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# **Volumes of Common Solids**

The volume of a rectangular solid is the product of its three dimensions. If a block of ice is 5 cm tall, 3 cm wide, and 2 cm deep, its volume is  $5 \times 3 \times 2 = 30$  cubic centimeters.



A cubic centimeter is the same as a milliliter. A milliliter of ice weighs about 0.92 grams. This means the block of ice would have a mass of  $30 \times 0.92 = 27.6$  grams.

#### Volume of a Sphere

A sphere with a radius of r has a volume of

$$v = \frac{4}{3}\pi r^3$$

(For completeness, the surface area of that sphere would be

$$a = 4\pi r^2$$

Note that a circle of radius r is one quarter of this:  $\pi r^2$ .)

## **Exercise 1** Flying Sphere

An iron sphere is traveling at 5 m/s (and is not spinning). The sphere has a radius of 1.5 m. Iron has a density of 7,800 kg per cubic meter. How much kinetic energy does the sphere have?



#### 1.1 Cylinders

The base and the top of a right cylinder are identical circles. The circles are on parallel planes. The sides are perpendicular to those planes.



#### Volume of a cylinder

The volume of the right cylinder of radius r and height h is given by:

$$v = \pi r^2 h$$

In other words, it is the area of the base times the height.

## Exercise 2 Tablet

Working Space

A drug company needs to create a tablet with volume of 90 cubic millimeters.

The tablet will be a cylinder with half spheres on each end. The radius will be 2mm.

How long do they need to make the tablet to be?



What if the base and top are identical, but the sides aren't perpendicular to the base? This is called *oblique cylinder*.



The volume is still the height times the area of the base. Note, however, that the height is measured perpendicular to the bottom and top.

Why is this the case?

#### 1.2 Volume, Area, and Height

On a solid with a flat base, the line that we use to measure height is always perpendicular to the plane of the base. We can take slices through the solid that are parallel to that base plane. For example, if we have a pyramid with a square base, each slice will be a square — small squares near the top, larger squares near the bottom.



We can figure out the area of the slice at every height *z*. For example, at z = 0, the slice would have area  $w^2$ . At z = h, the slice would have zero area. What about an arbitrary *z* in between? The edge of the square would be  $w(1 - \frac{z}{h})$ . The area of the slice would be  $w^2(1 - \frac{z}{h})^2$ 

The graph of this would look like this:



The volume is given by the area under the curve and above the axis. Once you learn integration, you will be extra good at finding the area under the curve. In this case, we will just tell you that the colored region in the picture is one third of the rectangle.

Thus, the area of a square-based pyramid is  $\frac{1}{3}hw^2$ .

In fact:

#### Volume of a pyramid

The volume of pyramid whose base has an area of b and height h is given by:

$$V = \frac{1}{3}hb$$

Regardless of the shape of the base.

Note that this is true even for oblique pyramids:

h

## Exercise 3 Hexagon-based Pyramid

Working Space

Answer on Page 38

There is a pyramid with a regular hexagon for a base. Each edge is 5 cm long. The pyramid is 13 cm tall. What is its volume?



Note that plotting the area of each slice and finding the area under the curve will let you find the area of many things. For example, let's say that you have a four-sided spiral, where each face has the same width *w*:



Every slice still has an area of  $w^2$ , which means this figure has a volume of  $hw^2$ .

### **Exercise 4** Volume of a building

Working Space

An architect is designing a hotel with a right triangular base; the base is 30 meters on each leg. The building gets narrower as you get closer to the top, and finally shrinks to a point. The spine of the building is where the right angle is. That spine is straight and perpendicular to the ground.

Each floor has a right isosceles triangle as its floor plan. The length of each leg is given by this formula:

$$w = 30\sqrt{1 - \frac{z}{100}}$$

So, the width of the building is 30 meters at height z = 0. At 100 meters, the building comes to a point. It will look like this:



What is the volume of the building in cubic meters?

## CHAPTER 2

## **Conic Sections**

In mathematics, conic sections (or simply conics) are curves obtained as the intersection of the surface of a cone with a plane. The three types of conic section are the hyperbola, the parabola, and the ellipse. The circle is a special case of the ellipse, though historically it was sometimes called a fourth type.

