Contents

Chapter 1

Basic Statistics

You live near a freeway, and someone asks you, “How fast do cars on that freeway drive?”

You say, “Pretty fast.”

They reply, “Can you be more specific?”

So, you pull out your radar gun you happen to always keep on you, and tell them, “That one is going 32.131 meters per second.”

To which they say, “I don’t want to know about that specific car. I want to know about all the cars.”

So, you spend the day beside the freeway measuring the speed of every car that goes by. And you get a list of a thousand numbers, including:

30.462 m/s	29.550 m/s	29.227 m/s
37.661 m/s	27.899 m/s	28.113 m/s
24.382 m/s	35.668 m/s	43.797 m/s
31.312 m/s	37.637 m/s	30.891 m/s

There are 12 numbers here. We say that there are 12 *samples*.

1.1 Mean

We often talk about the *average* of a set of samples, which is the same as the *mean*. To get the mean, sum up the samples and divide that number by the number of samples.

The numbers in that table sum to 388.599. If you divide that by 12, you find that the mean of those samples is 32.217 m/s.

We typically use the greek letter μ (“mu”) to represent the mean.

Definition of Mean

If you have a set of samples $x_1, x_2, \ldots, x_n$, the mean is:

$$\mu = \frac{1}{n}\sum_{i=1}^{n} x_i$$

This may be the first time you are seeing a summation ($\sum$). The equation above is equivalent to:

$$\mu = \frac{1}{n}(x_1 + x_2 + \ldots + x_n)$$

Exercise 1 Mean Grade

Working Space

Teachers often use the mean for grading. For example, if you took six quizzes in a class, your final grade might be the mean of the six scores. Find the mean of these six grades: 87, 91, 98, 65, 87, 100.

Answer on Page 45

If you tell your friend, “I measured the speed of 1000 cars, and the mean is 31.71 m/s”, your friend will wonder, “Are most of the speeds clustered around 31.71? Or are they all over the place and just happen to have a mean of 31.71?” To answer this question, we use variance.

1.2 Variance

Definition of Variance

If you have n samples $x_1, x_2, \ldots, x_n$ that have a mean of μ, the *variance* is defined to be:

$$v = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2$$

That is, you figure out how far each sample is from the median, you square that, and then you take the mean of all those squared distances.

x	$x - \mu$	$(x - \mu)^2$
30.462	-1.755	3.079
29.550	-2.667	7.111
29.227	-2.990	8.938
37.661	5.444	29.642
27.899	-4.318	18.642
28.113	-4.104	16.839
24.382	-7.835	61.381
35.668	3.451	11.912
43.797	11.580	134.106
31.312	-0.905	0.818
37.637	5.420	29.381
30.891	-1.326	1.757
$\sum x = 386.599$		$\sum (x - \mu)^2 = 323.605$
mean = 32.217		variance = 26.967

Thus, the variance of the 12 samples is 26.967. The bigger the variances, the farther the samples are spread apart; the smaller the variances, the closer samples are clustered around the mean.

Notice that most of the data points deviate from the mu by 1 to 5 m/s. Isn't it odd that the variance is a big number like 26.967? Remember that it represents the average of the squares. Sometimes, to get a better feel for how far the samples are from the mean, we use the square root of the variance, which is called *the standard deviation*.

The standard deviation of your 12 samples would be $\sqrt{26.9677} = 5.193$ m/s.

The standard deviation is used to figure out a data point is an outlier. For example, if you are asked, "That car that just sped past. Was it going freakishly fast?" You might respond, "No, it was within a standard deviation of the mean." or "Yes, its speed was 2 standard deviations more than the mean. They will probably get a ticket."

A singular μ usually represents the mean. σ usually represents the standard deviation. So σ^2 represents the variance.

Exercise 2 Variance of Grades

Working Space

Now, find the variance for your six grades. As a reminder, they were: 87, 91, 98, 65, 87, 100.

What is your standard deviation?

Answer on Page 45

1.3 Median

Sometimes you want to know where the middle is. For example, you want to know the speed at which half the cars are going faster and half are going slower. To get the median, you sort your samples from smallest to largest. If you have an odd number of samples, the one in the middle is the median. If you have an even number of samples, we take the mean of the two numbers in the middle.

In our example, you would sort your numbers and find the two in the middle:

24.382
27.899
28.113
29.227
29.550
30.462
30.891
31.312
35.668
37.637
37.661
43.797

You take the mean of the two middle numbers: $(30.462 + 30.891)/2 = 30.692$. The median speed would be 30.692 m/s.

Medians are often used when a small number of outliers majorly skew the mean. For example, income statistics usually use the median income because a few hundred billionares raise the mean a great deal.

Exercise 3 Median Grade

Working Space

Find the median of your six grades: 87, 91, 98, 65, 87, 100.

Answer on Page 45

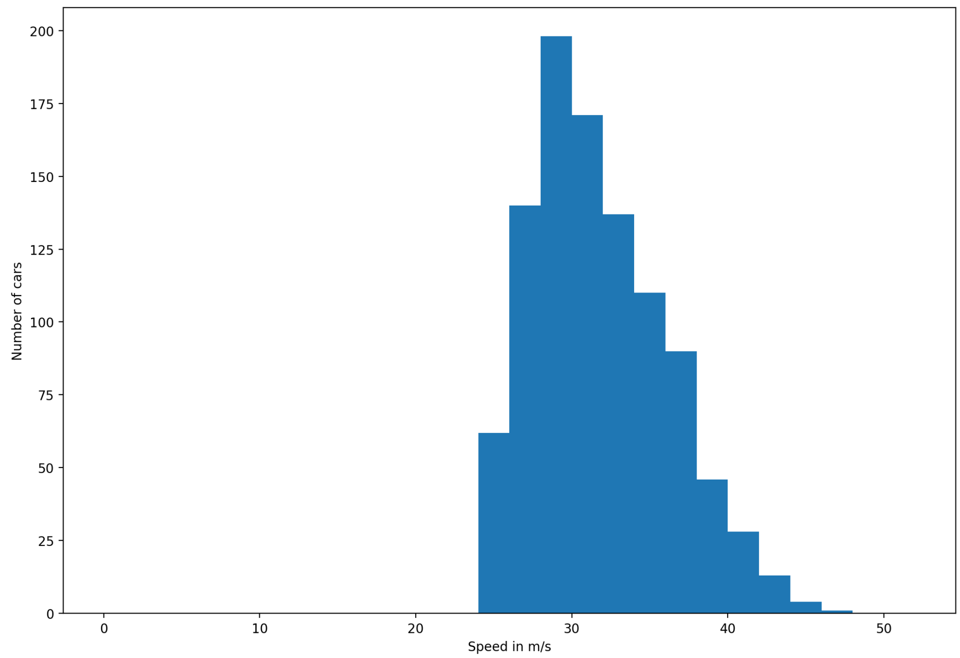
1.4 Histograms and Bell Curves

A histogram is a bar chart that shows how many samples are in each group. In our example, we group cars by speed. Maybe we count the number of cars going between 30 and 32 m/s. Next, we count the cars going between 32 and 34 m/2. Finally, we make a bar chart from that data.

