosion equals the velocity just before. If you know all the fragments but one, this equation tells you the missing one.

Exercise 16 **Three-Fragment Projectile Explosion***Working Space*

A projectile of mass 6.0 kg is launched from the ground and follows a parabolic arc. At the very top of its trajectory, its speed is measured to be 20 m/s, moving horizontally.

At that instant, it explodes into three fragments:

- Fragment A has mass 2.0 kg and flies off at 40° above the horizontal with a speed of 30 m/s.
- Fragment B has mass 1.0 kg and flies off at 60° below the horizontal with a speed of 25 m/s.
- Fragment C has mass 3.0 kg. Its velocity is unknown.

The explosion is so quick that gravity has no time to change the momentum during it. Everything moves in one vertical plane.

Find the **magnitude and direction** of the velocity of Fragment C.

Answer on Page 105

Momentum, impulse, collisions, and explosions all rest on one fact: when objects push only on each other, their total momentum never changes. Next, we will use these ideas to see how boats, planes, and helicopters move.

Exercise 17 **Sticking Together**

A 1 kg cart moving at 4 m/s hits a 3

Working Space

kg cart at rest, and they stick together.
What is their final velocity?

Answer on Page 106

Boats and Sailing

4.1 Boats

People were building boats thousands of years before anyone dreamed of airplanes. Along the way, boat builders worked out ideas about balance, steering, and the push of a moving fluid that later made airplanes and rockets possible. And a sailboat does something that seems impossible: it can sail *toward* the wind. By the end of this chapter, you will know how.

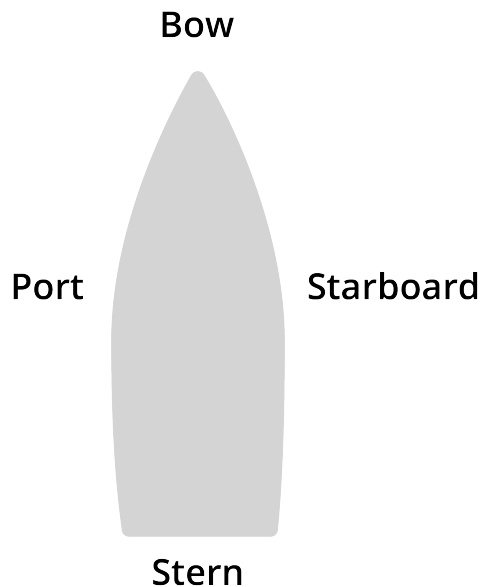
4.1.1 Basic Terminology

The front of a boat is called *the bow* (pronounced exactly the same as “bough”). The back of the boat is called *the stern*.

The underside of the boat is called *the hull*. The top of the boat is called *the deck*.

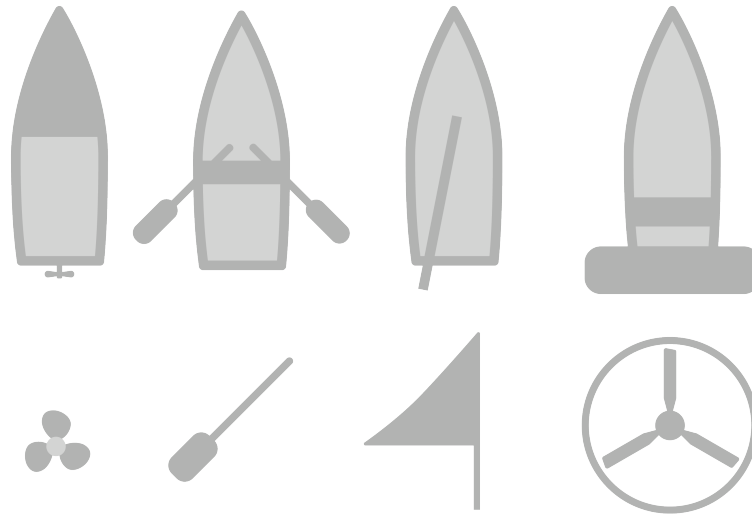


If you are standing at the stern and looking toward the bow, everything on your left is the *port* side. Everything on your right is the *starboard* side.



There are several different ways that boats are propelled:

- A motor turns a screw in the water, as in a motorboat. The screw is known as a *propeller*.
- A human pushes the water with a big spatula. If the spatula is attached to the boat with a pivot (as in a rowboat) it is an *oar*. If the spatula is not attached to the boat (as in a canoe), it is a *paddle*.
- A human pushes the ground beneath the water with a long pole. These boats are called *punts*, and are mainly used in more shallow waters where an oar or a paddle would be less convenient.
- The wind pushes the boat, as in a sailboat. The sails are held up by a *mast*.
- Some boats have a big fan that propel the boat. These are called *airboats*. Airboats are not the most efficient boats, but they can travel on waterways with water just a couple of inches deep.



In the terms of physics, each of these methods provide a *thrust vector* which is applied to the boat at a particular place and in a particular direction.

Nautical Miles and Knots

At sea, distances are measured in *nautical miles*, and speeds in *knots*: nautical miles per hour.

What is a nautical mile?

Latitude is how many degrees north or south of the equator you are. Sailors split each degree into 60 minutes and each minute into 60 seconds. If you are 12.45164° north of the equator, a sailor would say you are at latitude $12^\circ 27' 6''$ N.

A nautical mile is the distance of one minute of latitude. That makes navigation easy: at $12^\circ 27'$ N, you are $12 \times 60 + 27 = 747$ nautical miles from the equator. A nautical mile is a bit longer than the ordinary (*statute*) mile used on land: it is 1.151 statute miles, or 1.852 km.

4.1.2 Why Boats Float Upright

In the buoyancy chapter, you learned that the buoyant force equals the weight of the water a boat displaces. It pushes straight up, opposite gravity. But a boat also needs to stay right side up. How?

Center of Buoyancy

The buoyant force does not push on one spot; it pushes all over the underwater part of the hull. But we can treat it as a single push at one point, the *center of buoyancy*. To find it, imagine the hole in the water that the boat makes. Fill that hole with water. The center of mass of that water is the center of buoyancy.

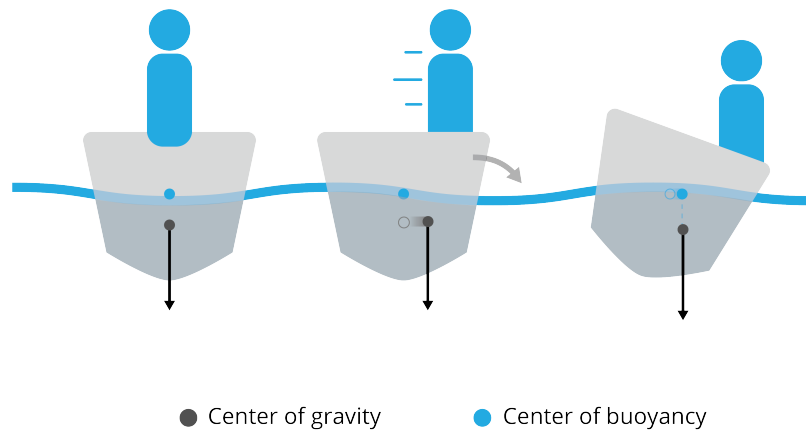
When the boat tilts, the shape of that hole changes, and the center of buoyancy moves toward the side that dipped deeper.

Center of Mass

Gravity pulls on the boat and everything in it as if all the weight sat at one point, the center of mass (also called the center of gravity).

Step to one side of a rowboat, and you move the center of mass with you. Now gravity pulls down on that side while buoyancy pushes up near the middle. The two forces twist the boat, and it leans. As it leans, the center of buoyancy slides toward the low side. The boat stops leaning when the center of buoyancy is straight below or above the center of mass.

If the water comes over the side before that happens, you are swimming.



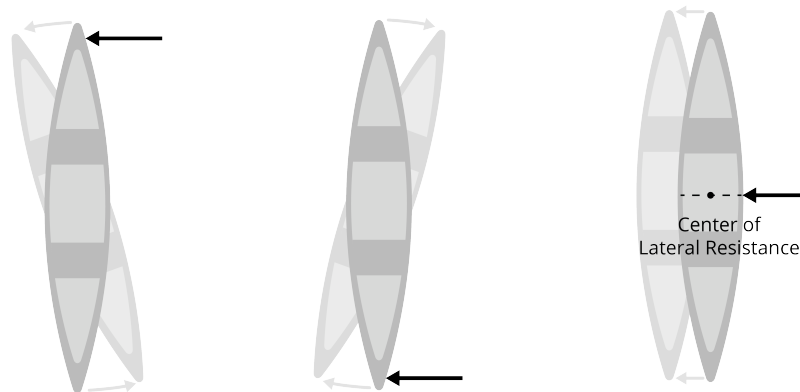
4.1.3 Center of Lateral Resistance

Floating is not enough. A useful boat must also go straight.

Imagine that you are standing knee-deep in a lake next to a canoe. If you push the front of the canoe away from you, it will rotate — the back end will actually swing toward you. There is a point near the middle of the canoe where if you push it will not rotate in either direction — the boat will just slide sideways. This point is known as the *center of lateral resistance*.

The trick to making a boat travel in a straight line is to make sure that the line that contains the thrust vector passes through the center of lateral resistance.

An outboard motor lets you aim the thrust vector. When the line of thrust passes to the starboard side of the center of lateral resistance, the boat turns to port.

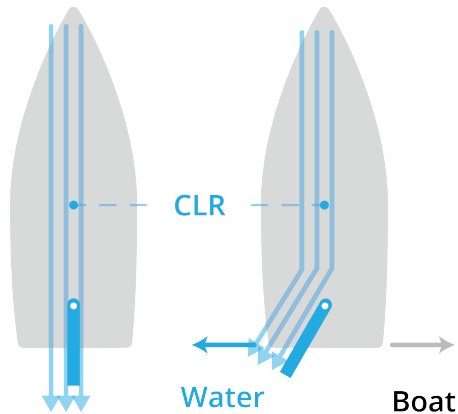


4.1.4 Steering with a Rudder

While outboard motors and airboats let you direct the thrust vector, most boats have a *rudder*. The rudder is a blade on a pivot near the back of the boat. The angle of the rudder can be adjusted so that water rushing past it gets pushed to one side or the other.

According to Newton's third law, when the water gets pushed to the left, the back of the boat gets pushed (with the same force) to the right. This causes the boat to rotate around its center of lateral resistance.

Note that a rudder only works when the boat is passing through the water.



4.1.5 Boat Length and Resistance

Every moving boat makes waves. The biggest one starts at the bow. Watch a boat speed up, and you will see something curious: that bow wave gets longer and longer.

Water waves have a strange property. Long waves travel faster than short ones. A deep-water wave with wavelength λ travels at

$$v = \sqrt{\frac{g\lambda}{2\pi}}$$

The bow wave travels with the boat, so the faster the boat goes, the longer its bow wave has to be. When the boat's speed makes the wave exactly as long as the boat's waterline, the stern sits in the trough of its own bow wave. The boat is tilted nose-up, trying to climb a hill of water it is making itself. Pushing faster takes enormous power. This speed is called the *hull speed*.

Set λ equal to the waterline length L :

$$v_{\text{hull}} = \sqrt{\frac{gL}{2\pi}}$$

With L in feet and v in knots, this works out to the sailor's rule of thumb:

$$v_{\text{hull}} \approx 1.34\sqrt{L}$$

Worked example. What is the hull speed of a boat with a 16 ft waterline?

Solution. $1.34\sqrt{16} = 1.34 \times 4 = 5.36$ knots.

Engineers compare boats of different sizes with the *Froude number*, $Fr = \frac{v}{\sqrt{gL}}$. At hull speed, $Fr = \frac{1}{\sqrt{2\pi}} \approx 0.4$, no matter how big the boat is. That is why a model ship in a test tank tells engineers how the real ship will behave: run the model at the same Froude number.

Longer boats are faster, which is one reason racing yachts and ocean liners are long. Some boats beat the hull speed by *planing*: a speedboat or a surfboard rises up and skims on top of the water instead of plowing through it.

Exercise 18 Longer Boat

What is the hull speed of a boat with a 36 ft waterline? How many times longer must a boat be to have twice the hull speed?

Working Space

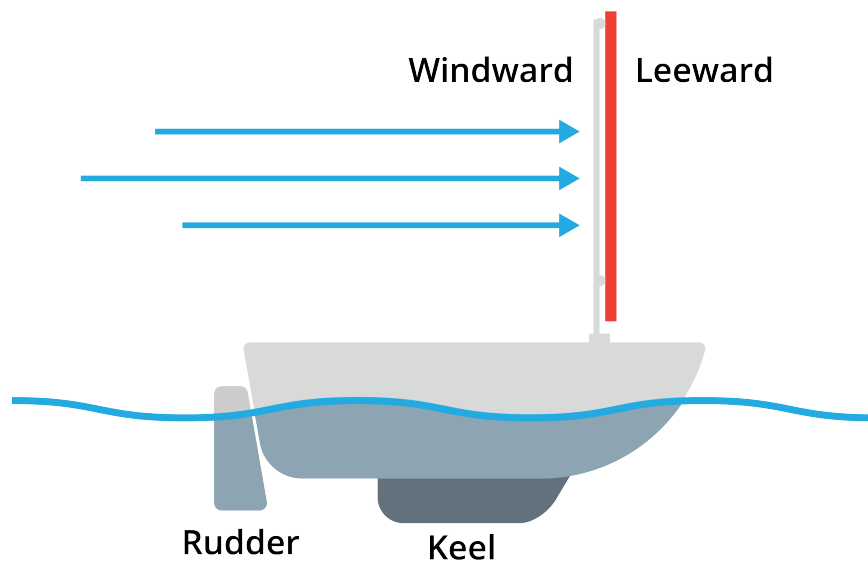
Answer on Page 106

4.2 Sailboats

Imagine that you have a canoe, and you are about to paddle from one island to another that is directly east of where you are standing. There is also a steady wind coming from the west, and you have a big piece of plywood. You might be inspired to use it as a sail.

This is sailing at its simplest: the wind comes from behind, hits the sail, and pushes the boat along with it.

The sail has two sides: The *windward* side is the one that is getting hit with the wind. The *leeward* side is the side away from the wind.



4.2.1 Magnitude of the Wind Force

The first natural question is: How much force will I have pushing my canoe through the water?

Wind Force

When the sail is perpendicular to the wind, the force of the wind on the sail in newtons (F_w) will be given by:

$$F_w = A \frac{d v^2}{2}$$

where A is the area of the sail in square meters d is the density of the gas in kg per cubic meter, and v is the wind speed in meters per second.

For air at STP, d is about 1.225 kg per cubic meter.

We call $\frac{d v^2}{2}$ the *wind pressure*: how much more pressure the windward side of the plywood feels than the leeward side. (The leeward side is turbulent, but its pressure stays close to 1 atmosphere.)

Let us say your canoe is standing still and the wind is 0.5 m/s. This means the wind

pressure is

$$P = \frac{1.225(0.5^2)}{2} = 0.153125 \text{ newtons per square meter}$$

Let us say your plywood sail is 2 meters tall and 1.5 meters wide. What will be the force of the wind?

$$F_w = AP = (3)(0.153125) \approx 0.46 \text{ newtons}$$

That is about the weight of a 47 gram candy bar. A gentle breeze does not push very hard!