#### 2.1 **Definitions**

Each type of conic sections can be defined as follows:

#### 2.1.1 Circle

A circle is the set of all points in a plane that are at a given distance (the radius) from a given point (the center). The standard equation for a circle with center (h, k) and radius r is:

$$(x-h)^2 + (y-k)^2 = r^2$$
(2.1)

#### 2.1.2 Ellipse

An ellipse is the set of all points such that the sum of the distances from two fixed points (the foci) is constant. The standard equation for an ellipse centered at the origin with semi-major axis a and semi-minor axis b is:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$
(2.2)

#### 2.1.3 Hyperbola

A hyperbola is the set of all points such that the absolute difference of the distances from two fixed points (the foci) is constant. The standard equation for a hyperbola centered at the origin is:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$
(2.3)

or

$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1 \tag{2.4}$$

depending on the orientation of the hyperbola.

#### 2.1.4 Parabola

A parabola is the set of all points that are equidistant from a fixed point (the focus) and a fixed line (the directrix). The standard equation for a parabola that opens upwards or downwards is:

$$y = a(x-h)^2 + k$$
 (2.5)

and that opens leftwards or rightwards is:

$$x = a(y-k)^2 + h$$
 (2.6)

where (h, k) is the vertex of the parabola.

CHAPTER 3

# Manufacturing

If you try to think of any man-made object, whether it was made from woods, metals, or plastics, chances are it was produced through a manufacturing process.

Over time, these processes have been refined to be more efficient, cost-effective, and faster at producing the goods that we use on a daily basis.

New methods are also constantly being developed by engineers and scientists, and today the range of options available means that choosing the most appropriate manufacturing method involves finding the sweet spot between cost-effectiveness, yield, and time needed.

#### 3.1 Woods and Metals Processes

Manufacturing methods that are able to process woods and metals are typically the processes that are used to construct the vast majority of the built world around us.

Infrastructure, transportation methods, and buildings would not exist without the advent of processes that allow us to accurately machine raw woods and metals into our desired forms.

The following sections will cover methods used to process both material types.

#### 3.2 Milling

Mills are powerful tools that allow us to carve out complex shapes from blocks of raw material. A tool bit follows a path to remove the desired material, which makes it a *subtractive* manufacturing process. The tool bit rotates at a very high speed, which allows it to process harder materials such as woods and metals.



There are various types of mills, ranging from a 3-axis mill, which can cut out simple shapes in the X, Y, and Z axes, all the way to 6-axis mills, which can also rotate about those axes to create more complex curvatures.



In manufacturing settings where speed and repeatability is paramount, mills are often computer controlled. This functionality is referred to as *Computer Numerical Control*, or CNC. CNC mills are able to repeatedly follow a tool path, resulting in consistent and accurate parts.

#### 3.3 Lathing

Lathes are tools that allow us to carve out complex cylindrical shapes from raw material. Like a mill, it also is a subtractive manufacturing process. However, lathes rotate the material itself at a high speed, rather than the tool bit. As the material rotates, the tool bit can be used to extract material layer by layer.



In the manufacturing industry, lathes are also often computer controlled. Alongside CNC mills, these CNC lathes are responsible for a majority of the objects that we interact with everyday.

#### 3.4 Metal-specific Processes

While mills and lathes are already able to cover the vast majority of manufacturing needs for woods and metals, there are certain processes that are specifically enabled by the unique properties of metals. More often than not, these processes leverage the malleability of metals at room temperature or higher.

#### 3.5 Sheet Metal Processes

While milling allows us to process blocks of metal to great effect, sometimes the application we need does not require material of such thickness.

This is where sheet metal comes in, as well as the methods we use to process it. One of the most common techniques in manufacturing is rolling, where a raw sheet is gradually rolled into the desired shape. This method is used to create many objects you may recognize, such as metal roofings, aircraft frames, and more.