Your 1000 cars would break up into these groups:

0 - 2 m/s	0 cars
2 - 4 m/s	0 cars
4 - 6 m/s	0 cars
...	...
20 - 22 m/s	0 cars
22 - 24 m/s	0 cars
24 - 26 m/s	65 cars
26 - 28 m/s	160 cars
28 - 30 m/s	175 cars
30 - 32 m/s	168 cars
32 - 34 m/s	150 cars
34 - 36 m/s	114 cars
36 - 38 m/s	79 cars
38 - 40 m/s	52 cars
40 - 42 m/s	20 cars
42 - 44 m/s	12 cars
44 - 46 m/s	4 cars
46 - 48 m/s	1 cars
48 - 50 m/s	0 cars

Next, we make a bar chart from that:



Often, a histogram will tell the story of the data. Here, you can see that no one is going less than 24 m/s, but a lot of people travel at 30 m/s. There are a few people who travel over 40 m/s, but there are also a couple of people who drive much faster than anyone else.

If we look closely at the shape of our histogram, we notice something interesting: it has a smooth, rounded hill in the middle and it tapers off on both sides. This is a common pattern in statistics, and it often suggests that the data follows what's called a **normal distribution**, also known as a **bell curve**.

A **bell curve** is a continuous curve that models how data is distributed when:

- Most values are near the average (mean), located at the peak of the curve,
- Fewer values are far from the mean, the tails of the curve,
- The distribution is roughly symmetric.

In our case, the majority of cars are traveling around 30–32 m/s. There are fewer cars going slower or faster than that. If we collected even more data (say, 10,000 cars), the histogram would start to resemble a smooth bell curve even more closely. The units of the x-axis (speed) would still be the same, but the y-axis would represent the *probability density* of finding a car at a certain speed, rather than just the count of cars in each speed range.

The peak of the bell curve represents the **mean speed**, and the spread of the curve is related to the **standard deviation** — a measure of how spread out the speeds are. Most data falls within:

- 1 standard deviation of the mean ($\mu \pm \sigma$): about 68% of the cars,
- 2 standard deviations: about 95%,
- 3 standard deviations: about 99.7%.

This means that if we know the mean speed and the standard deviation, we can predict how many cars will be within certain speed ranges. If you have ever heard of Six Sigma methodology, that means data falls within six standard deviations of the mean, which is a very high level of quality control (3.4 defects per million measurements!).

So when we say a dataset ”looks like a bell curve,” we mean that it has a predictable and symmetric structure, often meaning it is a reliable set of data with few outliers and consistent results.

1.5 Root-Mean-Squared

Scientists have a mean-like statistic that they love. It is named quadratic mean, but most just calls it Root-Mean-Squared, or RMS.

Definition of RMS

If you have a list of numbers $x_1, x_2, \ldots, x_n$, their RMS is

$$\sqrt{\frac{1}{n}\left(x_1^2 + x_2^2 + \ldots + x_n^2\right)}$$

You are taking the square root of the mean of squares of the samples, thus the name Root-Mean-Squared.

Using your 12 samples:

x	x^2
30.462	927.933
29.550	873.203
29.227	854.218
37.661	1418.351
27.899	778.354
28.113	790.341
24.382	594.482
35.668	1272.206
43.797	1918.177
31.312	980.441
37.637	1416.544
30.891	954.254
Mean of x^2	1064.875
RMS	32.632

Why is RMS useful? Let's say that all cars had the same mass m, and you need to know what the average kinetic energy per car is. If you know the RMS of the speeds of the cars is v_{rms}, the average kinetic energy for each car is

$$k = \frac{1}{2}mv_{rms}^2$$

(You don't believe me? Let's prove it. Substitute in the RMS:

$$k = \frac{1}{2}m\sqrt{\frac{1}{n}\left(x_1^2 + x_2^2 + \ldots + x_n^2\right)}^2$$

The square root and the square cancel each other out:

$$k = \frac{1}{2}m\frac{1}{n}\left(x_1^2 + x_2^2 + \ldots + x_n^2\right)$$

Use the distributive property:

$$k = \frac{1}{n}\left(\frac{1}{2}mx_1^2 + \frac{1}{2}mx_2^2 + \ldots + \frac{1}{2}mx_n^2\right)$$

That is all the kinetic energy divided by the number of cars, which is the mean kinetic enegy per car. Quod erat demonstrandum! (That is a Latin phrase that means "which is what I was trying to demonstrate". You will sometimes see "QED" at the end of a long mathematic proof.)

Now you are ready for the punchline: Kinetic energy and heat are the same thing. Instead of cars, heat is the kinetic energy of molecules moving around. More on this soon.

Video: Mean, Median, Mode: https://www.youtube.com/watch?v=5C9LBF3b65s

CHAPTER 2

Basic Statistics in Spreadsheets

When you completed the problems in the last section, you probably noticed how long it took to compute statistics like the mean, median, and variance by hand. Luckily, computers were designed to free us from these sorts of tedious tasks. The most basic tool for automating calculations is the spreadsheet program.

The first spreadsheet program (VisiCalc) was introduced in 1979 as a tool for finance people to play "what if" games. For example, a company might make a spreadsheet that told them how much more profit they would make if they changed from using an expensive metal to using a cheaper alloy.

There are lots of spreadsheet programs, including Microsoft's Excel, Google Sheets, and Apple's Numbers. Any spreadsheet program will work; they are all very similar. The instructions and screenshots here will be from Google Sheets — a free spreadsheet program you use through your web browser.

2.1 The Barrel Problem

In honor of this history, let's start by studying a business question: You have a friend who dreams of quitting her job to become a cooper. (A cooper makes barrels that are used for aging wine and whiskey.) According to her:

- It costs $45 dollars in materials to build one barrel.
- A barrel sells for $100 dollars.
- The workshop/warehouse she wants to rent costs $2000 per month.
- Taxes take 20% of her profits.
- She needs to make $4000 monthly after taxes.

She has asked you, "How many barrels do I need to make each month?"

2.2 Solving It Symbolically

Many problems can be solved two ways: symbolically or numerically. To solve this problem symbolically, you would write out the facts as equations or inequalities, then do symbol manipulations until you ended up with an answer. In this case, you would let b be the number of barrels and create the following inequality:

$$(1.0 - 0.2)\,(b(100 - 45) - 2000) \geq 4000$$

You would simplify it:

$$(0.8)\,(55b - 2000) \geq 4000$$

And simplify it more:

$$44b - 1600 \geq 4000$$

If that is true, then:

$$44b \geq 5600$$

And if that is true, then:

$$b \geq \frac{1400}{11}$$

1400/11 is about 127.27, so she needs to make and sell 128 barrels each month.