Exercise 19 A real sail

A small sailboat has a 10 square meter sail, held perpendicular to a 5 m/s wind. What is the wind pressure, and what is the force on the sail?

Working Space

Answer on Page 106

4.2.2 Direction and Location of the Wind Force

If there is low pressure on one side of the sail and high pressure on the other, the force vector will be perpendicular to the sail.

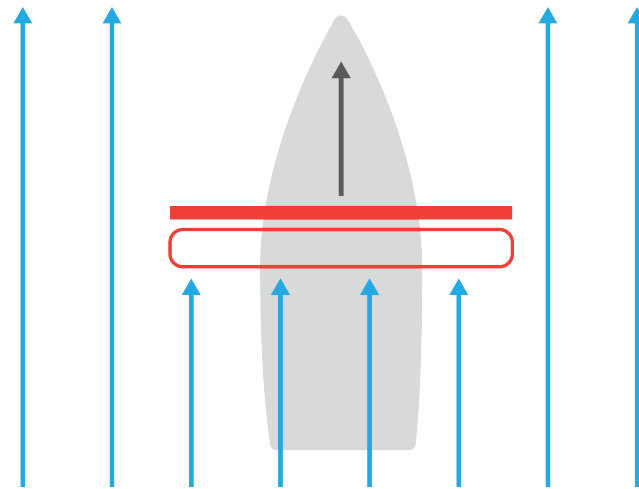
Where is this force vector applied? We can think of the force as being applied at the geometric center of the sail. This is called *the center of effort*. In this case, the center of effort is the exact center of the rectangular plywood.

The mast on a windsurfing board can be tilted from side to side. When the center of effort is over the center of lateral resistance, the board goes straight. To steer, the sailor moves the mast to one side of the center of lateral resistance, which rotates the board.

4.2.3 Beam Reach

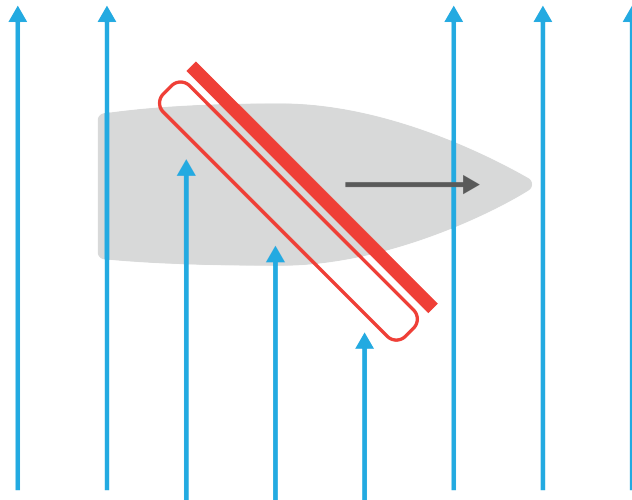
When you are sailing in the same direction as the wind, sailors say you are *running*.

Running



What if you want to go east and the wind (still 0.5 m per second) is now coming from the south? Sailing perpendicular to the wind is known as a *beam reach*.

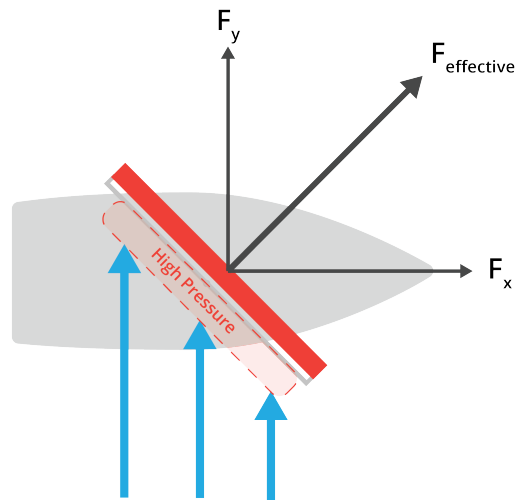
Beam Reach



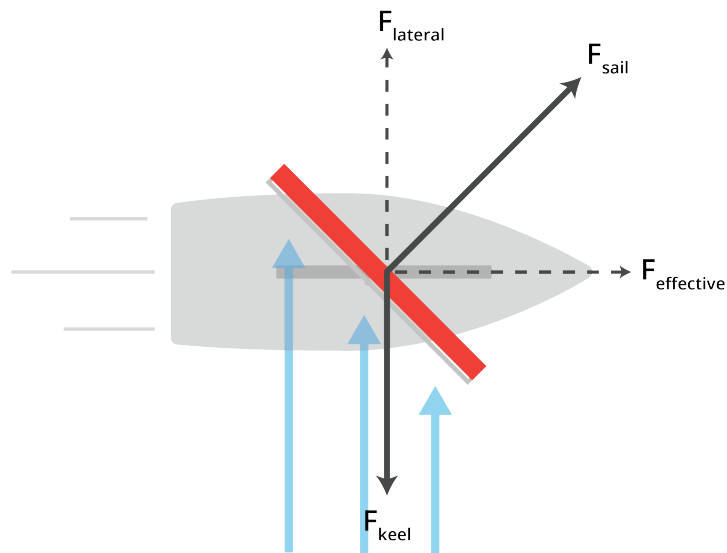
To do a beam reach, instead of mounting the plywood perpendicular to the boats direction of travel, you would mount it at a 45 degree angle. The wind pressure will build on the windward side of the plywood, and the plywood will experience a force pushing it at a 45° angle to the boat.

We can think of this force as having two components:

- One component pushes the boat forward (Yay!)
- One component pushes the boat sideways (Ugh!)



To minimize the effect of the sideways force, sailboats typically have a keel — a long fin on its underside that slows its sideways sliding.



Notice also that the “wind shadow” of the plywood is smaller when it is at a 45° angle to the wind. How much smaller? The effective area of your plywood has gone from 3 square meters to $\frac{3}{\sqrt{2}} \approx 2.12$ square meters. This means the force generated will be smaller, and some of it will be wasted pushing the boat sideways.

If we assume that the wind pressure is still 0.153125 newtons per square meter. The force on the plywood will be about

$$F_w = AP = \frac{3}{\sqrt{2}}(0.153125) \approx 0.325 \text{ newtons}$$

However, the direction of that force is not all in the direction you want to go, so the effective force is

$$F = F_w \frac{1}{\sqrt{2}} = \frac{3}{2}(0.153125) \approx 0.230 \text{ newtons}$$

Notice that we got twice as much effective force when we were running with the wind as when we are on a beam reach. However, any sailor will tell you that you can go much faster on a beam reach than you can running. Why?

4.2.4 Apparent Wind

When you are running, you can never go faster than the wind. As you speed up, the wind you feel gets weaker. For example, if you are going 0.2 m/s in a wind of 0.5 m/s, you (and your sail) will only experience wind at 0.3 m/s. We call the wind as experienced by the boat the *apparent wind*. The wind as observed by a stationary observer is called the *actual wind*.

If you are running with the wind, as you approach the speed of the wind, the force of that wind will decrease towards zero.

On a beam reach, speeding up does not weaken the wind. It makes the wind seem to swing toward the front of the boat, and it makes it *stronger*. You have felt this on a bike: ride through a crosswind, and the wind seems to blow from in front of you.

To find the apparent wind, subtract the boat's velocity from the actual wind's velocity. If you are going 0.2 m/s east and the actual wind blows 0.5 m/s from the south, the apparent wind is $[0, 0.5] - [0.2, 0] = [-0.2, 0.5]$. Its speed is $\sqrt{0.2^2 + 0.5^2} \approx 0.54$ m/s, more than the actual wind, and it seems to come from about $\arctan \frac{0.2}{0.5} \approx 22^\circ$ east of south. The faster you go, the stronger the apparent wind. That is why a beam reach beats running.

Exercise 20 Apparent wind

A boat sails east at 3 m/s. The actual wind blows from the south at 4 m/s.

Working Space

What are the speed and direction of the apparent wind?

Answer on Page 107

4.2.5 Close Reach

What if you want to go east, and the wind is coming from 40 degrees east of south? This would mean that you were sailing just 50 degrees away from straight into the wind. Is this possible?

If you put your sail at a 25 degree angle, you will still catch some wind and create some pressure on one side of the sail. Most of the resulting force would be trying to push your boat sideways, but some of it would be in the direction you were trying to travel.

The angle between the sail and the wind is called the *angle of attack*.

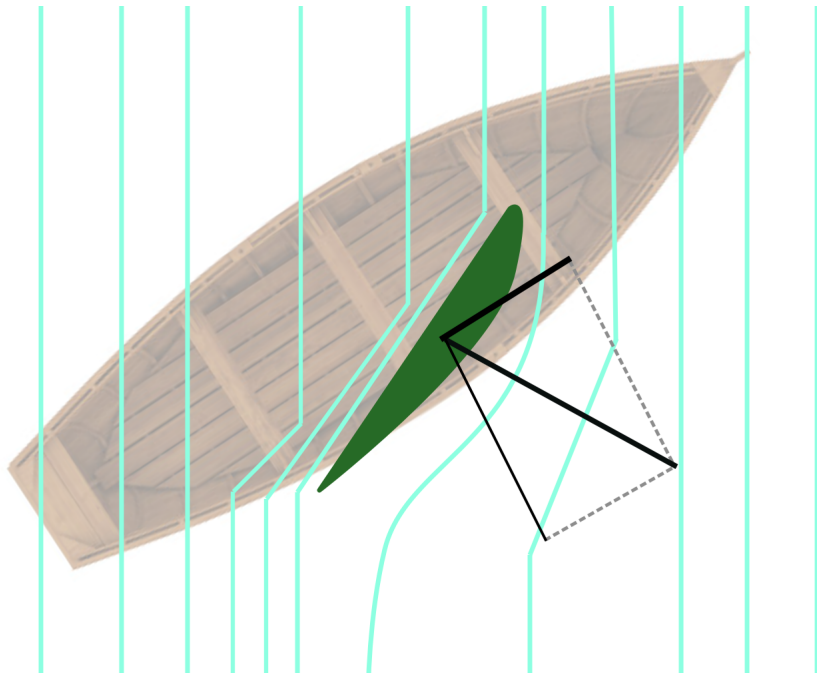
This is surprising: a boat can sail partly into the wind! It cannot sail straight into it. With each degree closer to the wind, the forward push shrinks and the sideways push grows. But a boat with a well-shaped sail and a good keel can point within about 45° of the wind.

4.2.6 Shaping the Sail

Most of the power of the wind can be captured with a piece of flat plywood. The wind hits it and creates a high pressure on the windward side. What about the other side of the plywood?

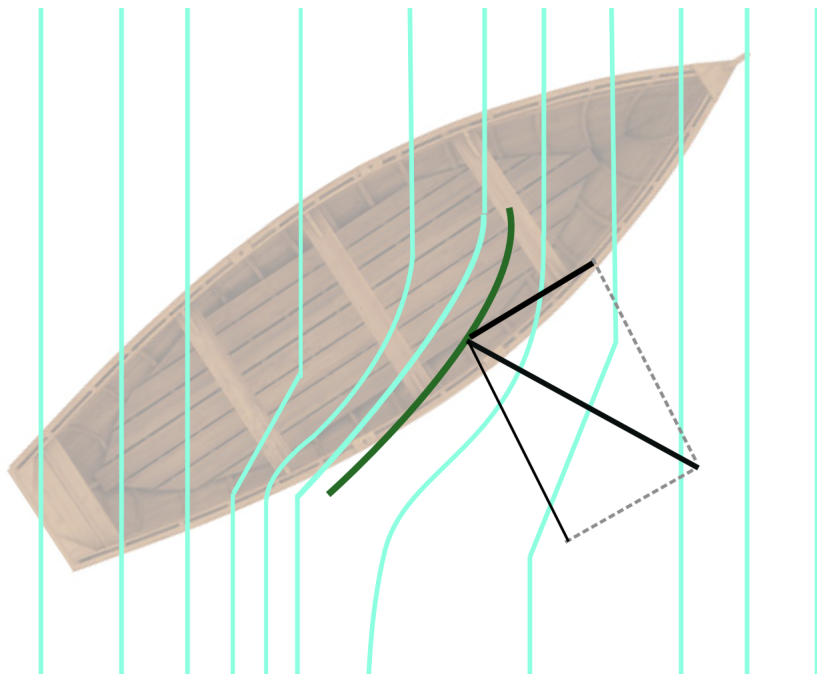
It turns out that if we can get the wind to travel smoothly over the back side of the plywood, the pressure on that side will be a little lower than if we had turbulence there. (We are not going to go too deeply into why. If you want to learn more, look up the Coanda effect.)

For example, if we were on a close reach, the very best sail we could have would gently pull the wind along its backside. It would look like this:



Of course, for the sail to work on either side of the boat, this asymmetrical design would not work. (Although, we should note that this design works great for airplane wings.)

When we make a sail out of cloth, we give it some curve known as *camber*. Slow winds require just a little camber; fast winds require more.



Some newer sailboats have wing sails that have two pieces that can be arranged to redirect the most air possible.

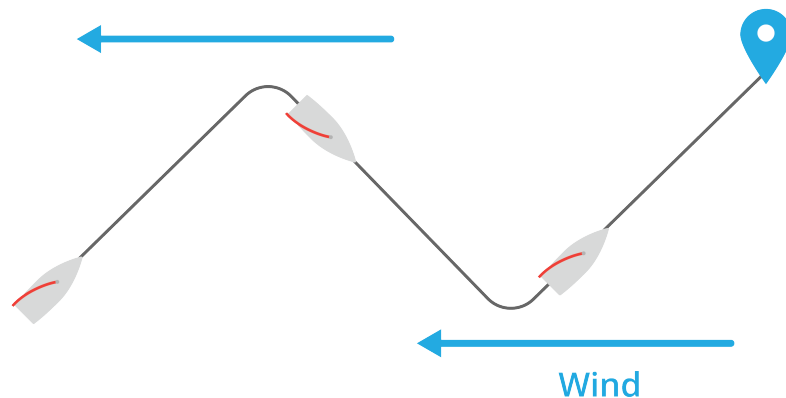
Note: when running with the wind, the turbulence on the leeward side of the sail is unavoidable. But when traveling perpendicular to the wind or on a close reach, the air should move smoothly over the leeward side of the sail. Many sailors have a piece of yarn taped on each side of the sail so they can see if the air is moving smoothly.

Many sailboats also have multiple sails. Besides the increase in the sail area, each sail also redirects the wind to pass smoothly over the leeward surface of the sail behind it.

4.2.7 Tacking into the Wind

What if the wind is coming from the east and you really need to go directly into the wind?