Another method is stamping, where a raw sheet is stamped into the desired shape. This allows us to create surfaces with complex geometries in an instant, and in large quantities. This method is often used for applications like the exterior panels of a car, where parts with compound curves are needed.



Lastly, there are also separate processes used to increase the strength of sheet metal parts. This falls under the category of sheet metal forming, and involves bending the edges of a part to form a reinforced flap. Almost all sheet metal parts are reinforced in this manner, as it is a relatively simple process and also helps to create a clean edge for a more finished look.

### 3.6 Casting

The last kind of metal-specific process we will cover is casting. Casting involves pouring molten metal into a mold, then letting the metal cool and set inside the mold. Smaller components with complex geometries and limited structural requirements (such as toys) are often cast, as it is an accurate and high-volume manufacturing method.



Casting also results in minimal material wastage, as it is not a subtractive manufacturing method where excess material is machined away, but rather only the specific amount of material required is poured in each time.

#### 3.7 Wood-specific Processes

Similar to metals, there are also certain manufacturing processes that are enabled by the unique qualities of wood. These make use of the water content inherent in wood, and the flexibility it enables.

#### 3.8 Bending

Turning raw wood into flat, workable pieces involves a variety of tools that you probably know of already, such as saws and drills. However, there are specific processes that help us create curved shapes with wood, in addition to the mill and the lathe mentioned earlier.

This is where bent lamination comes in. Bent lamination involves layering multiple thin veneers or strips of wood with adhesive, and clamping it to create the desired form while letting the glue dry. This method is often used for furniture production, enabling continuous curves in wood with tight radii.



Steam can also be used for bending, by helping soften the wood fibers to increase flexibility. Once the desired form is reached, the part can then cool down and harden. Unlike bent lamination, steam bending can be done without adhesives.

#### 3.9 Plastic-specific Processes

Although plastics only came into prominence in the mid 20th century, they have changed manufacturing and, by extension, the world as we know it. Easily manufacturable, durable, and cost-effective, they have come to permeate almost everything we use on a daily basis.

It must also be noted that these exact qualities have also resulted in the proliferation of plastics in our environment, and as such, usage of plastics should be well considered and limited. Alternative biodegradable materials are currently being trialed by scientists and would look to replicate many of the same qualities we expect from plastics, including its manufacturability.

### 3.10 Injection Molding

Injection molding is responsible for the vast majority of plastic products that you interact with on a daily basis. It is extremely quick, highly accurate, and has minimal material wastage, making it a popular and cost effective method of manufacturing plastic goods.

Similar to casting, injection molding involves injecting molten plastic into a mold, then allowing the part to cool and set inside the cavity.

You can often tell when a part was produced through injection molding, with telltale signs such as the parting line. This is where the parts of the mold meet, forming a visible line on the surface of a part.



#### 3.11 3D Printing

Injection molding has traditionally been the go-to technique for manufacturing plastic goods. However, new technologies result in the advent of new manufacturing methods. 3D printing is one such method, having come to prominence in the last few decades as a way to quickly prototype parts without having to create the molds needed for injection molding.

3D printing is an *additive* manufacturing process, where molten material is applied layer by layer to form the desired geometry. It allows for complex geometries, and whilst the accuracy may currently lag behind traditional injection molding, it is also improving rapidly.



### 3.12 Laser Cutting

Similarly, another manufacturing method enabled by new technologies is laser cutting. Like 3D printing, it has come into prominence as a method to quickly prototype parts. However, it is not an additive manufacturing process.

Instead laser cutting uses a laser beam to cut and etch through sheets of plastic, however it can also be used for other materials such as fabrics and card stocks. Laser cutting is mostly limited by material thickness, and as such can only cut through thinner sheets of material.



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CHAPTER 4

# **Falling Bodies**

Gravity exists all around us. If you throw a hammer straight up in the air, from the moment it leaves your hand until it hits the ground, it is accelerating toward the center of the earth at a constant rate.