That is a perfect answer, and we didn't need a spreadsheet at all. However:

- As problems get larger and more realistic, it gets much more difficult to solve them symbolically.
- As soon as you say "Yes, you need to make and sell 128 barrels each month." Your friend will ask "What if I make and sell 200 barrels? How much money will I make then?"

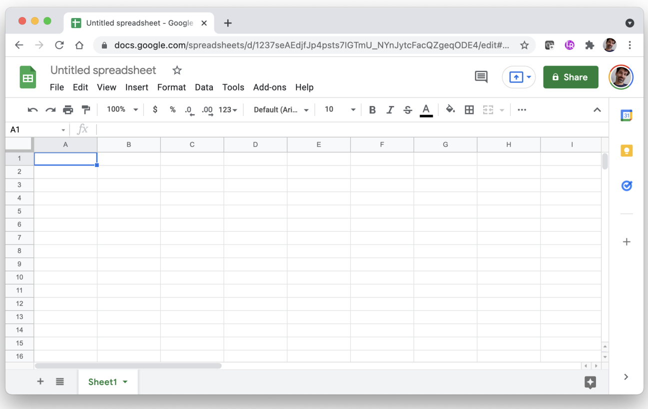
So, we use a spreadsheets to solve the problem numerically.

2.3 Your First Spreadsheet

Let's first make an inital spreadsheet.

In whatever spreadsheet program you are using, create a new spreadsheet document.

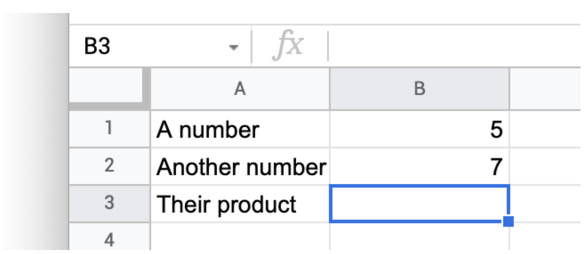
A spreadsheet is essentially a grid of cells. In each cell, you can put data (like numbers or text) and formulas.



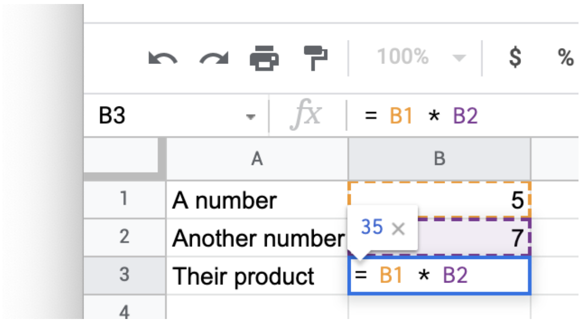
Let's put some labels in the column:

- Select the first cell (A1) and type "A number".
- Select the cell below it (A2) and type "Another number".
- Select the cell below that one (A3) and type "Their product".
- In the next column, type the number 5 in B1 and 7 in B2.

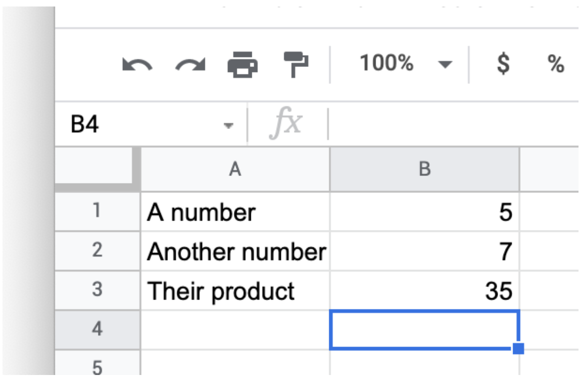
It should look like this:



Now, put a formula in cell B3. Select B3, and type "= B1 * B2". The spreadsheet knows this is a formula because it starts with '='. It will look like this as you type:



When you press Return or Tab, the spreadsheet will remember the formula, but display its value:



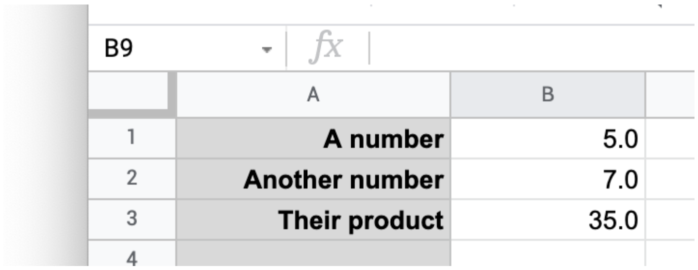
If you change the values of cell B1 or B2, the cell B3 will automatically be recalculated. Try it.

2.4 Formatting

Every spreadsheet lets you change the formatting of your columns and cells. They are all a little different, so play with your spreadsheet a little now. Try to do the following:

- Set the background of the first column to light gray.
- Right-justify the text in the first column.
- Make the text in the first column bold.
- Make the numbers in the second column have one digit after the decimal point.

It should look something like this:



That's a spreadsheet. You have a grid of cells, each of which can hold a value or a formula that uses values from other cells. The cells containing formulas automatically update as you edit the values in the other cells.

2.5 Comma-Separated Values

A large amount of data is exchanged in a file format called *Comma-Separated Values* or just CSV. Each CSV file holds one table of data. It is a text file, and each line of text corresponds to one row of data in the table. The data in each column is separated by a comma. The first line of a CSV is usually the names of the columns. A CSV might look like this:

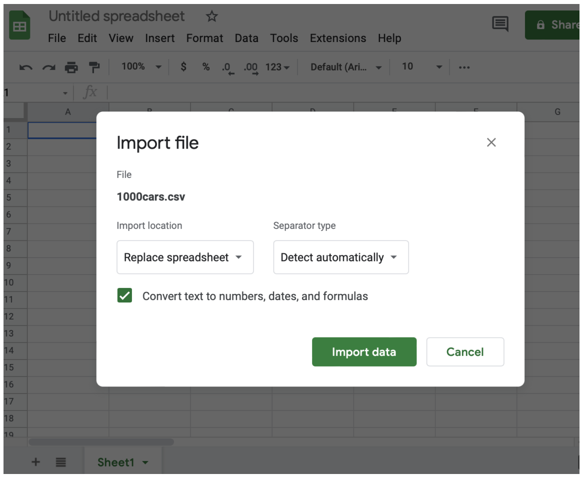
```
studentID,firstName,lastName,height,weight
1,Marvin,Sumner,260,45.3
2,Lucy,Harris,242,42.2
3,James,Boyd,261,44.2
```

In your digital resources for this module, you should have a file called `1000cars.csv`. It is a CSV with only one column called "speed". The first few lines look like this:

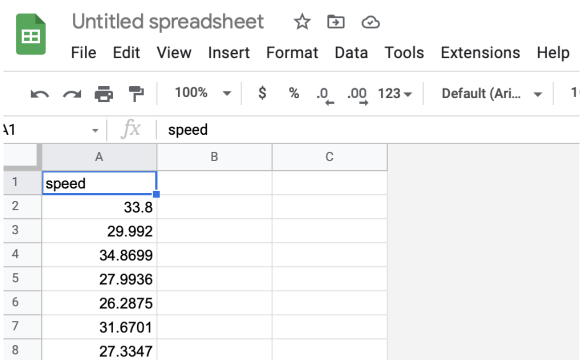
```
speed
33.8000
29.9920
34.8699
27.9936
```

There is a title line and 1000 data lines.