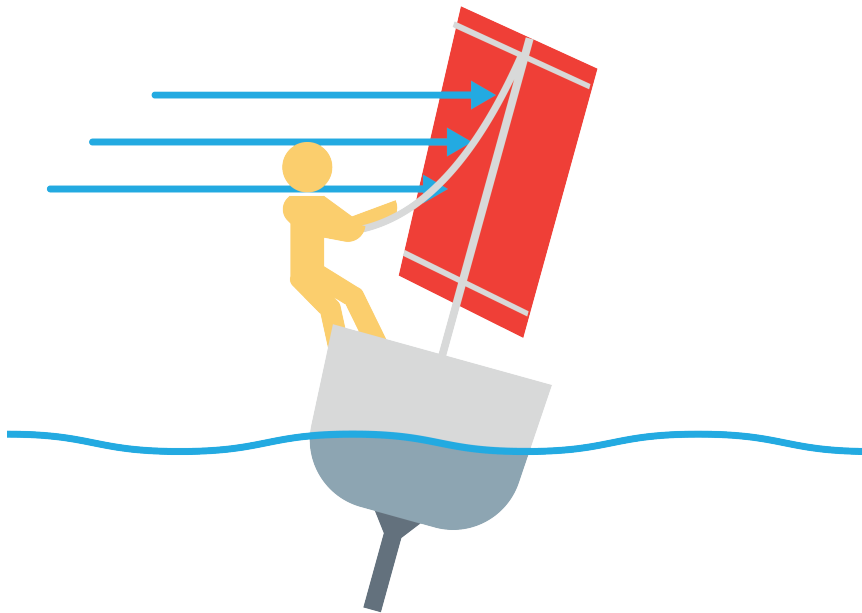
The boat will not travel straight into the wind, instead you will travel on a close reach with the wind coming from one side of the boat. You will then turn into the wind and continue turning until you are on a close reach with the wind coming from the other side. This is known as *tacking*.



4.2.8 Heeling

The center of effort is near the center of your sail, which is usually pretty high in the air. Yes, the force generated will push your boat, but it will also rotate your boat. Sailors call the resulting lean *heeling*. Heeling too far is problematic — the rudder gets pulled out of the water, which makes it hard to steer the boat.

If a boat is heeling too far, sailors will move their weight to the windward side of the boat — some even wear harnesses that push their weight out beyond the edge of the hull. If that does not work, they will reduce the sideways force on the sail.

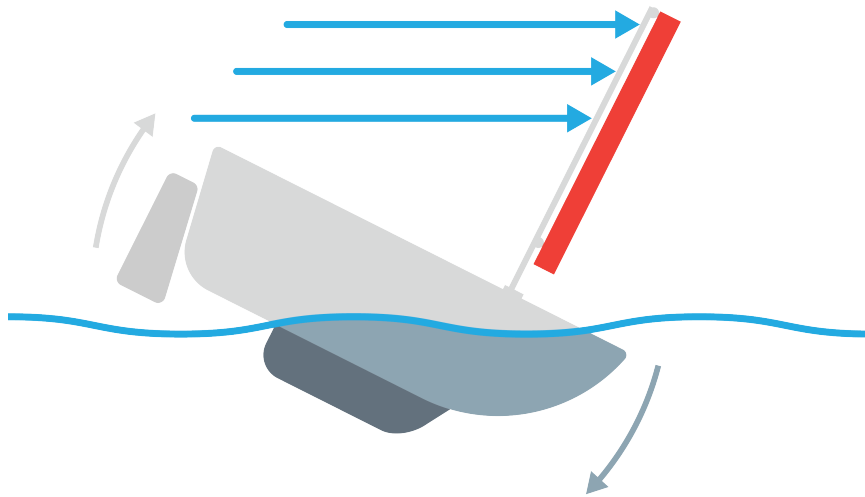


Can the boat flip over? (Sailors say *capsize*.) Larger sailboats are very hard to capsize. As the boat heels, the sail tips toward the water and catches less wind, while a heavy keel low in the water pulls the boat back upright. Small boats (think 3 or 4 meters long) can capsize, but they are small enough that the sailor can usually get them upright again without assistance.

There are boats with multiple hulls. A catamaran has two identical hulls that are side-by-side. As a result, the catamaran will not heel much at all. However, if a big gust of wind comes up, one hull can be pulled completely out of the water. Catamarans can be capsized.

If a boat is running when a big gust comes up, that rotating force will try to push the bow underwater. If the boat is going very fast, this can result in a somersault as the front of

the boat slows and dips suddenly and the back of the boat is pitched up over it.



Fluids in Motion: Bernoulli's Principle

Take a hot shower, and the curtain creeps inward and clings to your legs. Put your thumb over the end of a garden hose, and the water shoots across the yard. Two ships sailing side by side get pulled toward each other. A hurricane does not usually push a roof in; it lifts the roof off.

In the Buoyancy chapter, you studied fluids that sit still, where pressure depends only on depth. This chapter is about fluids that *move*. You will see one simple rule, discovered by Daniel Bernoulli in 1738, that explains all of the puzzles above: where a fluid flows faster, its pressure is lower.

A note on notation: the Buoyancy and Boats and Sailing chapters wrote density as d . From here on, we use the Greek letter ρ (rho), as physicists and engineers do, and as the Airplanes chapter does. So hydrostatic pressure is ρgh .

5.1 Continuity: What Goes In Must Come Out

Water is very hard to squeeze into a smaller volume, so in a pipe or hose, every liter that flows in one end must flow out the other. The volume passing a point each second, the *flow rate*, is the cross-sectional area times the speed: $Q = Av$. Since the flow rate is the same everywhere along the pipe,

The Continuity Equation

$$A_1 v_1 = A_2 v_2$$

Where the pipe narrows, the fluid must speed up. That is what your thumb does to a hose.

Example: A garden hose 2.0 cm across carries water at 1.5 m/s. You cover the end with your thumb, leaving an opening of 0.50 cm^2 . How fast does the water come out?

Solution: The hose's area is $\pi(1.0 \text{ cm})^2 \approx 3.14 \text{ cm}^2$. Then

$$v_2 = \frac{A_1 v_1}{A_2} = \frac{3.14 \times 1.5}{0.50} \approx 9.4 \text{ m/s}$$

more than six times faster. Rivers do the same thing: where a river narrows into rapids, the water speeds up.

5.2 Bernoulli's Equation

Why should faster flow mean lower pressure? Think about energy. Follow a small chunk of fluid with volume V as it moves from a wide part of a pipe to a narrow part. It speeds up, so its kinetic energy grows. Something has to do work on it to provide that energy, and the only thing pushing on it is the surrounding fluid. So the fluid behind must push harder than the fluid in front: the pressure must drop as the fluid speeds up.

Here is the calculation. A pressure P pushing on an area A makes a force PA ; pushing the chunk a distance x does work $PAx = PV$. So the net work done on the chunk by the fluid around it is $(P_1 - P_2)V$. By the work-energy theorem from the Work, Kinetic Energy, and Power chapter, that work equals the chunk's gain in kinetic plus gravitational potential energy. Its mass is ρV , so

$$(P_1 - P_2)V = \frac{1}{2}\rho V v_2^2 - \frac{1}{2}\rho V v_1^2 + \rho V g(h_2 - h_1)$$

Divide by V and collect each point's terms on one side:

Bernoulli's Equation

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

Along the flow, $P + \frac{1}{2}\rho v^2 + \rho g h$ stays the same.

Each term is something you have met. $\rho g h$ is the hydrostatic pressure from the Buoyancy chapter. $\frac{1}{2}\rho v^2$ is the wind pressure from the Boats and Sailing chapter, often called the *dynamic pressure*. Bernoulli's equation says they trade off with ordinary pressure P : at the same height, **where the speed goes up, the pressure goes down**. It is conservation of energy, written for fluids.

It holds exactly for an ideal fluid: one that cannot be compressed, has no internal friction, and flows smoothly. Water in pipes comes close. Air is compressible, but at speeds well below the speed of sound, it behaves as if it were not, so Bernoulli's equation works well for wind and airplanes too.

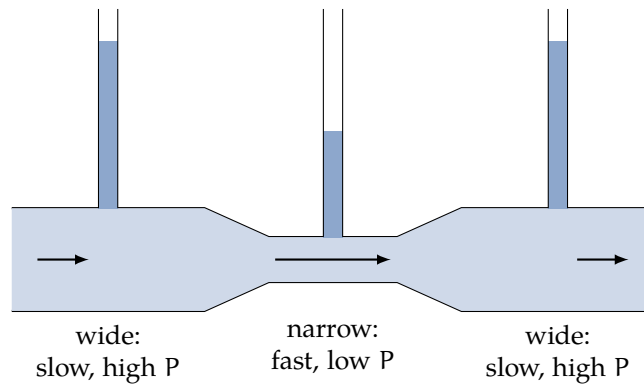


Figure 5.1: Water flowing through a pipe that narrows (a *Venturi tube*). The water speeds up in the narrow section (continuity), so its pressure drops there (Bernoulli). The vertical tubes show the pressure: the water climbs less high above the narrow section.

Example: Water flows at 2.0 m/s through a horizontal pipe that narrows from 4.0 cm² to 1.0 cm². How much lower is the pressure in the narrow part?

Solution: By continuity, the water speeds up to $v_2 = \frac{4.0}{1.0}(2.0) = 8.0$ m/s. The height is the same, so Bernoulli's equation gives

$$P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) = \frac{1}{2}(1000)(64 - 4.0) = 30,000 \text{ Pa}$$

That is enough to hold up a column of water about 3 meters tall. Measuring this pressure drop is how a *Venturi meter* measures how fast a fluid is flowing. Perfume sprayers and old car carburetors use the same effect in reverse: fast air over the top of a thin tube lowers the pressure there, and the liquid is pushed up the tube into the airstream.

5.3 Water Out of a Tank

Punch a hole in a water tank a depth h below the surface. The water at the surface is nearly still and the water in the jet is moving fast, but both are open to the air, so both are at atmospheric pressure. The P terms cancel, and Bernoulli's equation leaves

$$\frac{1}{2}\rho v^2 = \rho gh \implies v = \sqrt{2gh}$$

This is *Torricelli's law*: the water leaves exactly as fast as if it had fallen freely from the height of the surface. A barrel with a hole 1.8 m below the water line squirts at $\sqrt{2(9.8)(1.8)} \approx 5.9$ m/s.

As the tank drains, h shrinks, so the jet slows, and the tank drains more and more slowly. Continuity tells us how fast the surface drops: the volume leaving through the hole each second, $a\sqrt{2gh}$, equals the tank's area A times the speed of the surface. A computer can follow this in small time steps:

```

1 import math
2 import matplotlib.pyplot as plt
3
4 g = 9.8
5 tank_area = 0.5          # m^2, the tank's cross-section
6 hole_area = 2e-4         # m^2, a 2 cm^2 hole at the bottom
7 h = 1.8                  # m, starting water depth
8 dt = 0.1                 # s, one small time step
9
10 t = 0
11 times, depths = [0], [h]
12 while h > 0:
13     v = math.sqrt(2 * g * h)          # Torricelli: speed of the jet
14     flow = hole_area * v              # m^3 leaving each second
15     h -= flow / tank_area * dt         # the surface drops (continuity)
16     t += dt
17     times.append(t / 60)
18     depths.append(max(h, 0))
19
20 print(f"drains in {t / 60:.1f} minutes")
21 exact = tank_area / hole_area * math.sqrt(2 * 1.8 / g)
22 print(f"exact answer: {exact / 60:.1f} minutes")
23
24 plt.figure(figsize=(6, 3), dpi=200)
25 plt.plot(times, depths)
26 plt.xlabel("time (minutes)")
27 plt.ylabel("water depth (m)")
28 plt.tight_layout()
29 plt.savefig("draining_tank.png")

```

drains in 25.2 minutes
exact answer: 25.3 minutes

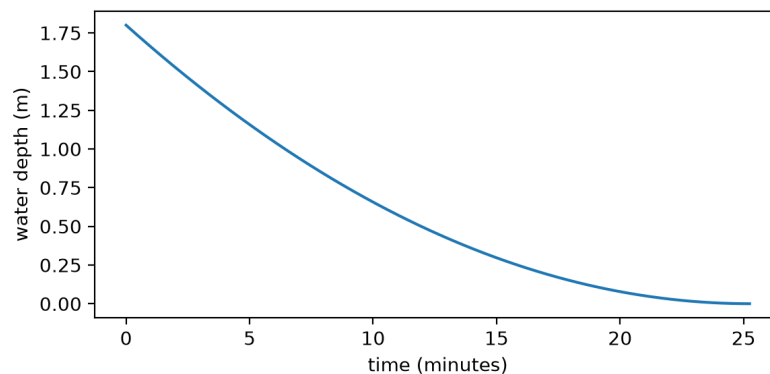


Figure 5.2: The water level falls fast at first and slowly at the end, because the jet weakens as the water gets shallower.

5.4 Explaining the Puzzles

The shower curtain. The spray drags air along with it, so the air inside the shower moves faster than the still air outside. Lower pressure inside, higher pressure outside: the curtain is pushed in.

Passing ships. Water rushing through the narrow channel between two ships moves faster than the water on their outer sides. The pressure between them drops, and the water outside pushes them together. Ship captains are trained to keep their distance for exactly this reason.

A roof in a hurricane. Wind at 50 m/s sweeps over a roof, while the air inside the house is still. The pressure difference is $\frac{1}{2}\rho v^2 = \frac{1}{2}(1.2)(50^2) = 1,500$ Pa, which seems small. But over a 150 m² roof, it adds up to an upward force of 225,000 N, the weight of about 23 tonnes. Roofs are torn off from below, which is why hurricane building codes require strong straps tying the roof to the walls.

5.5 What About Wings?

You may have heard this explanation of how wings make lift: the top of the wing is curved, so air flowing over the top has farther to go; it must go faster to “meet up” with the air under the wing; faster means lower pressure; so the wing is pushed up.

The middle of that story is false. Nothing makes the two streams meet up. In wind tunnels, the air over the top of a wing actually arrives at the back edge *first*, well before the air underneath. And as the Airplanes chapter points out, a wing with a flat top or an upside-down wing can still fly, if it meets the air at an angle.

Here is what really happens. A wing tilted into the air flow turns the air downward behind it. By Newton’s third law, if the wing pushes air down, the air pushes the wing up. Turning the flow also changes its speed: the air speeds up over the top and slows down under the bottom. So Bernoulli’s equation is right that the pressure is lower on top. It is just not the cause: Newton’s laws and Bernoulli’s equation are two ways of describing the same flow, and both give the same lift.

Example: A 1,000 kg small plane has 16 m² of wing. In level flight, the lift must equal its weight. What pressure difference does that take?

Solution: $\Delta P = \frac{(1000)(9.8)}{16} \approx 610$ Pa, about 0.6% of atmospheric pressure. If the air under the wing moves at 50 m/s, Bernoulli’s equation says the air over the top needs to move only about 59 m/s: $\frac{1}{2}(1.2)(59^2 - 50^2) \approx 610$ Pa. A small speed difference over a large area holds a plane in the sky.

The Airplanes chapter’s lift equation, $L = \frac{1}{2}\rho v^2 SC_L$, is built on this idea: the dynamic

pressure $\frac{1}{2}\rho v^2$ times the wing area, times a number C_L that measures how well the wing's shape and angle turn that dynamic pressure into a pressure difference.

Exercise 21 A River Narrows

A river 30 m wide and 2.0 m deep flows at 1.0 m/s. Downstream, it squeezes through a gorge 10 m wide and 3.0 m deep. How fast does the water move in the gorge?