Acceleration can be defined as change in velocity. If the hammer leaves your hand with a velocity of 12 meters per second upward, one second later, it will be rising, and its velocity will have slowed to 2.2 meters per second. One second after that, the hammer will be falling at a rate of 7.6 meters per second. Every second the hammer's velocity is changing by 9.8 meters per second, and that change is always toward the center of the earth. When the hammer is going up, gravity is slowing it down by 9.8 meters per second, each second it is in the air. When the hammer is coming down, gravity is increasing the speed of its descent by 9.8 meters per second.



Acceleration due to gravity on earth is a constant negative 9.8 meters per second per second:

a = -9.8

(Why is it negative? We are talking about height, which increases as you go away from the center of the earth. Acceleration is changing the velocity in the opposite direction.)

### 4.1 Calculating the Velocity

Given that the acceleration is constant, it makes sense that the velocity is a straight line. Assuming once again that the hammer leaves your hand at 12 meters per second, then the upwards velocity at time t is given by:

$$v = 12 - 9.8t$$

Note that the velocity of the hammer is being given as a function. Here is its graph:



## **Exercise 5** When is the apex of flight?



At this point, we need to acknowledge air resistance. Gravity is not the only force on the hammer; as it travels through the air, the air tries to slow it down. This force is called *air resistance*, and for a large, fast-moving object (like an airplane) it is GIGANTIC force. For a dense object (like a hammer) moving at a slow speed (what you generate with your hand), air resistance doesn't significantly affect acceleration.

#### 4.2 Calculating Position

If you let go of the hammer when it is 2 meters above the ground, the height of the hammer is given by:

$$p = -\frac{9.8}{2}t^2 + 12t + 2$$

Here is a graph of this function:



How do we know? The change in position between time 0 and any time t is equal to the area under the velocity graph between x = 0 and x = t.

Let's use the velocity graph to figure out how much the position has changed in the first second of the hammer's flight. Here iss the velocity graph with the area under the graph for the first second filled in:



The blue filled region is the area of the dashed rectangle minus that empty triangle in its upper left. The height of the rectangle is twelve and its width is the amount of time the hammer has been in flight (t). The triangle is t wide and 9.8t tall. Thus, the area of the blue region is given by  $12t - \frac{1}{2}9.8t^2$ .

That's the change in position. Where was it originally? 2 meters off the ground. This means the height is given by  $p = 2 + 12t - \frac{1}{2}9.8t^2$ . We usually write terms so that the exponent decreases, so:

$$p = -\frac{1}{2}9.8t^2 + 12t + 2$$

Finding the area under the curve like this is called *integration*. We say "To find a function that gives the change in position, we just integrate the velocity function." Much of the study of calculus is learning to integrate different sorts of functions.

One important note about integration: Any time the curve drops under the x-axis, the area is considered negative. (Which makes sense, right? If the velocity is negative, the hammer's position is decreasing.)



#### 4.3 Quadratic functions

Functions of the form  $f(x) = ax^2 + bx + c$  are called *quadratic functions*. If a > 0, the ends go up. If a < 0, the ends go down.



The graph of a quadratic function is a *parabola*.

### 4.4 Simulating a falling body in Python

Now you are going to write some Python code that simulates the flying hammer. First, we are just going to print out the position, speed, and acceleration of the hammer for every 1/100th of a second after it leaves your hand. (Later, we will make a graph.)

Create a file called falling.py and type this into it:

```
# Acceleration on earth
acceleration = -9.8 \# m/s/s
# Size of time step
time_step = 0.01 # seconds
# Initial values
speed = 12 # m/s upward
height = 2 # m above the ground
current_time = 0.0 # seconds after release
# Is the hammer still aloft?
while height > 0.0:
    # Show the values
    print(f"{current_time:.2f} s:")
    print(f"\tacceleration: {acceleration:.2f} m/s/s")
    print(f"\tspeed: {speed:.2f} m/s")
    print(f"\theight: {height:.2f} m")
    # Update height
    height = height + time_step * speed
    # Update speed
    speed = speed + time_step * acceleration
    # Update time
    current_time = current_time + time_step
print(f"Hit the ground: Complete")
When you run it, you will see something like this:
```

```
0.00 s:

acceleration: -9.80 m/s/s

speed: 12.00 m/s

height: 2.00 m

0.01 s:

acceleration: -9.80 m/s/s

speed: 11.90 m/s

height: 2.12 m

0.02 s:

acceleration: -9.80 m/s/s

speed: 11.80 m/s
```

```
height: 2.24 m

0.03 s:

acceleration: -9.80 m/s/s

speed: 11.71 m/s

height: 2.36 m

...

2.60 s:

acceleration: -9.80 m/s/s

speed: -13.48 m/s

height: 0.20 m

2.61 s:

acceleration: -9.80 m/s/s

speed: -13.58 m/s

height: 0.07 m

Hit the ground: Complete
```