Import this CSV into your spreadsheet program. In Google Sheets, it looks like this:



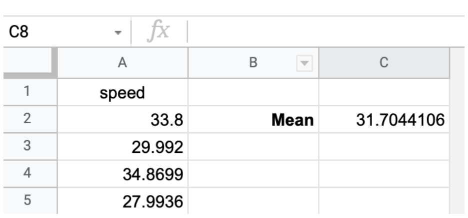
You should see a long, long column of data appear. (Mine goes from cell A2 through A1001.)



2.6 Statistics in Spreadsheets

Let's take the mean of all 1000 numbers. In cell B2, type in a label: "Mean". (Feel free to format your labels as you wish. Bolding is recommended.)

In cell C2, enter the formula "=AVERAGE(A2:A1001)". When you press return, the cell will show the mean: 31.70441, if done correctly.



Notice that by specifying that the function `AVERAGE` was to be performed on a range of cells: cells A2 through (":") A1001.

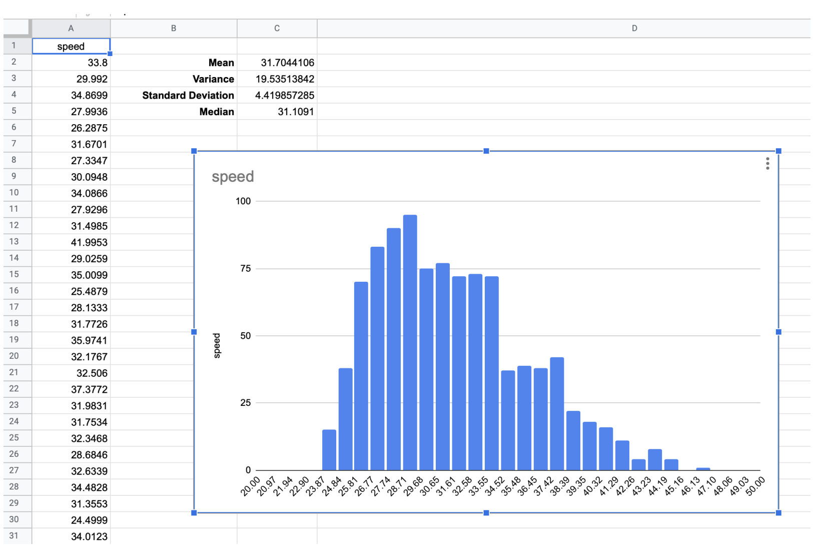
Do the calculations for variance, standard deviation, and median.

- The function for variance is `VAR`.
- The function for standard deviation is `STDEV`.
- The function for median is `MEDIAN`.

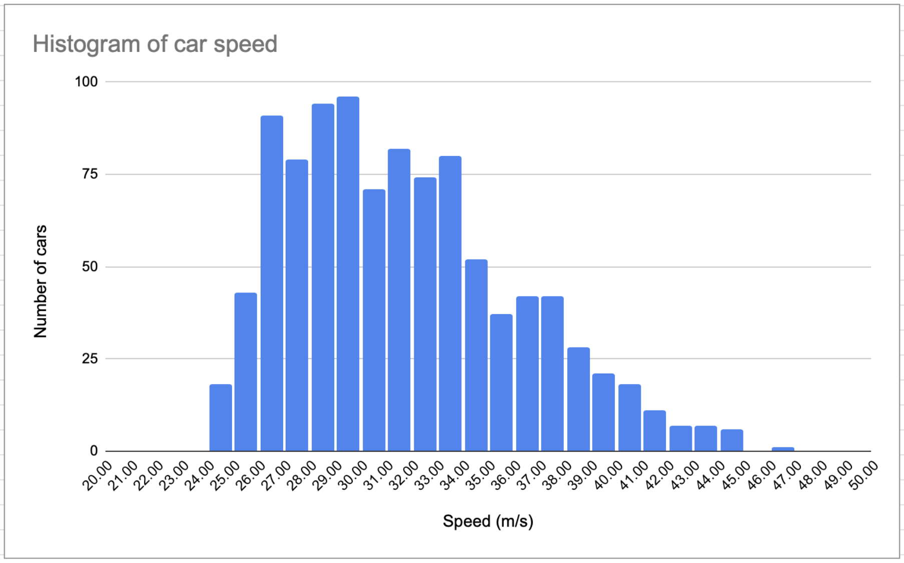
	A	B	C
1	speed		
2	33.8	Mean	31.7044106
3	29.992	Variance	19.53513842
4	34.8699	Standard Deviation	4.419857285
5	27.9936	Median	31.1091
6	26.2875		
7	31.6701		

2.7 Histogram

Most spreadsheets have the ability to create a histogram. In Google Sheets, you select the entire range A2:A1001 by selecting the first cell and then shift-clicking the last. Next, you choose Insert→Chart. In the inspector, change the type of the chart to a histogram (at the bottom under "other"). This will get you a basic histogram.



Play with the formatting to see how unique you can make data. Here is an example:



Exercise 4 RMS

In your spreadsheet, calculate the quadratic mean (the root-mean-squared) of the speeds.

You will need the following three functions:

- `SUMSQ` returns the sum of the squares of a range of cells.
- `COUNT` returns the number of cells in a range that contains numbers.
- `SQRT` returns the square root of a number.

Working Space

Answer on Page 45

2.8 The Return of the Barrel Problem

Let's get back to our example. Put labels in the A column:

- Barrels produced (per month)
- Materials cost (per barrel)
- Sale price (per barrel)
- Pre-tax earnings (per month)
- Taxes (per month)
- Take home pay (per month)

Format them any way you like. It should look something like this:

	A
1	**Barrels Produced** (per month)
2	**Materials cost** (per barrel)
3	**Sale price** (per barrel)
4	**Rent** (per month)
5	**Pretax Earnings** (per month)
6	**Taxes** (per month)
7	**Take home pay** (per month)
8	

In the B column, the first four cells are values (not formulas):

- 115 formatted as a number with no decimal point
- 45 formatted as currency
- 100 formatted as currency
- 2000 formatted as currency

It should look something like this:

	A	B
1	**Barrels Produced** (per month)	115
2	**Materials cost** (per barrel)	$45.00
3	**Sale price** (per barrel)	$100.00
4	**Rent** (per month)	$2,000.00

The next three cells in the B column will have formulas:

- B1 * (B3 - B2) - B4
- 0.2 * B5

- B5 - B6

It should look something like this:

	A	B
1	**Barrels Produced** (per month)	115
2	**Materials cost** (per barrel)	$45.00
3	**Sale price** (per barrel)	$100.00
4	**Rent** (per month)	$2,000.00
5	**Pretax Earnings** (per month)	$4,325.00
6	**Taxes** (per month)	$865.00
7	**Take home pay** (per month)	$3,460.00
8		

Now you can share this spreadsheet with your friend, and she can put different values into the cells for what-if games, such as "If I can get my materials cost down to $42 per barrel, what happens to my take home pay?"