Working Space

Answer on Page 107

Exercise 22 Water Tower

The water level in a town's water tower is 20 m above a faucet. Ignoring friction in the pipes, how fast would water come out of the open faucet? Why is the real speed lower?

Working Space

Answer on Page 107

Exercise 23 A Venturi Meter

Water moves at 3.0 m/s in a horizontal pipe with a cross-section of 6.0 cm^2 . It flows into a section with a cross-section of 2.0 cm^2 . Find the speed in the narrow section and the drop in pressure.

Working Space

Answer on Page 107

Exercise 24 **Blowing Between Two Sheets**

Hold two sheets of paper a few centimeters apart, hanging down, and blow between them. Predict what happens, and explain why.

Working Space

Answer on Page 107

Exercise 25 **An Open Window?**

Some people say you should open your windows during a hurricane to “equalize the pressure.” Using the roof example, explain what the pressure difference across a roof comes from, and why opening windows is not a reliable fix. (Real advice from engineers: keep windows closed and protected.)

Working Space

Answer on Page 107

Airplanes

A loaded Boeing 747 weighs about 400,000 kg, as much as 300 cars. Yet it lifts off the runway and cruises at 900 km per hour. In this chapter, you will see how. Good news: if you understood how a sail works, you already understand most of an airplane.

6.1 Drag and Thrust

As an airplane flies, the air pushes back on it, trying to slow it down. Pilots call this force *drag*. You can think of drag as air friction.

Drag grows with the square of the speed. Double your speed, and the drag becomes four times as large. (You have felt this: stick your hand out of a car window at 30 km/h, then at 60 km/h.)

To overcome drag, an airplane needs *thrust*, a forward push. Airplanes get thrust by throwing air toward the back of the plane. By Newton's third law, the air pushes the plane forward.

All that pushing takes a lot of fuel. A Boeing 747 burns about 4 liters of fuel every second, and a flight from London to New York needs about 70,000 kg of it. At takeoff, the fuel can weigh nearly twice as much as all the passengers.

6.2 Lift

A hot air balloon floats because it is lighter than the air it displaces. An airplane is far heavier than the air it displaces, so gravity would pull it from the sky without another upward force. That force is *lift*.

Lift works just like sailing on a close reach. The wing meets the air at a slight tilt, its *angle of attack*. Air piles up under the wing, creating high pressure. Air flowing smoothly over the curved top of the wing leaves low pressure there. The difference in pressure pushes the wing up. A shape designed to do this is called an *airfoil*. Wings, propeller blades, kites, and sails are all airfoils.

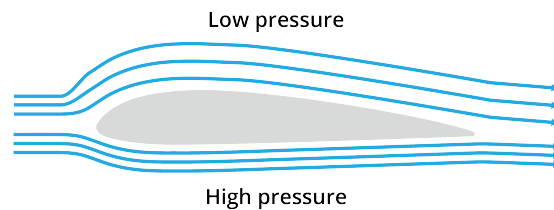


Figure 6.1: Air flowing around a wing. The wing deflects air downward, leaving high pressure below and low pressure above.

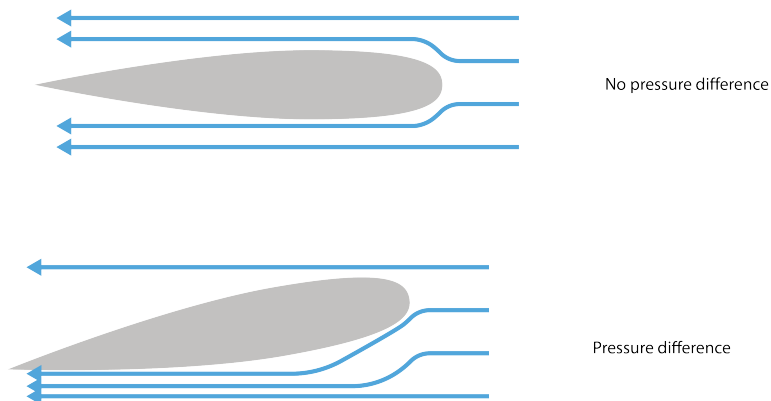


Figure 6.2: A symmetric airfoil meeting the air head-on (top) creates no pressure difference. Tilted to an angle of attack (bottom), it does.

You may have heard that lift comes mostly from the rounded top of the wing. That explanation oversells the curve. Stunt planes fly upside down for minutes at a time, which would be impossible if the curved top did all the work. The angle of attack matters

most; the curve helps the air stay smoothly attached to the wing.

Engineers capture all of this in the *lift equation*:

$$L = \frac{1}{2}\rho v^2 S C_L$$

where ρ is the density of the air, v is the airspeed, S is the wing area, and C_L is the *lift coefficient*, a number that depends on the wing's shape and angle of attack. Notice the $\frac{1}{2}\rho v^2$: it is exactly the wind pressure from the sailing chapter, which the Fluids in Motion chapter called the dynamic pressure. That chapter also explains how Bernoulli's equation and Newton's third law describe the same lift.

Worked example. A small plane has a wing area of 20 m^2 and flies at 50 m/s through air with $\rho = 1.2 \text{ kg/m}^3$. Its lift coefficient is 0.5 . How much lift do the wings produce?

Solution. $L = \frac{1}{2}(1.2)(50)^2(20)(0.5) = 15,000 \text{ N}$. That is enough to hold up about $\frac{15,000}{9.8} \approx 1,500 \text{ kg}$, the mass of a small plane with its passengers.

Exercise 26 Doubling the Speed

Using the same wing and angle of attack, what is the lift at 100 m/s ?

Working Space

Answer on Page 108

High in the sky, the air is thinner. If two identical planes fly at the same speed at different altitudes:

- The higher plane feels less drag.
- The higher plane's wings make less lift.
- The higher plane's engines get less oxygen per liter of air, which changes how the fuel burns.

Airliners cruise at about 11 km because the thin air means less drag and better fuel economy; they make up for the weaker lift by flying fast.

Exercise 27 Thin air

At 11 km , the air density is about 0.36

Working Space

kg/m^3 . How fast must the plane from the worked example fly to make the same 15,000 N of lift (with the same lift coefficient)?

Answer on Page 108

6.3 Control: Roll, Pitch, Yaw

A plane can rotate three ways. Nose up and down is *pitch*. One wing up and the other down is *roll*. Nose left and right is *yaw*. The pilot has a control for each.

The first control is the throttle, which sets the thrust.

The pilot also has a stick. Pulling back on the stick tilts the *elevator* on the tail up. Air hitting the elevator pushes the tail down and the nose up. Pushing the stick forward does the opposite. The elevator controls pitch.

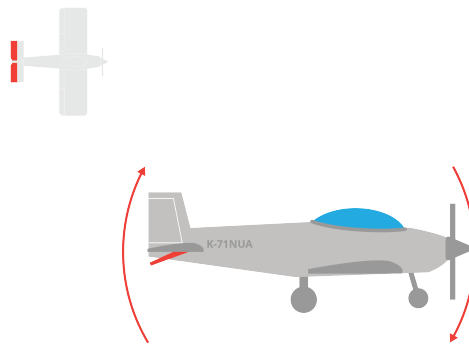


Figure 6.3: Top and side views of the elevator and the pitch it produces.

The stick also moves left and right. Pushing it left raises the *aileron* on the left wing and lowers the aileron on the right wing. The left wing drops and the right wing rises. The ailerons control roll.

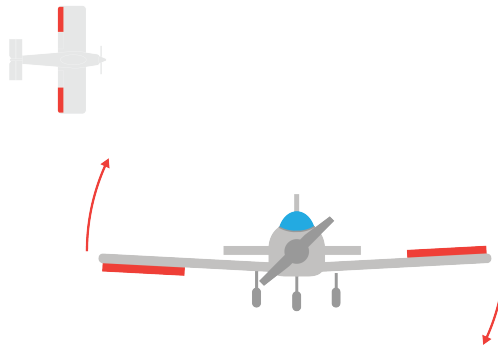


Figure 6.4: Aileron locations on the plane and their effect on roll.

The pilot works the *rudder* on the tail with foot pedals. Pushing the right pedal swings the rudder to the right. Air pushes the tail to the left, and the nose swings to the right. The rudder controls yaw. It works exactly like a boat's rudder.

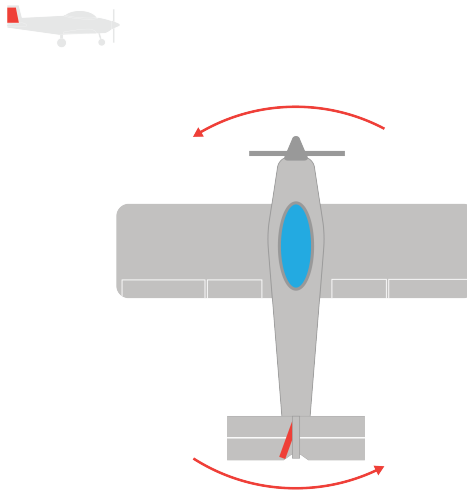


Figure 6.5: The rudder controls yaw, the left-right swing of the nose.

Flaps are panels on the back edges of the wings. For takeoff and landing, the pilot extends them down, which gives the wings more lift (and more drag) at low speeds.

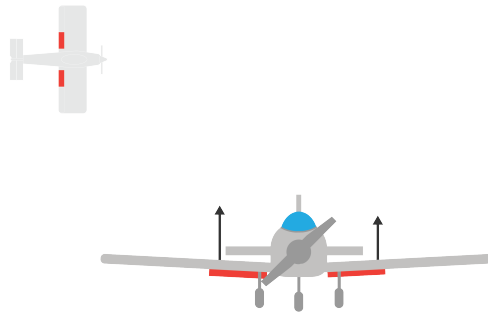


Figure 6.6: The location and effect of the flaps.

6.4 Thrust

There are three common ways to make thrust.

A *propeller* is a spinning set of small wings. Each blade is an airfoil turned on its side, so its “lift” points forward. Small planes spin their propellers with piston engines much like car engines.

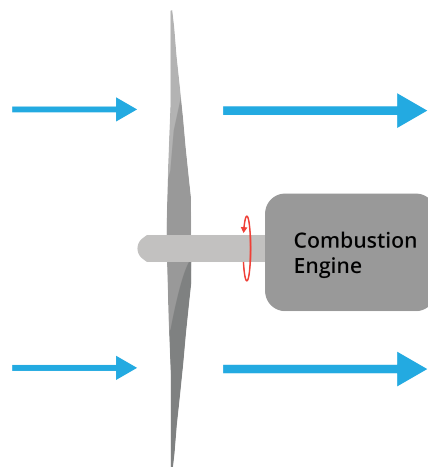


Figure 6.7: Each propeller blade is a small spinning wing whose lift points forward.

A *jet engine* sucks in air, squeezes it with a spinning compressor, sprays in fuel, and burns the mixture. The hot gas roars out the back much faster than the air came in, and that pushes the plane forward. On its way out, the gas spins a turbine, which drives the compressor at the front.

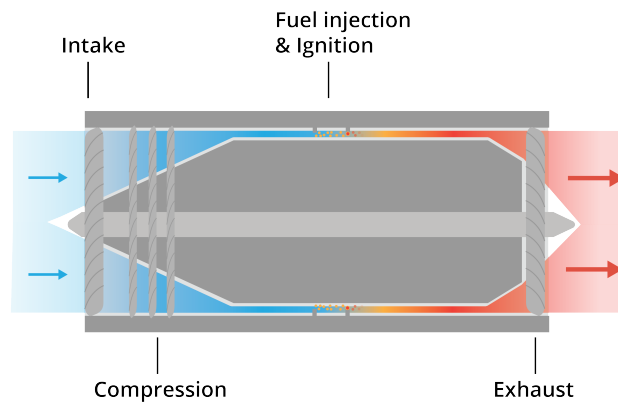


Figure 6.8: Cross section of a jet engine.

A *turboprop* uses a jet engine's turbine to spin a propeller. Turboprops are efficient at the lower speeds and shorter hops flown by regional airlines.

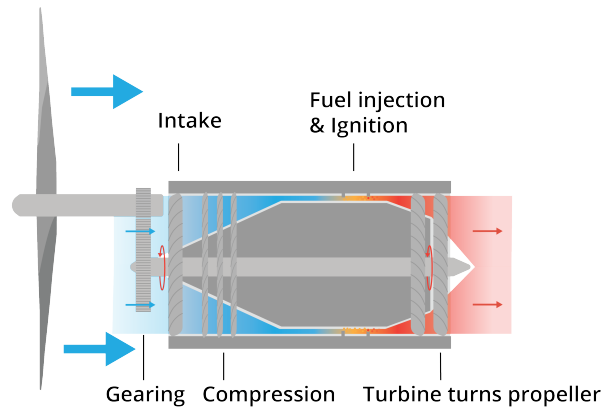


Figure 6.9: Cross section of a turboprop.

6.5 Gliders

A glider has no engine at all. It starts high in the air with lots of potential energy and trades height for speed, sliding gently downhill through the air. A good glider can travel 40 meters forward for every meter it descends; this is its *glide ratio*. To stay up for hours, glider pilots hunt for rising air, such as warm air rising from a sunny parking lot, and circle in it to regain height.

Exercise 28 **How far can it glide?**

A glider with a glide ratio of 40 is released at an altitude of 1,500 m. In still air, how far can it glide before it reaches the ground?

Working Space

Answer on Page 108

Airplanes need runways and a steady stream of air over their wings. The next chapter, Drones, Helicopters, and Rockets, looks at machines that can hover, and at machines that fly where there is no air at all.

Drones, Helicopters, and Rockets

A drone the size of your hand can hover perfectly still, then dart sideways without tilting its nose. A helicopter can land on a hospital roof. A rocket climbs where there is no air at all to push against. In the Airplanes chapter, you saw how wings make lift by turning air downward. Rotorcraft use the same idea with spinning wings, and rockets throw their own mass backward instead of pushing on air.

7.1 Rotorcraft

7.1.1 Quadcopters

Let us start with *quadcopters*, the four-rotor aircraft most people call drones. (You may also hear rotorcraft, or UAVs, for unmanned aerial vehicles.)

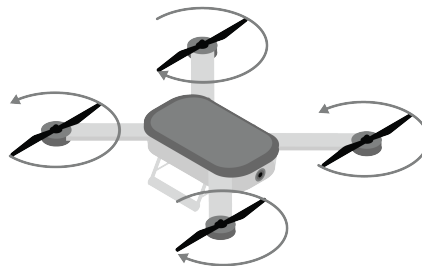


Figure 7.1: A quadcopter: four rotors on a central body, with one end chosen as the front.

A few terms will come up again and again:

Rotor The rotor is the spinning part that makes lift. It has a central *hub* with two or more *blades* attached. People often say propeller, but rotor is the better word for a part that lifts the aircraft.

UAS / UAV Unmanned Aircraft System or Unmanned Aerial Vehicle: a drone, plus the software and controllers that fly it.

Fuselage The main body, or frame.

Torque A twisting force. Push on a door near its hinges and it barely moves; push near the handle and it swings easily. The second push has more torque. You will study torque in detail later. For now, know that every spinning rotor twists the drone the opposite way, and those twists must be balanced.