Note that the acceleration isn't changing at all, but it is changing the speed, and the speed is changing the height.

We can see that the hammer in our simulation hits the ground just after 2.61 seconds.

#### 4.4.1 Graphs and Lists

Now, we are going to graph the acceleration, speed, and height using a library called matplotlib. However, to make the graphs, we need to gather all the data into lists.

For example, we will have a list of speeds, and the first three entries will be 12.0, 11.9, and 11.8.

We create an empty list and assign it to a variable like this:

x = []

Next, we can add items like this:

```
x.append(3.14)
```

To get the first time back, we can ask for the object at index 0.

y = x[0]

Note that the list starts at 0. If you have 32 items in the list, the first item is at index 0; the

last item is at index 31.

Duplicate the file falling.py and name the new copy falling\_graph.py

We are going to make a plot of the height over time. At the start of the program, you will import the matplotlib library. At the end of the program, you will create a plot and show it to the user.

In falling\_graph.py, add the bold code:

```
import matplotlib.pyplot as plt
# Acceleration on earth
acceleration = -9.8 \# m/s/s
# Size of time step
time_step = 0.01 # seconds
# Initial values
speed = 12 # m/s upward
height = 2 # m above the ground
current_time = 0.0 # seconds after release
# Create empty lists
accelerations = []
speeds = []
heights = []
times = []
# Is the hammer still aloft?
while height > 0.0:
    # Add the data to the lists
    times.append(current_time)
    accelerations.append(acceleration)
    speeds.append(speed)
    heights.append(height)
    # Update height
    height = height + time_step * speed
    # Update speed
    speed = speed + time_step * acceleration
    # Update time
    current_time = current_time + time_step
```

```
# Make a plot
fig, ax = plt.subplots()
fig.suptitle("Falling Hammer")
ax.set_xlabel("Time (s)")
ax.set_ylabel("Height (m)")
ax.plot(times, heights)
plt.show()
```

When you run the program, you should see a graph of the height over time.



It is more interesting if we can see all three: acceleration, speed, and height. So, let's make three stacked plots. Change the plotting code in falling\_graph.py to:

```
# Make a plot with three subplots
fig, ax = plt.subplots(3,1)
fig.suptitle("Falling Hammer")
# The first subplot is acceleration
ax[0].set_ylabel("Acceleration (m/s/s)")
ax[0].plot(times, accelerations)
# Second subplot is speed
ax[1].set_ylabel("Speed (m/s)")
ax[1].plot(times, speeds)
# Third subplot is height
```

```
ax[2].set_xlabel("Time (s)")
ax[2].set_ylabel("Height (m)")
ax[2].plot(times, heights)
plt.show()
```

You will now get plots of all three variables:



Falling Hammer

This is what we expected, right? The acceleration is a constant negative number. The speed is a straight line with a negative slope. The height is a parabola.

A natural question at this point is "When exactly will the hammer hit the ground?" In other words, when does height = 0? The values of t where a function is zero are known as its *roots*. Height is given by a quadratic function. In the next chapter, you will get the trick for finding the roots of any quadratic function.

# **Projectile Motion**

A projectile is an object that, once thrown or dropped, continues to move only under the influence of gravity. Throwing a baseball, shooting a cannon, and diving off a high diving board are all examples. NASA flight planners use projectile motion to plan flight paths for space vehicles, such as sending rovers to Mars. You've already learned how to describe and model one-dimensional projectile motion in the Falling Bodies chapter. Now, we consider projectiles that also have horizontal motion, and therefore are moving in two dimensions.