Sometimes it is nice to show a range of values for a variable or two. In this case, it might be nice to show your friend what the numbers look like if she produces 115, 120, 125, 130, 135, or 140 barrels per month.

We have one column, and now we need six. How do we duplicate cells?

1. Click B1 to select it, then shift-click on B7 to select all seven cells.
2. Copy them. (Depending on what program you are using, there is likely a menu item for this.)
3. Click C1 to select it
4. Paste them.

	A	B	C
1	**Barrels Produced** (per month)	115	115
2	**Materials cost** (per barrel)	$45.00	$45.00
3	**Sale price** (per barrel)	$100.00	$100.00
4	**Rent** (per month)	$2,000.00	$2,000.00
5	**Pretax Earnings** (per month)	$4,325.00	$4,325.00
6	**Taxes** (per month)	$865.00	$865.00
7	**Take home pay** (per month)	$3,460.00	$3,460.00
8			

We want the first cell in the new column to be 120. You could just type in 120, but let's do something more clever. Put a formula into that cell: = B1 + 5. Now, the cell should show 120.

Why did we put in a formula? When we duplicate this column, this cell will always have 5 more barrels than the cell to its left.

Next, let's duplicate the second column a few times. The easy way to do this is to select the cells as you did before, then drag the lower-right corner to the right until column G is in the selection. When you end the drag, the copies will appear:

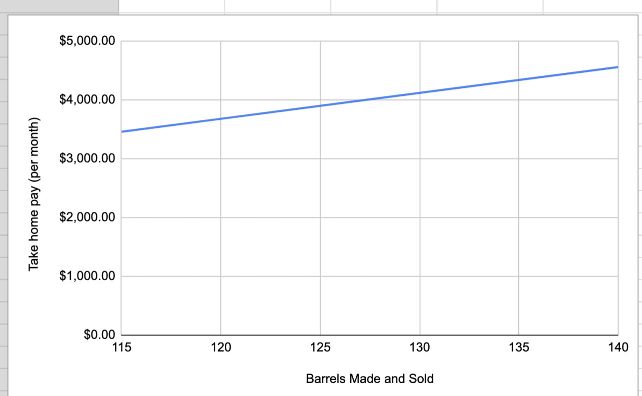
	A	B	C	D	E	F	G
1	**Barrels Produced** (per month)	115	120	125	130	135	140
2	**Materials cost** (per barrel)	$45.00	$45.00	$45.00	$45.00	$45.00	$45.00
3	**Sale price** (per barrel)	$100.00	$100.00	$100.00	$100.00	$100.00	$100.00
4	**Rent** (per month)	$2,000.00	$2,000.00	$2,000.00	$2,000.00	$2,000.00	$2,000.00
5	**Pretax Earnings** (per month)	$4,325.00	$4,600.00	$4,875.00	$5,150.00	$5,425.00	$5,700.00
6	**Taxes** (per month)	$865.00	$920.00	$975.00	$1,030.00	$1,085.00	$1,140.00
7	**Take home pay** (per month)	$3,460.00	$3,680.00	$3,900.00	$4,120.00	$4,340.00	$4,560.00

Nice, right? Now your friend can easily see how many barrels correspond to how much take-home pay. But do you know what would be even more helpful? A graph.

2.9 Graphing

Graphing is a little different on every different platform. Here is what you want the graph to look like:

	A	B	C	D	E	F	G
1	**Barrels Produced** (per month)	115	120	125	130	135	140
2	**Materials cost** (per barrel)	$45.00	$45.00	$45.00	$45.00	$45.00	$45.00
3	**Sale price** (per barrel)	$100.00	$100.00	$100.00	$100.00	$100.00	$100.00
4	**Rent** (per month)	$2,000.00	$2,000.00	$2,000.00	$2,000.00	$2,000.00	$2,000.00
5	**Pretax Earnings** (per month)	$4,325.00	$4,600.00	$4,875.00	$5,150.00	$5,425.00	$5,700.00
6	**Taxes** (per month)	$865.00	$920.00	$975.00	$1,030.00	$1,085.00	$1,140.00
7	**Take home pay** (per month)	$3,460.00	$3,680.00	$3,900.00	$4,120.00	$4,340.00	$4,560.00



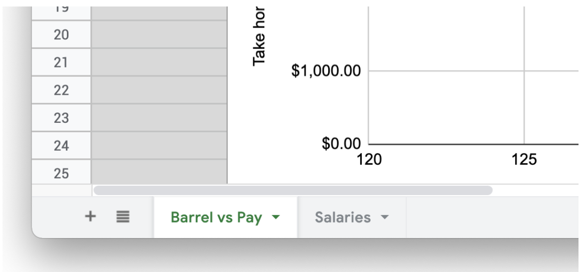
On Google Sheets:

1. Select cells B7 through G7.
2. Choose the menu item Insert -> Chart.
3. Choose the chart type (Line)

4. Add the X-axis to be B1 through G1.
5. Under the Customize tab, Set the label for the X-axis to be "Barrels Made and Sold".
6. Delete the chart title (which is the same as the Y-axis label).

2.10 Other Things You Should Know About Spreadsheets

Your spreadsheet document can have several "Sheets". Each has its own grid of cells. The sheet has a name; usually, you call it something like "Salaries". When you need to use a value from the "Salaries" sheet in another sheet, you can specify "Salaries!A2" — that is, cell A2 on sheet "Salaries". To flip between the sheets, there is usually a tab for each at the bottom of the document.



By default, the cell references are relative. In other words, when you write a formula in cell H5 that references the value in cell G4, the cell remembers "The cell that is one up and one to the left of me." Thus, if you copy that formula into B9, now that formula reads the value from A8.

If you want an absolute reference, you use $. If H5 references G4, G4 will be used no matter where on the sheet the formula is copied to.

You can use the $ on the row or column. In $A4, the column is absolute and the row is relative. In A$4, the row is absolute and the column is relative.

2.11 Challenge: Make a spreadsheet

You have a company that bids on painting jobs. Make a spreadsheet to help you do bids. Here are the parameters:

- The client will tell you how many square meters of wall needs to be painted.
- Paint costs $0.02 per square meter of wall
- On average, a square meter of wall takes 0.02 hours to paint.

- You can hire painters at $15 per hour.
- You add 20% to your estimated costs for a margin of error and profit.