7.1.2 VTOL

An airplane needs a runway to build up enough speed for its wings to lift it. A quadcopter's rotors make lift while the drone sits still. Each rotor throws air downward, and by Newton's third law the air pushes the rotor up. So a quadcopter can take off and land straight up and down: this is called *Vertical Take-Off and Landing*, or VTOL.

Two forces act vertically: the total thrust of the rotors, upward, and the weight mg , downward. Newton's second law says

$$F_{\text{rotors}} - mg = ma$$

To climb, the rotors push harder than the weight. To hover, they push exactly as hard as the weight:

$$F_{\text{rotors}} = mg \tag{7.1}$$

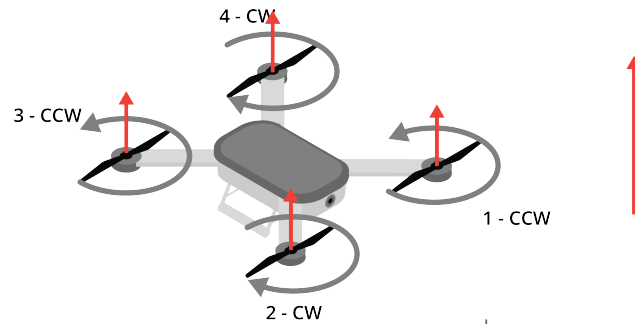


Figure 7.2: A quadcopter climbing straight up. All four rotors push equally (red arrows).

Exercise 29 Hover and climb

A 1.2 kg drone hovers. How much total thrust do its rotors make? How much thrust does each rotor make? How much total thrust would it need to accelerate upward at 2 m/s^2 ?

Working Space

Answer on Page 108

7.1.3 Rotor Structure

How does a rotor make lift? Each blade is a small wing. As it spins, it meets the air at an angle of attack and pushes the air down.

Each blade has a *root*, which connects to the hub, and a *tip* at its outer end. Most blades are *twisted*: steeply angled near the root and flatter near the tip, like the thread of a screw. Why? The tip travels around a much bigger circle than the root, so it moves much faster

through the air. A flatter angle at the tip keeps every part of the blade pushing about equally.

Spin a rotor backward, and it pushes air the other way.

Rotors are described by their *diameter*, their *pitch*, and their *number of blades*. A “10x5” rotor has a 10 inch diameter and a 5 inch pitch.

The **diameter** is the width of the circle the tips sweep. A bigger rotor pushes a bigger column of air more slowly. For the same thrust, that takes less power. This is why a helicopter uses one huge, slow rotor, while a small drone, which must stay compact, uses several small, fast ones.

The **pitch** is how far the rotor would screw itself forward in one full turn, if the air were solid like wood. A higher pitch takes a bigger bite of air with each turn: more thrust per turn, but the motor must work harder, so it cannot spin as fast. A lower pitch spins faster but moves less air per turn.

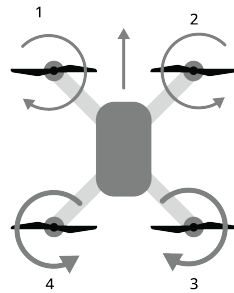
Most rotors have **two blades**. They balance easily and are cheap to make. More blades give more thrust at the same size and speed, but each blade partly flies through the churned-up air left by the blade ahead of it, so each extra blade adds less than the one before.

7.1.4 Movement, Spin Direction, and Torque

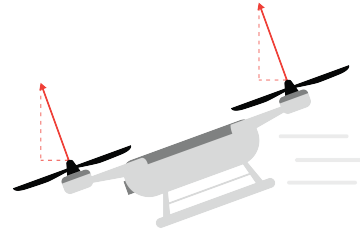
Two rotors spin clockwise (CW) and two spin counterclockwise (CCW). Rotors diagonally across from each other spin the same way; neighbors spin opposite ways. Each spinning rotor twists the body the other way. If all four spun the same direction, the drone would whirl around like a top. With two of each, the twists cancel, and the drone can hover perfectly still.

Many drones offer *headless mode*, where “forward” always means “away from the pilot,” no matter which way the drone faces. It is easier for beginners. Normally, though, one end of the drone is its front, or head, and forward means the direction the head points. In the rest of this section, we ignore drag and assume the center of mass is in the middle of the body.

To fly forward, the drone *itches* its nose down. The two back rotors spin faster than the two front rotors, so the back rises. Now the rotors’ thrust tilts forward. Part of it still holds the drone up, and part of it pushes the drone forward. To fly backward, the front rotors spin faster instead.



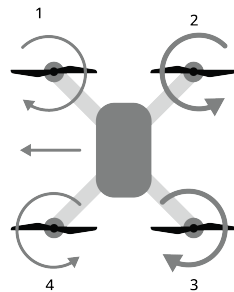
(a) The back rotors (3 and 4) spin faster.



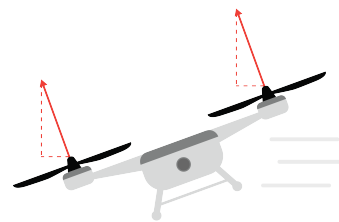
(b) The drone tips forward, and its thrust tilts forward too.

Figure 7.3: Flying forward.

To move sideways, the drone *rolls*. To go left, the rotors on the right spin faster. The right side rises, the drone tips left, and the tilted thrust carries it left.



(a) The right rotors (2 and 3) spin faster.



(b) The drone tips left and slides left.

Figure 7.4: Flying sideways.

Turning in place is *yaw*. Here the drone uses the twisting effect on purpose. Speed up the two clockwise rotors and slow down the two counterclockwise ones, keeping the total thrust the same. The clockwise rotors now twist the body harder, so the body turns counterclockwise (to the left). Speed up the counterclockwise pair, and it turns to the right.

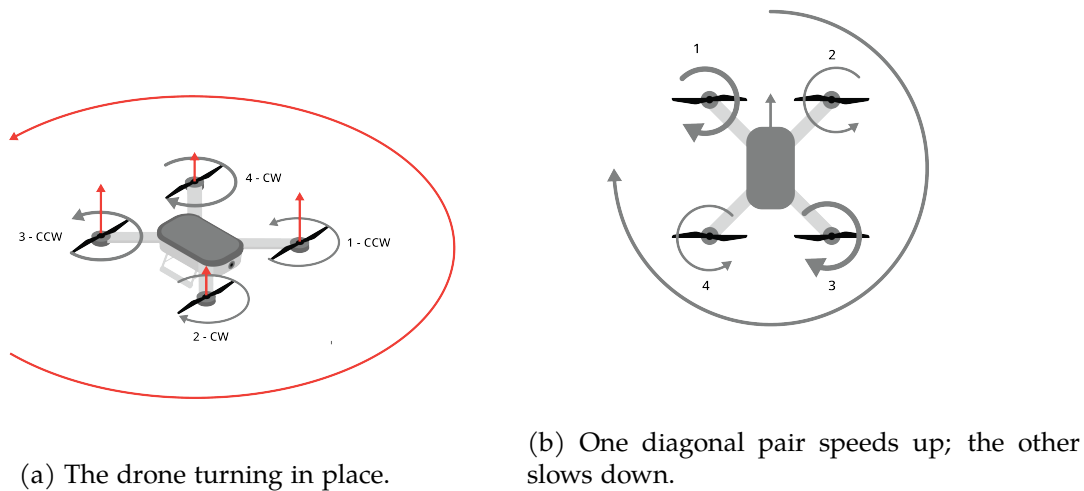


Figure 7.5: Yaw.

Your digital resources include a guide to choosing rotors for different kinds of drones.

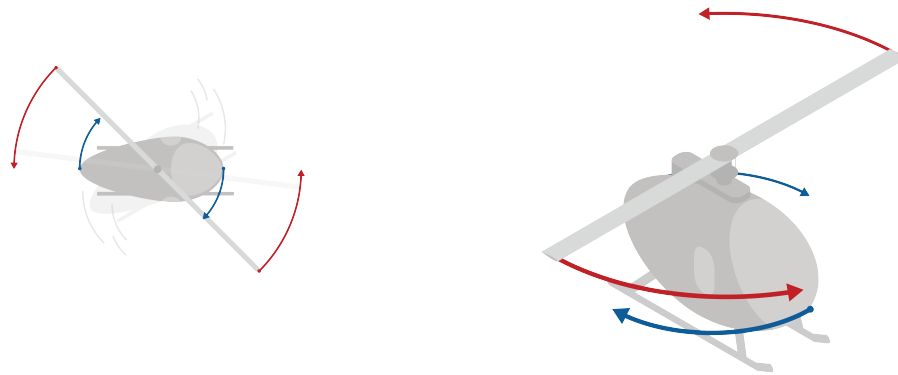
7.1.5 Helicopters

A *helicopter* lifts itself with one big *main rotor*. The physics is the same as a quadcopter's: the blades throw air down, and the air pushes the helicopter up. But one big rotor creates two new problems.

First, the main rotor's twist has nothing to cancel it. Second, a helicopter cannot steer by speeding up some rotors and slowing others, because it has only one. Let us take these one at a time.

Tail Rotor

On most American helicopters, the main rotor turns counterclockwise when seen from above. By Newton's third law, the rotor twists the body the other way, clockwise. With nothing to stop it, the body would spin faster and faster until the helicopter crashed.



(a) Seen from above: the rotor turns one way (red), the body the other (blue).

(b) The body keeps spinning faster unless something stops it.

Figure 7.6: A helicopter with no tail rotor.

The fix is a small *tail rotor* at the end of a long tail. It spins in a vertical plane and pushes air sideways. Because it is so far from the main rotor, even a small push makes a big counter-twist (just as a door is easy to push far from its hinges).

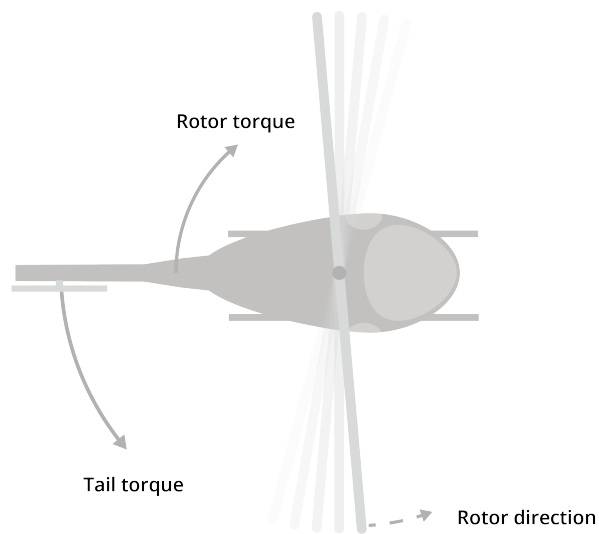


Figure 7.7: The tail rotor's push (tail torque) balances the twist from the main rotor (rotor torque).

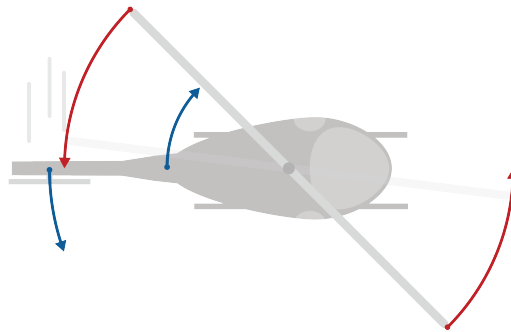


Figure 7.8: With a tail rotor, the twists balance and the body holds still.

The tail rotor also steers. The pilot's foot pedals change the pitch of the tail rotor blades. To turn left, the pilot increases the pitch. The tail rotor pushes harder, overpowering the main rotor's twist, and the nose swings left. To turn right, the pilot decreases the pitch, and the main rotor's twist swings the nose right (see figure 7.9).

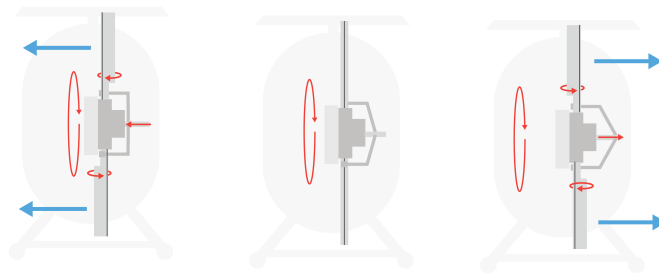


Figure 7.9: Seen from behind: changing the tail rotor's pitch changes how hard it pushes, which controls yaw.

Collective and Cyclic

The main rotor spins at a nearly constant speed. To control it, the pilot changes the pitch (angle of attack) of the blades as they spin. The clever device that does this is the *swash plate*: a pair of plates around the rotor shaft. The bottom plate does not spin. The top plate spins with the rotor, and links connect it to each blade. Push the plates up or tilt them, and you change the blades' pitch.

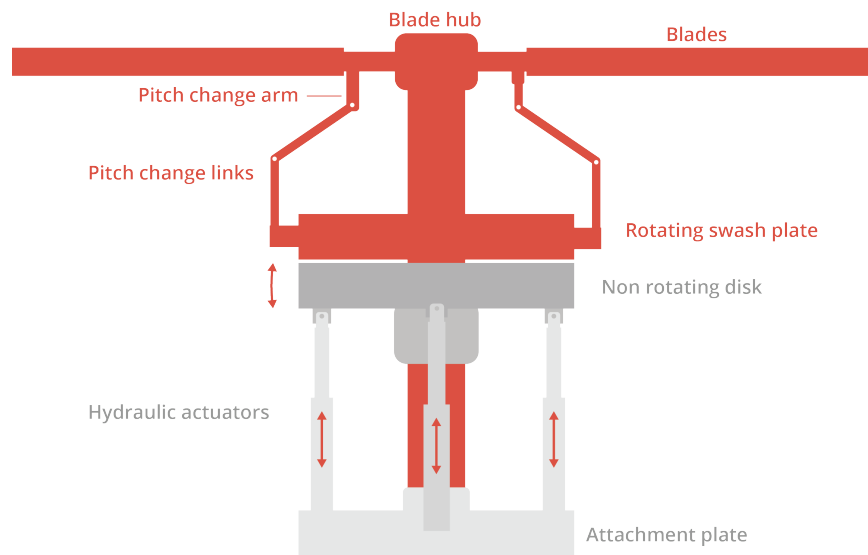
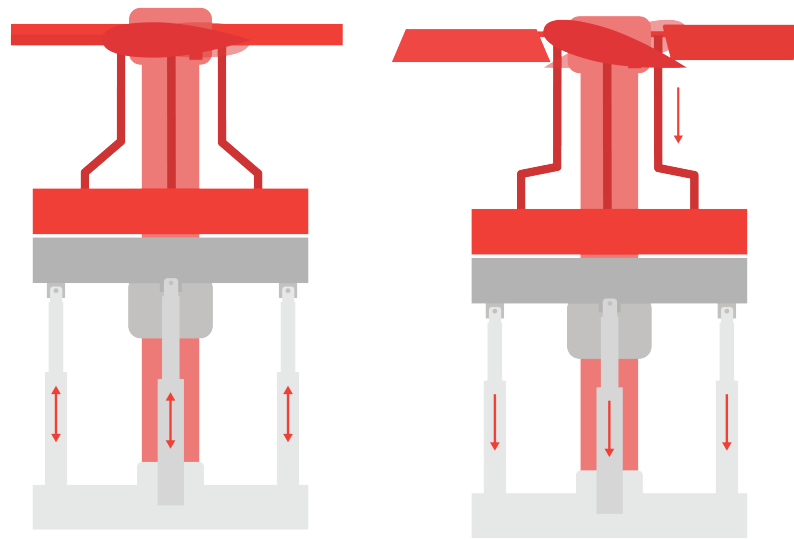


Figure 7.10: The swash plate. Actuators move the non-rotating plate; the rotating plate above it passes the motion to the blades through the pitch change links.