First, we will compare the motion of projectiles that are dropped versus horizontally launched from the same height. This will frame our discussion of the important concept of independence of motion: the vertical and horizontal motions of a projectile can be considered and described independent from each other. This will allow you to predict how far horizontally launched objects will travel before hitting the ground. Next, you'll learn to describe the motion of projectiles launched at an angle (like some heavy ground artillery). Finally, you'll use what you've learned to create a model of any projectile motion.

### 5.1 Comparing Projectiles

PhET or video showing horizontally-launched vs dropped objects time to move vertically is not affected by the addition of horizontal motion

### 5.2 Independence of Motion

we can describe the x and y motion separately. you already know how to describe the y motion from falling bodies show 2D kinematics equations graphs comparing x motions and y motions

## 5.3 Horizontally-launched Projectiles

example

exercise

exercise - Newton's cannon

### 5.4 Projectiles launched at an Angle

#### 5.4.1 From the Ground

separating vertical and horizontal components of initial motion with trigonometry example

exercise - how far does the object travel?

exercise - at what angle should you launch for an object to go the furthest given a maximum launch velocity?

exercise - I have a target x-meters away, I must launch at v-miles per hour, what angle will allow me to hit my target, if any?

### 5.5 Simulating Projectile Motion

## Answers to Exercises

#### Answer to Exercise 1 (on page 4)

The volume of the sphere (in cubic meters) is

$$\frac{4}{3}\pi(1.5)^3 = 4.5\pi \approx 14.14$$

The mass (in kg) is  $14.14 \times 7800 \approx 110,269$ 

The kinetic energy (in joules) is

$$k = \frac{110269 \times 5^2}{2} = 1,378,373$$

About 1.4 million joules.

#### **Answer to Exercise 2 (on page 5)**

In your mind, you can disassemble the tablet into a sphere (made up of the two ends) and a cylinder (between the two ends).

The volume of the sphere (in cubic millimeters) is

$$\frac{4}{3}\pi(2)^3 = \frac{32}{3}\pi \approx 33.5$$

Thus, the cylinder part has to be 90-33.5 = 56.5 cubic mm. The cylinder part has a radius of 2 mm. If the length of the cylinder part is x, then

$$\pi 2^2 x = 56.5$$

Thus  $x = \frac{56.5}{4\pi} \approx 4.5$  mm.

The cylinder part of the table needs to be 4.5mm. Thus the entire tablet is 8.5mm long.

## Answer to Exercise 3 (on page 8)

First, you need to find the area of the base, which is a regular hexagon:



All the angles in this picture are 60° or  $\frac{\pi}{3}$  radians. This means each line is 5 cm long.

This tells us that we need to find the area of one of these triangles and multiply that by six.

Every triangle has a base of 5cm. How tall are they?



 $5\sin 60^\circ = 5\frac{\sqrt{3}}{2}$ 

Which is about 4.33 cm.

So, the area of single triangle is

$$\frac{1}{2}(5)\left(5\frac{\sqrt{3}}{2}\right) = 25\frac{\sqrt{3}}{4}$$

And the area of the whole hexagon is six times that:

$$75\frac{\sqrt{3}}{2}$$

Thus, the volume of the pyramid is:

$$\frac{1}{3}hb = \frac{1}{3}13\left(75\frac{\sqrt{3}}{2}\right)$$

About 281.46 cubic centimeters.

## Answer to Exercise 4 (on page 10)

The area at height z is given by:

$$a = \frac{1}{2}w^2 = \frac{1}{2}\left(30\sqrt{1-\frac{z}{100}}\right)^2 = \frac{1}{2}900\left(1-\frac{z}{100}\right)$$

If we plot that, it looks like this:



What is the area of the blue region?  $\frac{1}{2}(900)(100) = 45,000$ 

The building will be 45 thousand cubic meters.

## Answer to Exercise 5 (on page 27)

Solve for when the velocity is zero.

 $t = \frac{12}{9.8} = 1.22$  seconds after release.



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