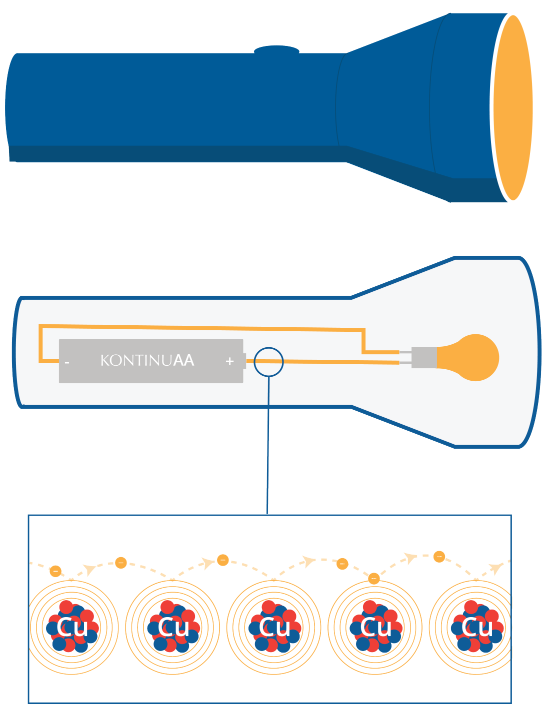
Make a spreadsheet such that when you type in the square meters to be painted, the spreadsheet tells you how much you will spend on paint and labor. It should also tell you what your bid should be.

Chapter 3

Introduction to Electricity

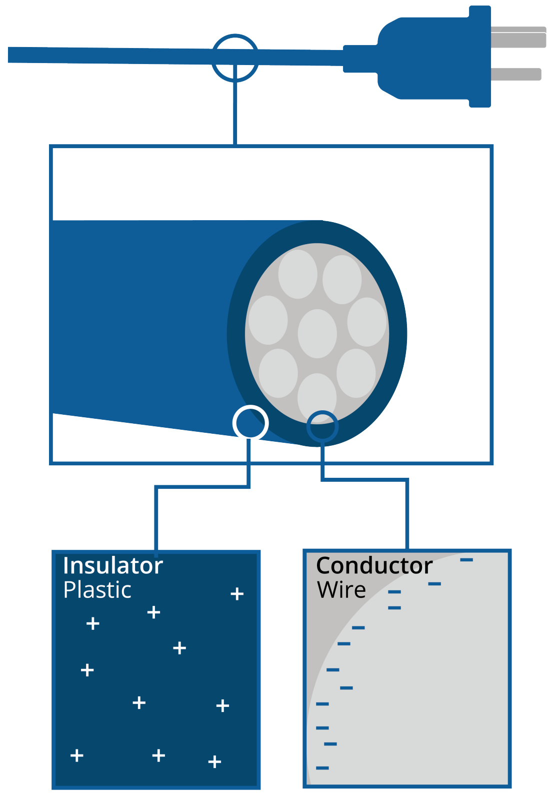
What happens when you turn on a flashlight? The battery in the flashlight acts as an electron pump, and the electrons flow through the wires to the lightbulb (or LED). As the electrons pass through the lightbulb, they excite the molecules within, which gives off light and heat. (LEDs also give off light and heat, but they give off much less heat.) The electrons then return to the battery to be pumped around again.

When electricity is flowing through a copper wire, the protons and neutrons of the copper stay put, while the electrons jump between the atoms on their way from the battery to the lightbulb and back again.



In some materials, like copper and iron, electrons are loosely bound to their nuclei, forming a sea of electrons, which allows energy to flow. These are good *electrical conductors*. In other materials, like glass and plastic, electrons don't leave their nuclei as easily. This makes them are terrible electrical conductors – we call these materials *electrical insulators*.

For example, the plastic around a wire is used for electrical insulation.



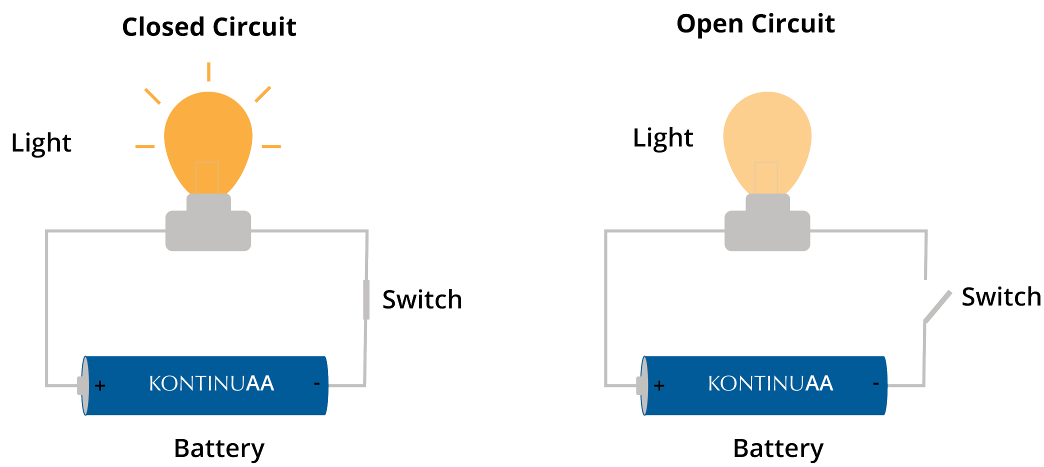
3.1 Units

Electrons are very small, so to study them, scientists came up with a unit that represents *a lot* of electrons. 1 *coulomb* is about 6,241,509,074,460,762,608 electrons. When 5 coulombs enter one end of the wire every second (and simultaneously 5 coulombs exit the other end), we say "This wire is carrying 5 amperes of current."

(Truthfully, we usually shorten ampere to just "amp". This is sometimes a little awkward, because we also often shorten the word "amplifier" to "amp", but you should generally be able to tell which is which from the context.)

If you look at the circuit breakers or fuses for your home's electrical system, you'll see that each one is rated in amps. For example, maybe the circuit that supplies power to your kitchen has a 10 amp circuit breaker. If, for some reason, more than 10 amps tries to pass through that wire, the circuit breaker will turn off the whole circuit.

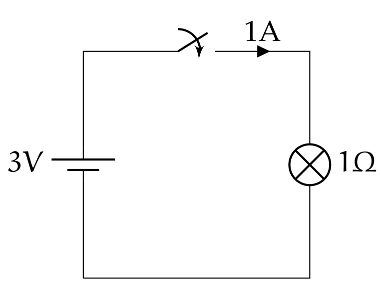
When your flashlight is on, it pushes about 1 amp of current through the lightbulb (When it is off, there is no current in the lightbulb).



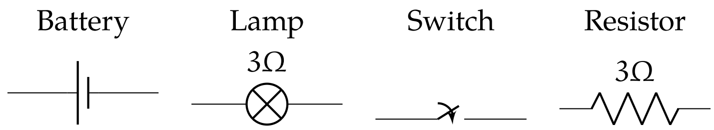
The lightbulb creates *resistance* that the current pushes through. Think of it like plumbing: The current is the amount of water passing through a pipe. The resistance is something that tries to stop the current – like a ball of hair, similar how different surfaces apply friction when pushing a box. The battery is what allows the current to push through the resistance; we call that pressure *voltage*. Especially in physics, voltage is often referred to as *electromagnetic potential*.