The *collective* raises or lowers the whole swash plate. That changes the pitch of all the blades together, so the whole rotor makes more lift or less. Raise the collective, and the helicopter climbs.



Collective

Figure 7.11: Collective: the swash plate moves straight up or down, changing every blade's pitch by the same amount.

The *cyclic* tilts the swash plate. Now each blade's pitch changes as it goes around: steep on one side of the circle, flat on the other. One side of the rotor makes more lift than the other, so the spinning disk tips, and its thrust tilts, just like a quadcopter tipping forward. Push the cyclic forward, and the helicopter flies forward. (One surprise: a spinning rotor acts like a gyroscope, so it tips about a quarter turn after the point where the extra lift is added. The linkage is built to account for that.)

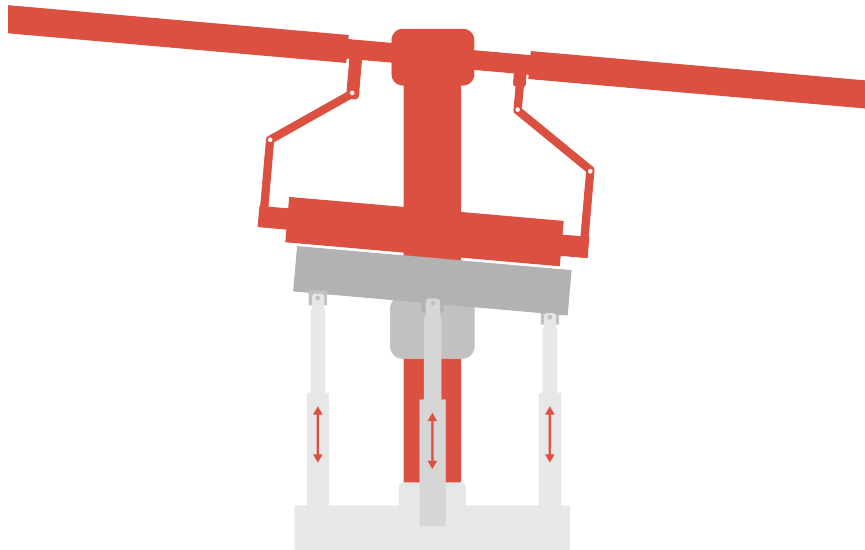


Figure 7.12: Cyclic: the swash plate tilts, so each blade's pitch rises and falls once per turn.

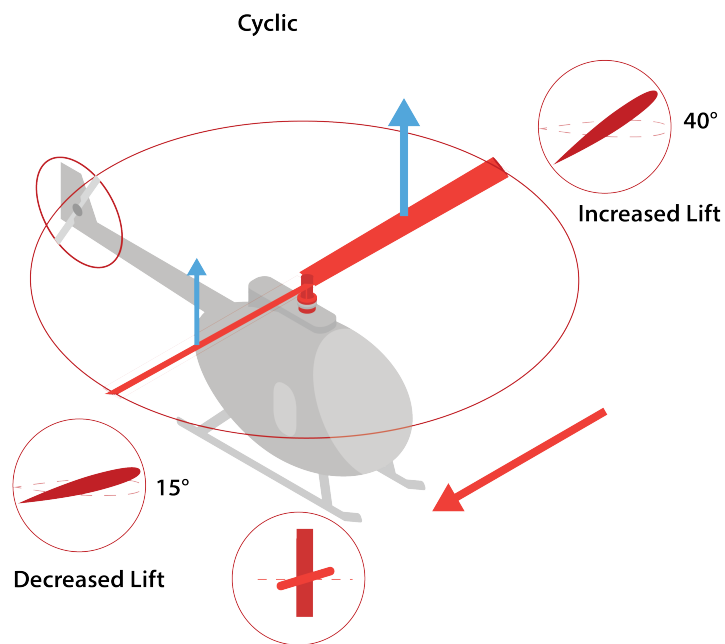
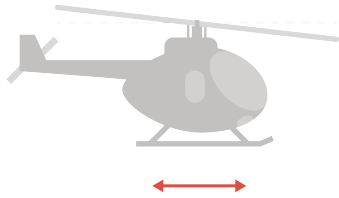
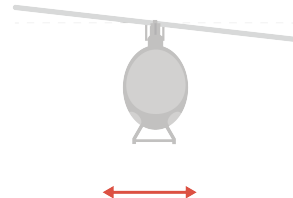


Figure 7.13: With cyclic applied, one blade meets the air at a steep angle (more lift) while the opposite blade meets it at a shallow angle (less lift).

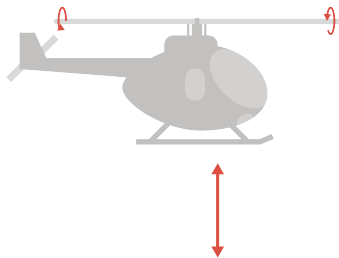
Putting it all together:



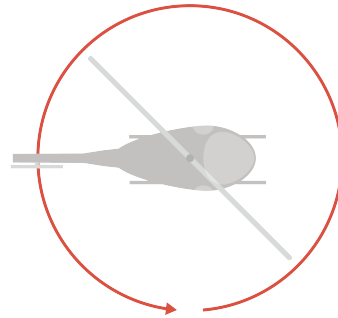
(a) Forward and back: cyclic.



(b) Side to side: cyclic.



(c) Up and down: collective.



(d) Turning: tail rotor pedals.

Figure 7.14: The helicopter's controls.

7.2 Rocketry

Out in space, there is no air to push against. So how does a rocket move?

Imagine floating in space holding a bowling ball. Throw the ball away from you, and you drift the other way. That is Newton's third law, and momentum conservation: the ball's momentum one way equals your momentum the other way.

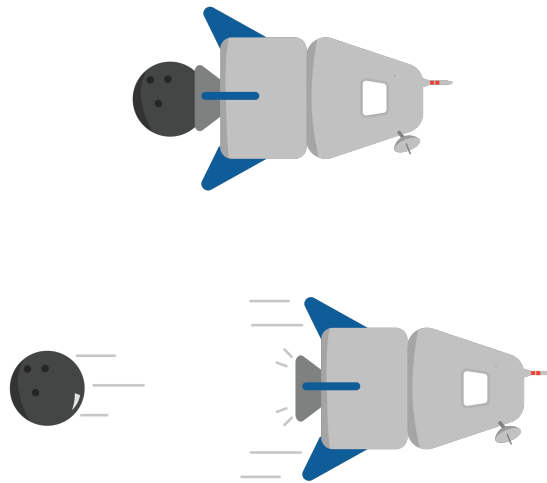


Figure 7.15: Throwing mass backward pushes the spacecraft forward.

A rocket does the same thing, but instead of bowling balls, it throws enormous numbers of hot gas molecules out the back at very high speed. There is no air in space to supply oxygen for burning, so rockets carry their own *oxidizer* along with their fuel.

7.2.1 Types of Rocket Fuels

There are two main kinds of chemical rockets.

A *solid fuel rocket* burns a solid mix of fuel and oxidizer, like a giant firework. Once lit, it cannot be turned off until the fuel is gone.

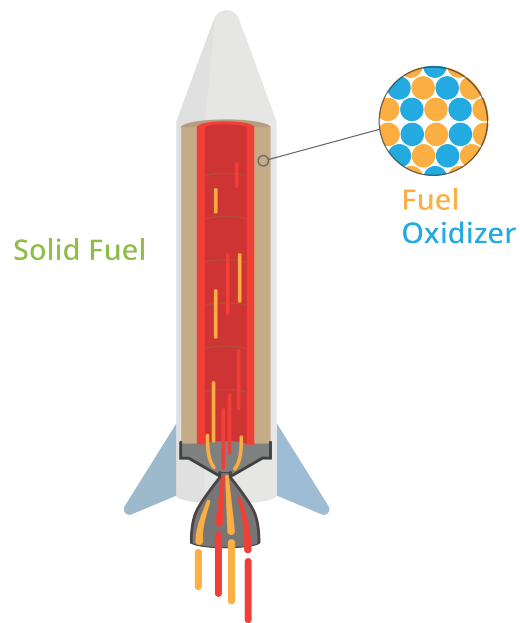


Figure 7.16: A solid fuel rocket.

A *liquid fuel rocket* keeps liquid fuel and liquid oxidizer in separate tanks. Pumps feed them into a combustion chamber, where they burn and blast out the nozzle. Most liquid fuel engines can be throttled up and down, and some can be shut off and restarted.

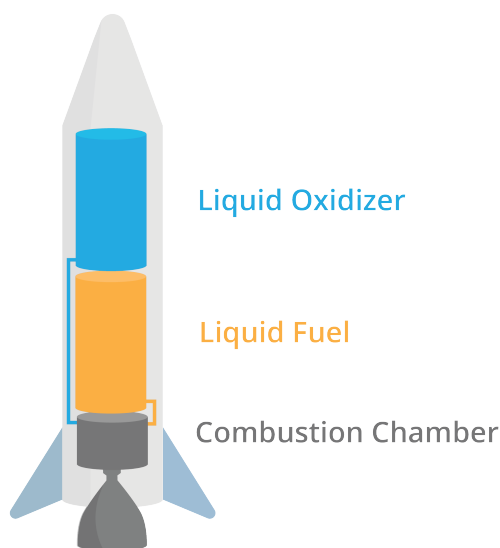


Figure 7.17: A liquid fuel rocket: separate tanks of oxidizer and fuel feed the combustion chamber.

7.2.2 The Tyranny of the Rocket Equation

A rocket must carry all its fuel. But fuel is heavy, so the rocket needs more fuel to lift that fuel, which is heavy, so it needs more fuel...

The *rocket equation* tells you how bad this gets:

$$\Delta v = v_e \ln \left(\frac{m_0}{m_f} \right)$$

Here Δv is the change in the rocket's speed, v_e is the speed of the exhaust, m_0 is the starting mass (with fuel), and m_f is the final mass (fuel used up). The $\ln$ is the natural logarithm. You will study it in a later volume; for now, use the $\ln$ button on your calculator, or undo it with e^x .

Worked example. Reaching low Earth orbit takes a Δv of about 9.4 km/s. A good kerosene engine has $v_e \approx 3$ km/s. What fraction of the rocket must be fuel?

Solution. $\frac{m_0}{m_f} = e^{9.4/3} \approx 23$. The rocket must start 23 times heavier than it ends. So about 96% of the rocket at launch is fuel and oxidizer, and only 4% is everything else: tanks, engines, and the satellite. That is the tyranny.

One way to fight back is *staging*. A staged rocket is several rockets stacked together. When the bottom stage runs out of fuel, it drops away, and the rocket no longer has to haul its empty tanks and heavy engines.

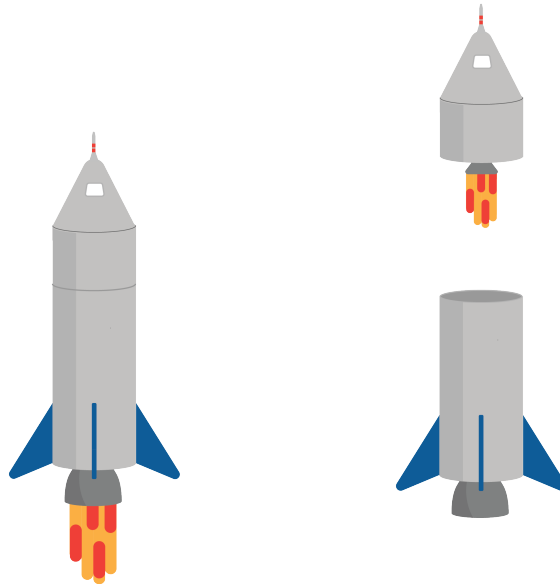


Figure 7.18: A two-stage rocket drops its first stage once that stage's fuel is used up.

7.2.3 Control in the Atmosphere

Sensors on board track which way the rocket points, and a computer adjusts the controls many times a second to keep it on course. There are three common controls.

Movable fins work like an airplane's control surfaces.

A *gimbaled engine* can pivot. Swinging the nozzle changes the direction of the thrust, which steers the rocket. This is called *thrust vectoring*. Most modern rockets steer this way.

Vernier engines are small extra engines that fine-tune the direction. They add weight, so they are rarely used today.

- **Movable fins**

- **Pros:** Simple, reliable, light, and effective at high speed in the atmosphere.
- **Cons:** Useless in a vacuum; weak at low speed or in thin air.

- **Gimbaled engine (thrust vectoring)**
 - **Pros:** Precise; works in air and in vacuum; responds quickly.
 - **Cons:** Mechanically complex, heavier, more expensive, more things that can fail.
- **Vernier engines**
 - **Pros:** Fine control; a backup if something fails.
 - **Cons:** Heavy and less efficient.

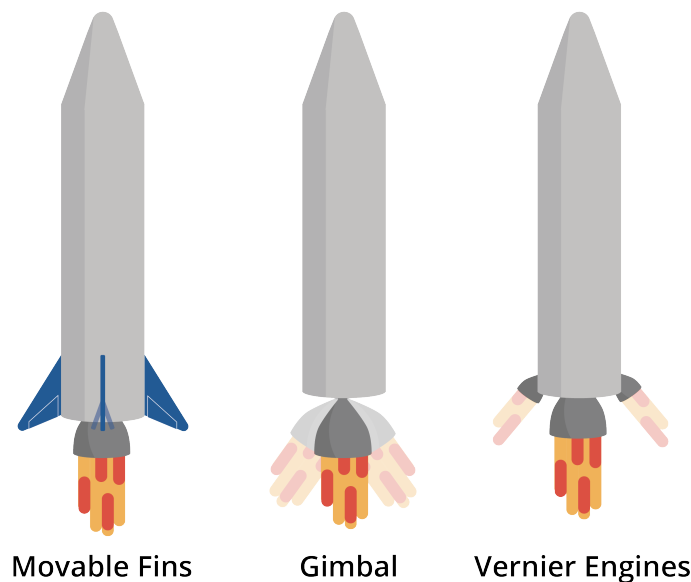


Figure 7.19: Three ways to steer a rocket in the atmosphere.

7.2.4 Control in Space

Once in space, a spacecraft still needs to turn and adjust its position.

A *reaction control system* (RCS) is a set of small thrusters, often mounted in clusters of 3 or 4, that can nudge the spacecraft in any direction. RCS thrusters are used for docking, turning, and small orbit changes.