3.2 Circuit Diagrams

Here is a circuit diagram of your flashlight:



The lines are wires. The symbols that we will use are:



The battery pushes the electrons from the positive end (the larger line) to the negative end (the smaller line), so the circuit must go around in a circle for the current to flow. This is why the current stops flowing when the switch breaks the circuit.

You can think of a switch as having zero resistance when it is closed and infinite resistance when it is open.

For our purposes, a lamp is just a resistor that gives off light.

3.3 Ohm's Law

Resistance is measured in *ohms*, and we use a Greek capital omega for that: Ω

Voltage is measured in *volts*.

Ohm's Law

Whenever a voltage V is pushing a current I through a resistance of R, the following is true:

$$V = IR$$

where V is in volts, I is in amps, and R is in ohms.

3.4 Power and Watts

Joule's Law

When a current I is passing through a resistance R, the power consumed is

$$W = I^2R$$

where W is in watts, I is in amps, and R is in ohms.

Of course $V = IR$, so we can extend this to:

$$W = I^2R = IV = \frac{V^2}{R}$$

Your flashlight's batteries provide about 3 volts. How much battery power is the flashlight using when it is on? The power (in watts) produced by the battery is the product of the voltage (in volts) and the current (in amps). This means your flashlight is giving off $3volts \times 1amp = 3watts$ of power. Some of that power is given off as light, some as heat.

A watt is 1 joule of energy per second. We say that a watt is a measure of *power*.

When we talk about how much energy is stored in a battery, we use a unit like a kilowatt-hour. A kilowatt-hour is equivalent to 3.6 million joules.

3.5 Another great use of RMS

In many electrical problems, the voltage fluctuates a great deal. For example, the fluctuations in voltage makes the sound that comes out of an audio speaker.

You can use the root-mean-squared of the voltage to figure out the average power your speaker is consuming.

Let's say that the RMS of the voltage you are sending to the speaker is V_{rms} and the resistance of the speaker is R ohms. This means the power consumed by the speaker is:

$$P = \frac{V_{rms}^2}{R}$$

Similarly, if you know the RMS of the current you are pushing through the speaker is I_{rms}, then the power consumed by the speaker is:

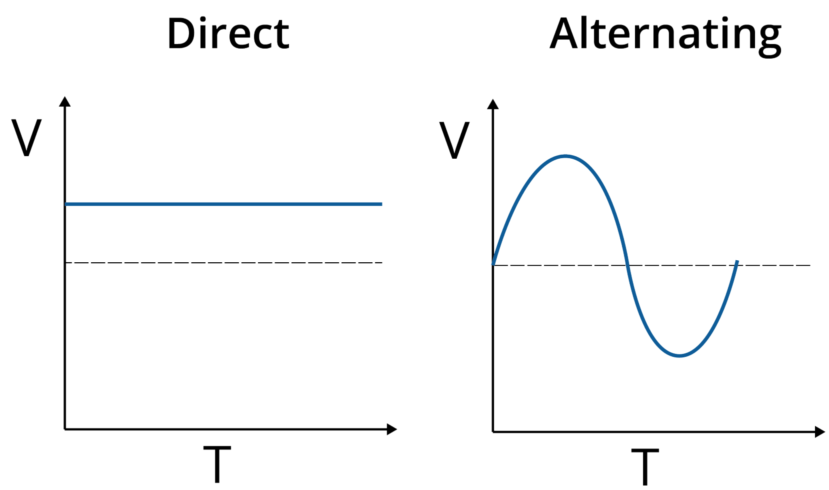
$$P = I_{rms}R$$

3.6 Electricity Dangers

Large amounts of electricity moving through your body can hurt or even kill you. You must be careful around electricity.

That said, your body is not a very good conductor, so low-voltage systems (like a flashlight) don't have enough voltage to move significant amounts of current through your body.

However, the electricity in a power outlet has much more voltage. The voltage in these outlets is fluctuating between positive and negative, so we call it *Alternating Current* or AC.



In most countries, the RMS of the voltage between 110 and 240 V. (The peak voltage is always $\sqrt{2}$ times the RMS value. In the US, for example, people say "Our outlets supply 120 V." They mean that the RMS of the voltage difference between the wire and the earth is 120V. The peak voltage is almost 170V.)

How much current can a human handle? Not much. You can barely feel 1 mA moving through your body, but at 16 mA, your muscles will clench and you won't be able to relax them — many people die from electrocution because they grab a wire which pushes enough current through their body to prevent them from letting go of the wire. At 20 mA, a human's respiratory muscles become paralyzed.

The fuse breaker in a house will often allow 20 A to flow through the circuit before it shuts off the power. Always be very sure to shut off the power before touching any of the wiring in your house.

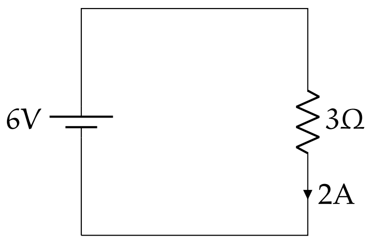
While water is actually a mediocre conductor, it can still deliver enough current to kill you. If you see a wire in a puddle, you should not touch the puddle. Interestingly, because of the salt, sea water is more than 100 times better at conducting electricity than the water you drink.

If you hold a wire in each hand, how many Ohms of resistance will your body have? Once it gets past your skin, you will look like a bag of salt water to the electricity. After the skin, your body will have a resistance of about 300Ω. However, the skin is a pretty good insulator. If you have dry, calloused hands, your skin may add 100,000Ω to the resistance.

Chapter 4

DC Circuit Analysis

In the most basic circuit, you have only a battery and a resistor:

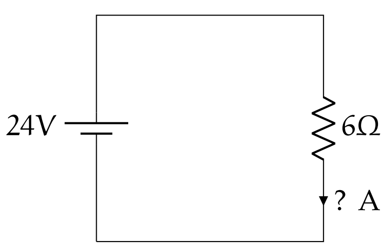


For this situation, you only need Ohm's Law: $V = IR$. In this case, $6V = 3\Omega \times 2A$.

Exercise 5 Ohm's Law

Working Space

How many amps are going around the circuit?

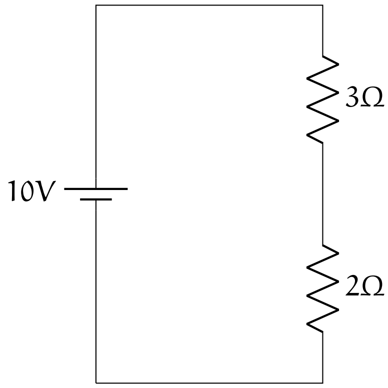


Answer on Page 46

4.1 Resistors in Series

When you have two resistors wired together in a long line, we say they are "in series." If you have two resistors R_1 and R_2 wired in series, the total resistance is $R_1 + R_2$.