Reaction wheels are heavy wheels inside the spacecraft. Spin a wheel one way, and the spacecraft turns the other way. (Try it on a spinning office chair: hold a bicycle wheel and spin it, and the chair turns the other way.) With three wheels on three axes, the spacecraft can point anywhere without using any fuel.

A *magnetorquer* is an electromagnet that pushes against Earth's magnetic field, the way a compass needle turns to line up with it. Magnetorquers are popular on small satellites in low orbit. For large spacecraft or deep space, Earth's magnetic field is too weak to help.

- **RCS thrusters**

- **Pros:** Precise; can both move and turn the spacecraft; reliable.
- **Cons:** Use up propellant; add weight and complexity.

- **Reaction wheels**

- **Pros:** No propellant; precise; long-lasting; quiet.
- **Cons:** Limited turning force. A wheel can only spin so fast; once it maxes out, thrusters must unload it.

- **Magnetorquers**

- **Pros:** No propellant; light; simple.
- **Cons:** Only work near a planet with a magnetic field; weak.

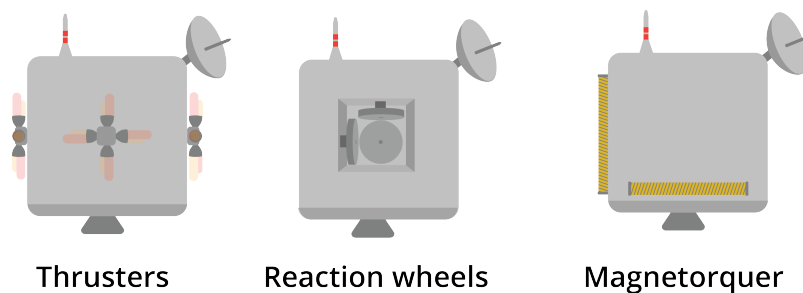


Figure 7.20: Three ways to steer a spacecraft in space.

7.2.5 Alternative Propulsion

A *solar sail* is a huge, thin, shiny sheet. Light carries momentum, so each photon that bounces off the sail gives it a tiny push. The push is feeble, but it never stops, and the spacecraft needs no fuel.

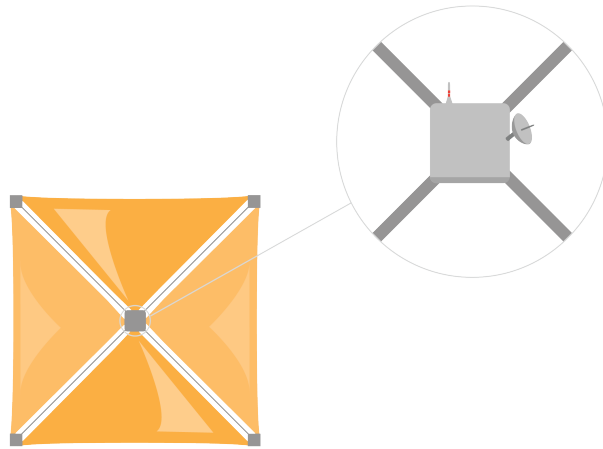


Figure 7.21: A spacecraft with a solar sail.

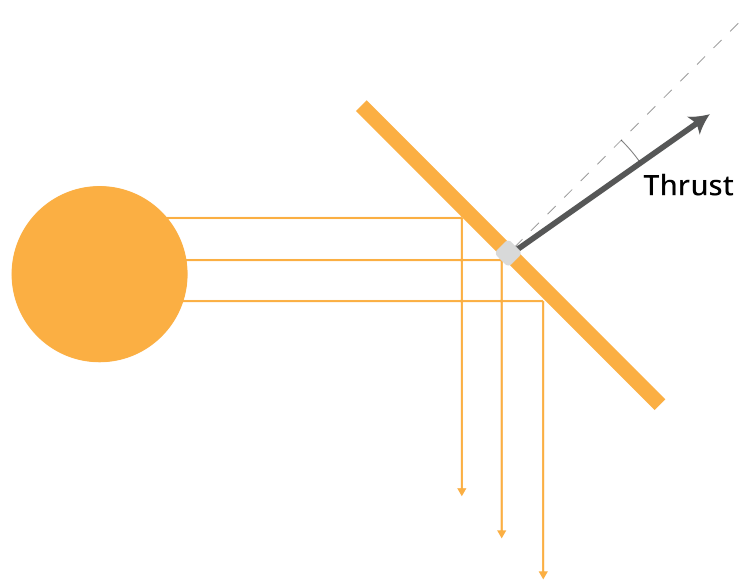


Figure 7.22: Sunlight reflecting off a tilted sail pushes it at a right angle to its surface.

A perfectly reflective sail would be pushed straight out from its surface, at a right angle to the sail. Real sails absorb a little of the light, and absorbed photons push along the direction the light was traveling. So the real thrust is tilted slightly away from the right angle, toward the direction of the sunlight.

Ion propulsion uses electricity instead of burning. An ion thruster strips electrons from atoms of a gas (usually xenon) and uses electric fields, or electric and magnetic fields together, to fling the charged atoms out the back at tens of kilometers per second. That exhaust is ten times faster than a chemical rocket's, so an ion thruster gets far more push out of each kilogram of propellant. The catch: the thrust is tiny, about the weight of a sheet of paper. Ion thrusters must run for months, which is fine for a probe on a long trip through the solar system.

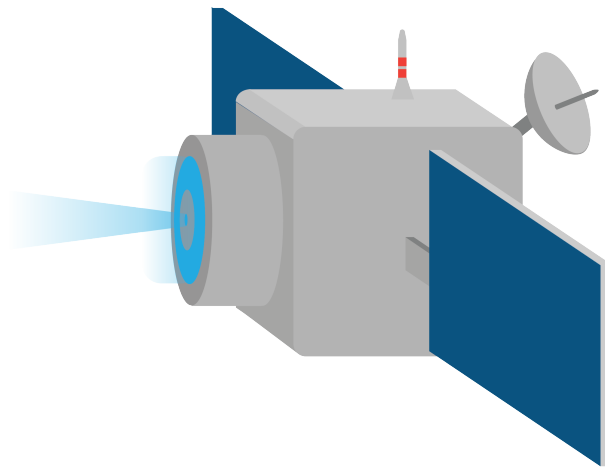


Figure 7.23: An ion thruster.

Exercise 30 **Faster exhaust**

An ion thruster has an exhaust speed of 30 km/s. Using the rocket equation, what fraction of a spacecraft's starting mass must be propellant to gain 9.4 km/s? Compare with the chemical rocket in the worked example.

Working Space

Answer on Page 108

Answers to Exercises

Answer to Exercise 1 (on page 9)

- (a) The normal force must be equal to the perpendicular component of the gravitational force, since there is no acceleration in the y-direction of our rotated y-axis:

$$F_N = mg \cos(\theta) = (30 \text{ kg})(9.8 \text{ m/s}^2) \cos(30^\circ) \approx 254.6 \text{ N}$$

- (b) The frictional force can be found using the coefficient of kinetic friction and the normal force:

$$F_F = \mu_k F_N = (0.3)(254.6 \text{ N}) \approx 76.4 \text{ N}$$

- (c) The acceleration can be found by setting up the equation for the sum of forces in the x-direction:

$$\sum F_x = F_F - mg \sin(\theta) = ma_x$$

Solving for a_x :

$$\begin{aligned} a_x &= \frac{F_F - mg \sin(\theta)}{m} \\ &= \frac{76.4 \text{ N} - (30 \text{ kg})(9.8 \text{ m/s}^2) \sin(30^\circ)}{30 \text{ kg}} \\ &= \frac{76.4 - 147}{30} \\ &\approx \frac{-70.6}{30} \\ &\approx -2.35 \text{ m/s}^2 \end{aligned}$$

Answer to Exercise 2 (on page 10)

For a box to be on the verge of sliding, the frictional force (static) must equal the component of gravitational force parallel to the incline. Thus, we can set up the following

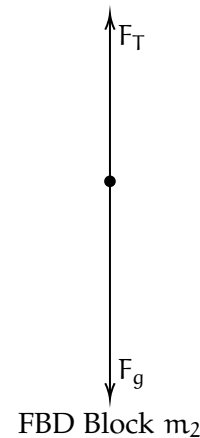
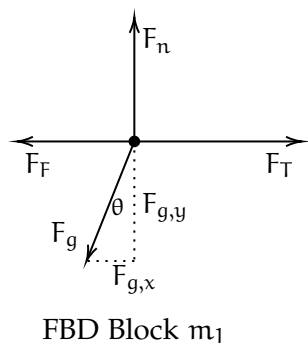
equation:

$$\begin{aligned}\sum F_x &= mg \sin(\theta) - F_F = 0 \\ mg \sin(\theta) &= F_F \\ mg \sin(\theta) &= \mu_s F_N \\ mg \sin(\theta) &= \mu_s mg \cos(\theta) \\ \tan(\theta) &= \mu_s\end{aligned}$$

Solving for θ , we get $\theta = \tan^{-1}(\mu_s)$. Plugging in the value of μ_s , we get $\theta = 24.89^\circ$

Answer to Exercise 3 (on page 10)

Let us create free body diagrams for both objects.



Note that the dashed lines are components of forces, not forces themselves. We can see that the common force uniting the two objects is F_T . We can solve for the acceleration in the x -direction of the block of mass m_1 and the acceleration in the y -direction of m_2 .

$$\begin{aligned}
F_{\text{net},m_1,x} &= F_T - m_1 g \sin \theta - \mu_k m_1 g \cos \theta \\
&= m_1 a_x \\
F_{\text{net},m_2,y} &= m_2 g - F_T \\
&= m_2 a \\
F_{\text{net},m_1,x} + F_{\text{net},m_2,y} &= m_2 g - m_1 g \sin \theta - \mu_k m_1 g \cos \theta = (m_1 + m_2) a \\
a &= \frac{m_2 g - m_1 g \sin \theta - \mu_k m_1 g \cos \theta}{m_1 + m_2}
\end{aligned}$$

Both masses have the same size of acceleration, because the string ties them together: m_2 moves straight down while m_1 moves up along the ramp. Notice that the acceleration is much smaller than g . The hanging block's weight has to drag both masses, fight m_1 's weight along the ramp, and overcome friction.

Answer to Exercise 4 (on page 16)

(a) To find how far from the base of the cliff the rock lands, we can use the horizontal motion equation:

$$\begin{aligned}
\Delta x &= v_{0x} t \\
\Delta x &= (8 \frac{\text{m}}{\text{s}})(3\text{s}) = 24\text{m}
\end{aligned}$$

(b) To find the height of the cliff, we can use the vertical motion equation. Since $v_{0y} = 0$, and the initial vertical position is at the top of the cliff (we can call this $y_0 = 0$), we have:

$$\begin{aligned}
y_f &= v_{0y} t + \frac{1}{2}(-g)t^2 \\
y_f &= 0 + \frac{1}{2}(-10)(3)^2 \\
y_f &= -45\text{m}
\end{aligned}$$

We get a negative value for y_f because the rock is falling downward from its initial position, the top of the cliff which we selected as the origin. Therefore, the height of the cliff is 45 meters.

So the rock lands 24 meters from the base of the cliff, and the cliff is 45 meters high.

Answer to Exercise 5 (on page 19)

Let us first find the horizontal and vertical components of the initial velocity:

$$v_{0x} = (25) \cos(40^\circ) \approx 19.15 \text{ m/s},$$

$$v_{0y} = (25) \sin(40^\circ) \approx 16.07 \text{ m/s}.$$

(a) To find the peak height, we can use the vertical motion equations. Taking it in two steps, let us first find the time to reach the peak height, where the vertical velocity $v_y = 0$:

$$v_y = v_{0y} + (-g)t$$

$$0 = 16.07 - 10t$$

$$t_{\text{peak}} = \frac{16.07}{10} \approx 1.607 \text{ s}$$

Now, we can use this time to find the peak height using the vertical displacement equation:

$$\Delta y = v_{0y}t + \frac{1}{2}(-g)t^2$$

$$\Delta y = (16.07)(1.607) + \frac{1}{2}(-10)(1.607)^2$$

$$\Delta y \approx 12.91 \text{ m}$$

(b) To find the total horizontal distance traveled, we first need the total time of flight. Since the motion is symmetric, the total time will be twice the time to reach the peak height:

$$t_{\text{total}} = 2t_{\text{peak}} \approx 2(1.607) \approx 3.214 \text{ s} = 3.21 \text{ s}$$

Since there is no horizontal acceleration, we can use the horizontal motion equation to find the horizontal distance:

$$\Delta x = v_{0x}t_{\text{total}} \approx (19.15)(3.21) \approx 61.5 \text{ m}$$

So, the peak height of the pumpkin is approximately 12.91 m, and the horizontal distance it travels before hitting the ground is approximately 61.5 m.

Answer to Exercise 6 (on page 19)

There is a trade-off. Aim low, and the ball moves fast sideways but lands quickly. Aim high, and it stays up a long time but barely moves sideways. The best compromise is 45° . Here is the proof.

Let the initial launch speed be v and the launch angle be θ . The horizontal and vertical velocity components are

$$v_x = v \cos \theta, \quad v_y = v \sin \theta.$$

The ball lands when its height returns to zero: $v \sin \theta t - \frac{1}{2}gt^2 = 0$, or

$$0 = t \left(v \sin \theta - \frac{1}{2}gt \right)$$

This has two solutions: $t = 0$ (the time at launch) and

$$t = \frac{2v \sin \theta}{g}.$$

Thus, the horizontal range is

$$R_x = v_x \cdot t = v \cos \theta \cdot \frac{2v \sin \theta}{g} = \frac{v^2}{g} \sin(2\theta).$$

The function $\sin(2\theta)$ reaches its maximum value of 1 when $2\theta = 90^\circ$, or $\theta = 45^\circ$. Therefore, the optimal launch angle for maximum horizontal distance is 45° .

(b) Since $\sin(2\theta) = \sin(180^\circ - 2\theta) = \sin(2(90^\circ - \theta))$, the angles θ and $90^\circ - \theta$ give the same range. There are two ways to get a certain horizontal launch distance, one at an angle less than 45° and one at an angle greater than 45° . For example, a projectile launched at 30° will travel the same horizontal distance as one launched at 60° , assuming the same initial velocity. However, the projectile launched at 30° will spend less time in the air and have a lower peak height than the one launched at 60° .

Answer to Exercise 7 (on page 20)

Using the horizontal distance equation, R , with $v_0 = 14$ m/s and $R = 100$ m, we have:

$$\begin{aligned} R &= \frac{v^2 \sin(2\theta)}{g} \\ \sin(2\theta) &= \frac{Rg}{v^2} \\ \sin(2\theta) &= \frac{(100)(10)}{14^2} \approx 5.10 \end{aligned}$$

Since the sine of an angle cannot be greater than 1, it is impossible to hit the target with a launch speed of 14 m/s.

Instead, we can calculate the minimum launch speed needed to hit the target at a 45° angle, while satisfying the 100 m requirement. Assuming the $\sin(2\theta) = 1$ for a 45° launch angle, we can rearrange the range equation to solve for v :

$$v = \sqrt{\frac{Rg}{\sin(2\theta)}}$$
$$v = \sqrt{\frac{(100)(10)}{1}} = \sqrt{1000} \approx 31.62 \text{ m/s}$$

So at an assumed launch angle of 45° , a minimum launch speed of approximately 31.62 m/s is needed to hit the target 100 meters away.

Answer to Exercise 8 (on page 22)

At 45° , the range drops from 40.8 m to about 31.36 m. Trying angles shows the best angle is now a little lower: 43° gives about 31.43 m. Drag steals speed the whole time the ball is in the air, so a slightly flatter, shorter flight wins. That is why the best launch angle for a real soccer ball or golf ball is below 45° .

Answer to Exercise 9 (on page 23)

$$R = \frac{v^2 \sin 90^\circ}{g} = \frac{400}{9.8} \approx 40.8 \text{ m.}$$

Answer to Exercise 10 (on page 26)

The momentum of the first car is 12,000 kg m/s in the north direction.

The momentum of the second car is 24,000 kg m/s in the east direction.

The new object will be moving northeast. What is the angle compared with the east?

$$\theta = \arctan \frac{12,000}{24,000} \approx 0.4636 \text{ radians} \approx 26.565 \text{ degrees north of east}$$

The magnitude of the momentum of the new object is $\sqrt{12,000^2 + 24,000^2} \approx 26,833 \text{ kg m/s}$

Its mass is 2,500 kg. So the speed will be $26,833/2,500 = 10.73 \text{ m/s}$.

Answer to Exercise 11 (on page 28)

The original forward momentum was 1.2 kg m/s. The original kinetic energy is $(1/2)(0.4)(3^2) = 1.8$ joules.

Let s be the post-collision speed of the ball that had been at rest. Let x and y be the forward and sideways speeds (post-collision) of the other ball. Conservation of kinetic energy says

$$(1/2)(0.4)(s^2) + (1/2)(0.4)(x^2 + y^2) = 1.8$$

Forward momentum is conserved:

$$0.4 \frac{s}{\sqrt{2}} + 0.4x = 1.2$$

Which can be rewritten:

$$x = 3 - \frac{s}{\sqrt{2}}$$

Sideways momentum stays zero:

$$(0.4) \frac{s}{\sqrt{2}} - 0.4y = 0.0$$

Which can be rewritten:

$$y = \frac{s}{\sqrt{2}}$$

Substituting into to the conservation of kinetic energy equation above:

$$(1/2)(0.4)(s^2) + (1/2)(0.4) \left(\left(3 - \frac{s}{\sqrt{2}} \right)^2 + \left(\frac{s}{\sqrt{2}} \right)^2 \right) = 1.8$$

Which can be rewritten:

$$s^2 - \frac{3}{\sqrt{2}}s + 0 = 0$$

There are two solutions to this quadratic: $s = 0$ (before collision) and $s = \frac{3}{\sqrt{2}}$. Thus,

$$y = \frac{3}{2}$$

and

$$x = 3 - \frac{3}{2} = \frac{3}{2}$$

So both balls careen off at 45° angles (on opposite sides of the original path) at the same speed, $\frac{3}{\sqrt{2}} \approx 2.12$ m/s. Notice that they leave at right angles to each other. Pool players know this: when a moving ball hits a resting ball off-center, the two always split at 90° .

Answer to Exercise 12 (on page 29)

The impulse is the change in momentum: $(50)(4) = 200$ kg m/s, upward. Stiff-legged: $F = \frac{200}{0.02} = 10,000$ N. Knees bent: $F = \frac{200}{0.25} = 800$ N. (On top of this, the ground must also hold up the student's weight, about 490 N.) Bending your knees cuts the force by a factor of more than 10.

Answer to Exercise 13 (on page 31)

Because both carts collide elastically, the Conservation of Kinetic Energy and Conservation of Momentum both apply.

We can rearrange the Conservation of Kinetic Energy in the following ways:

$$\begin{aligned}
 \frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 &= \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2 \\
 \frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 - \frac{1}{2}m_1v_{1f}^2 - \frac{1}{2}m_2v_{2f}^2 &= 0 \\
 m_1v_{1i}^2 + m_2v_{2i}^2 - m_1v_{1f}^2 - m_2v_{2f}^2 &= 0 \\
 m_1v_{1i}^2 - m_1v_{1f}^2 + m_2v_{2i}^2 - m_2v_{2f}^2 &= 0 \\
 m_1(v_{1i}^2 - v_{1f}^2) + m_2(v_{2i}^2 - v_{2f}^2) &= 0 \\
 m_1(v_{1i} - v_{1f})(v_{1i} + v_{1f}) + m_2(v_{2i} - v_{2f})(v_{2i} + v_{2f}) &= 0
 \end{aligned} \tag{1}$$

And rearranging Conservation of Momentum:

$$\begin{aligned}
 m_1v_{1i} + m_2v_{2i} &= m_1v_{1f} + m_2v_{2f} \\
 m_1v_{1i} + m_2v_{2i} - m_1v_{1f} - m_2v_{2f} &= 0 \\
 m_1(v_{1i} - v_{1f}) + m_2(v_{2i} - v_{2f}) &= 0 \\
 m_1(v_{1i} - v_{1f}) &= -m_2(v_{2i} - v_{2f})
 \end{aligned} \tag{2}$$

Substituting Equation (2) into Equation (1):

$$\begin{aligned}
 m_1(v_{1i} - v_{1f})(v_{1i} + v_{1f}) + m_2(v_{2i} - v_{2f})(v_{2i} + v_{2f}) &= 0 \\
 -m_2(v_{2i} - v_{2f})(v_{1i} + v_{1f}) + m_2(v_{2i} - v_{2f})(v_{2i} + v_{2f}) &= 0 \\
 m_2(v_{2i} - v_{2f}) [(v_{2i} + v_{2f}) - (v_{1i} + v_{1f})] &= 0 \\
 (v_{2i} + v_{2f}) - (v_{1i} + v_{1f}) &= 0 \text{ assuming } v_{2i} \neq v_{2f} \\
 v_{2f} - v_{1f} &= v_{1i} - v_{2i}
 \end{aligned} \tag{3}$$

Equation 3 tells us that, in an elastic collision, the relative speed of separation is equal to relative speed of approach. This allows us to do the following plug-ins:

$$\begin{aligned}
 m_1v_{1i} + m_2v_{2i} &= m_1v_{1f} + m_2(v_{1f} + v_{1i} - v_{2i}) \\
 m_1v_{1i} + m_2v_{2i} &= (m_1 + m_2)v_{1f} + m_2v_{1i} - m_2v_{2i} \\
 (m_1 - m_2)v_{1i} + 2m_2v_{2i} &= (m_1 + m_2)v_{1f} \\
 v_{1f} &= \frac{m_1 - m_2}{m_1 + m_2}v_{1i} + \frac{2m_2}{m_1 + m_2}v_{2i}
 \end{aligned} \tag{4}$$

The same process results in v_{2f} :

$$v_{2f} = \frac{2m_1}{m_1 + m_2}v_{1i} + \frac{m_2 - m_1}{m_1 + m_2}v_{2i} \quad (5)$$

Equations (4) and (5) solve every head-on elastic collision. Try $m_1 = m_2$ and $v_{2i} = 0$: you get $v_{1f} = 0$ and $v_{2f} = v_{1i}$, the billiard stop shot. Now plug in our values:

$$v_{1f} = \frac{2 - 1}{2 + 1}(3.0) = \frac{1}{3}(3.0) = 1.0 \text{ m/s}$$

$$v_{2f} = \frac{2(2)}{2 + 1}(3.0) = \frac{4}{3}(3.0) = 4.0 \text{ m/s}$$

Answer to Exercise 14 (on page 32)

Using the Conservation of Momentum theorem:

$$\begin{aligned} m_A v_{A,i} + m_B v_{B,i} &= m_A v_{A,f} + m_B v_{B,f} \\ (1000)(20) + (1500)(-10) &= (1000)(5) + (1500)v_{B,f} \\ 20000 - 15000 - 5000 &= (1500)v_{B,f} \\ 0 &= (1500)v_{B,f} \\ v_{B,f} &= 0 \text{ m/s} \end{aligned}$$

Car B ends up stopped.

Before the crash, the kinetic energy is $\frac{1}{2}(1000)(20^2) + \frac{1}{2}(1500)(10^2) = 200,000 + 75,000 = 275,000 \text{ J}$. Afterward, it is $\frac{1}{2}(1000)(5^2) = 12,500 \text{ J}$. The crash destroyed 262,500 J of kinetic energy (into bent metal, heat, and noise), so it is inelastic. The cars did not stick together, so it is not perfectly inelastic.

Answer to Exercise 15 (on page 33)

(a) Using the conservation of momentum, we can say that:

$$\begin{aligned} m_{\text{bullet}}(v_{\text{bullet}}) + m_{\text{block}}(0) &= (m_{\text{bullet}} + m_{\text{block}})v_f \\ (0.01)(500) + (2)(0) &= ((0.01) + (2))v_f \\ v_f &= \frac{(500)(0.01)}{2.01} \\ v_f &\approx 2.49 \text{ m/s} \end{aligned}$$

The *block-bullet system* moves at a speed of 2.49 m/s, in the same direction the bullet was fired.

(b) To find the initial kinetic energy, we use the initial velocity:

$$KE_i = \frac{1}{2}(0.01)(500)^2 = 1250$$

And, to find the final kinetic energy, we use the summed masses and the calculated final velocity

$$KE_f = \frac{1}{2}(2.01)(2.49)^2 \approx 6.23$$

So the net energy lost is the difference:

$$\Delta KE = KE_i - KE_f \approx 1250 - 6.23 \approx 1243.7 \text{ J}$$

(c)

$$\text{Fraction lost} = \frac{\Delta KE}{KE_i} = \frac{1243.7}{1250} \approx 0.99496$$

About 99.5% of the kinetic energy is lost, almost all of it to heat and splintered wood. The bullet and block stick together, so the collision is perfectly inelastic.

Answer to Exercise 16 (on page 34)

The total momentum immediately before the explosion has two components:

$$p_{i,x} = (6)(20) = 120 \text{ kg m/s} \quad p_{i,y} = 0$$

Let us analyze Fragment A:

$$p_{Ax} = m_A v_{Ax} = 2(30 \cos 40^\circ), \quad p_{Ay} = m_A v_{Ay} = 2(30 \sin 40^\circ)$$

And Fragment B,

$$p_{Bx} = m_B v_{Bx} = 1(25 \cos 60^\circ), \quad p_{By} = m_B v_{By} = 1(-25 \sin 60^\circ)$$

Since the Conservation of Momentum applies to the components:

$$\begin{aligned}p_{x,\text{final}} &= p_{Ax} + p_{Bx} + p_{Cx} = 120 \text{ kg}\cdot\text{m/s} \\p_{Cx} &= 120 - (p_{Ax} + p_{Bx}) \\&= 120 - 2(30 \cos 40^\circ) - 1(25 \cos 60^\circ) \\v_{Cx} &= \frac{120 - 2(30 \cos 40^\circ) - 1(25 \cos 60^\circ)}{3.0} \\&= 20.51 \text{ m/s}\end{aligned}$$

And same for the Y-components

$$\begin{aligned}p_{y,\text{final}} &= p_{Ay} + p_{By} + p_{Cy} = 0 \text{ kg}\cdot\text{m/s} \\p_{Cy} &= -(p_{Ay} + p_{By}) \\&= -(2(30 \sin 40^\circ) + 1(-25 \sin 60^\circ)) \\v_{Cy} &= \frac{-(38.57 - 21.65)}{3} \\&= -5.64 \text{ m/s}\end{aligned}$$

So Fragment C moves at $\sqrt{20.51^2 + (-5.64)^2} = 21.3 \text{ m/s}$ with a direction of $\theta = \tan^{-1} \left(\frac{-5.64}{20.51} \right) = -15^\circ$ or 15° below the horizontal.

Answer to Exercise 17 (on page 34)

$(1)(4) = (1 + 3)v$, so $v = 1 \text{ m/s}$.

Answer to Exercise 18 (on page 44)

$1.34\sqrt{36} = 8.04$ knots. Hull speed grows with the square root of length, so doubling the speed takes 4 times the length.

Answer to Exercise 19 (on page 46)

$P = \frac{1.225(5^2)}{2} \approx 15.3$ newtons per square meter. $F_w = (10)(15.3) \approx 153$ newtons. Ten times the wind speed gave a hundred times the pressure, because pressure grows with the square of the speed.

Answer to Exercise 20 (on page 50)

Apparent wind = $[0, 4] - [3, 0] = [-3, 4]$, so its speed is $\sqrt{9 + 16} = 5$ m/s. It seems to come from $\arctan \frac{3}{4} \approx 37^\circ$ east of south.

Answer to Exercise 21 (on page 62)

The cross-sections are $30 \times 2.0 = 60 \text{ m}^2$ and $10 \times 3.0 = 30 \text{ m}^2$. By continuity, $v_2 = \frac{60 \times 1.0}{30} = 2.0$ m/s.

Answer to Exercise 22 (on page 62)

By Torricelli's law, $v = \sqrt{2(9.8)(20)} \approx 19.8$ m/s. Real water slows down because of friction with the pipe walls and in the faucet itself, which turns some of the energy into heat. Bernoulli's equation ignores that friction.

Answer to Exercise 23 (on page 62)

By continuity, $v_2 = \frac{6.0}{2.0}(3.0) = 9.0$ m/s. By Bernoulli, $P_1 - P_2 = \frac{1}{2}(1000)(81 - 9.0) = 36,000$ Pa.

Answer to Exercise 24 (on page 63)

The sheets move *together*, not apart. The fast-moving air between them has lower pressure than the still air outside, so the outside air pushes them inward. It is the same effect that pulls passing ships together.

Answer to Exercise 25 (on page 63)

The pressure difference comes from the fast wind over the roof (lower pressure on top, by Bernoulli's equation) compared with the still air inside. Opening windows does not make the wind slower over the roof, and wind entering a house through an opening on the side facing the wind can *raise* the pressure inside, pushing up on the roof even harder. The roof's attachment to the walls is what keeps it on.

Answer to Exercise 26 (on page 67)

Lift depends on v^2 , so doubling the speed multiplies the lift by four: $L = 60,000$ N.

Answer to Exercise 27 (on page 67)

Solve $15,000 = \frac{1}{2}(0.36)v^2(20)(0.5)$: $v^2 = \frac{15,000}{1.8} \approx 8,333$, so $v \approx 91$ m/s. The air is one third as dense, so the plane must fly $\sqrt{3} \approx 1.7$ times as fast.

Answer to Exercise 28 (on page 72)

$40 \times 1,500 = 60,000$ m, or 60 km.

Answer to Exercise 29 (on page 75)

Hovering: $F = mg = (1.2)(9.8) \approx 11.8$ N in total, or about 2.9 N per rotor. Climbing: $F = mg + ma = 1.2(9.8 + 2) \approx 14.2$ N.

Answer to Exercise 30 (on page 93)

$\frac{m_0}{m_f} = e^{9.4/30} \approx 1.37$, so the final mass is $\frac{1}{1.37} \approx 73\%$ of the starting mass. Only about 27% of the spacecraft needs to be propellant, compared with 96% for the chemical rocket. (An ion thruster could never lift off from Earth, though; its thrust is far too weak.)



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