In this diagram, for example, the total resistance is 5Ω.



The current flowing through the circuit, then, is $10/5 = 2A$.

By Ohm's law, the voltage drop across the upper resistor is $IR = 2A \times 3\Omega = 6V$.

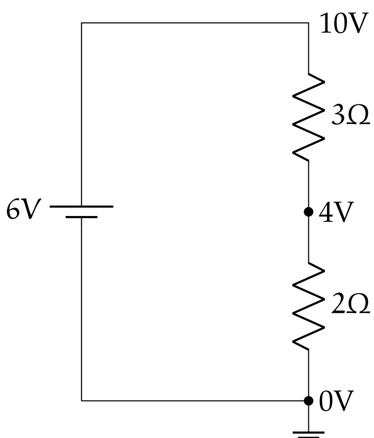
The voltage drop across the lower resistor is $IR = 2A \times 2\Omega = 4V$.

Notice that the battery pumps the voltage up to 10V, then the two resistors drop it by exactly 10V. This is known as "Kirchhoff's Voltage Law":

Kirchhoff's Voltage Law

As you make a loop around a circuit, the sum of the voltage increase must equal the sum of the voltage decrease.

The negative end of the battery is connected to "ground" (it has zero voltage). We can then draw a diagram with the voltages (That symbol in the lower right represents a connection to ground).

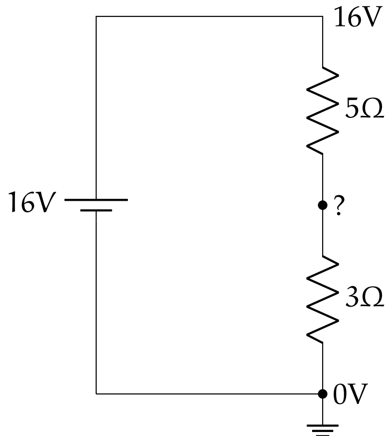


Exercise 6 Resistors In Series

Working Space

What is the current going around the circuit?

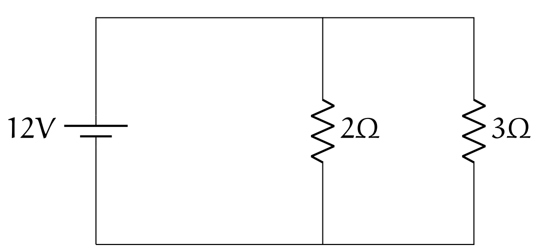
What is the voltage drop across each resistor?



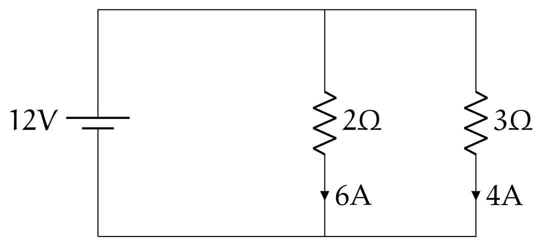
Answer on Page 46

4.2 Resistors in Parallel

Observe the following circuit. Note that the current can go two different paths.



There is 12 volts pushing current through both resistors. So 6A will go through the 2Ω resistor and 4A will go through the 3Ω resistor. Note that even though there is a path of least resistance (2Ω), the current is still devided evenly among both branches.



Thus, a total of 10 A will be going through the battery.

Imagine you are a battery. You can't see that you have two resistors. What does it feel like to you? $\frac{V}{I} = R$, and $V = 12$ and $I = 10$. This means the effective resistance of the two resistors in parallel is $\frac{12}{10}$ or $\frac{6}{5}\Omega$.

Resistance in Parallel

If you have several resistances $R_1, R_2, \ldots, R_n$ wired in parallel, their effective resistance R_t is given by

$$\frac{1}{R_t} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}$$

In our example:

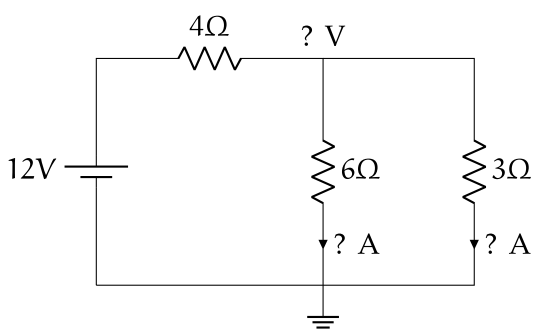
$$\frac{1}{R_t} = \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$$

Thus, $R_t = \frac{6}{5}\Omega$.

Exercise 7 Resistors In Parallel

Working Space

What is the current going through the battery? What is the drop over the 4Ω resistor? What is the current in each branch?



Answer on Page 46

4.3 Kirchhoff's Current Law

States that all currents coming into a junction (or node) must equal the currents leaving that junction. Why? Because first of all, energy is always conserved, meaning it cannot be lost or destroyed within the equations in the first place. Secondly, each circuit is conservative, meaning it cannot lose voltage, and thus, cannot lose current. As you go around a loop, the starting point must have the same chargable amount as it did when you began, so any increases and decreases along the loop have to cancel out for a total change of zero.

Kirchhoff's Current Law

The sum of current into a junction equals the sum of current out of the junction.

Chapter 5

Charge

If you rub a balloon against your hair, then place it next to a wall, it will stick. This is because it stole some electrons from your hair, and now the ballon has slightly more electrons than protons. We say that it has gotten an *electrical charge*. In this case, the balloon has a negative electrical charge.

Objects with slightly more protons than electrons have a positive charge.

This charge is measured in coulombs. The charge of a single proton is about 1.6×10^{-19} coulombs.

An object with a negative charge and an object with a positive charge will be attracted to each other. Two objects with the same charge will be repelled by each other.

Coulomb's Law

If two objects with charge q_1 and q_2 (in coulombs) are r meters from each other, the force of attraction or repulsion is given by

$$F = K\frac{|q_1 q_2|}{r^2}$$

where F is in newtons and K is Coulomb's constant: about 8.988×10^9.

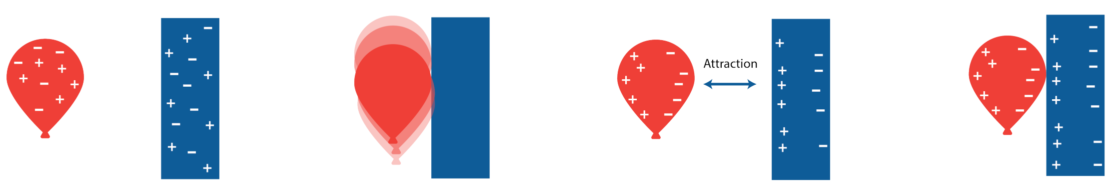
Exercise 8 Coulomb's Law

Working Space

Two balloons are charged with an identical quantity and type of charge: -5×10^{-9} coulombs. They are held apart at a separation distance of 12 cm. Determine the magnitude of the electrical force of repulsion between them.

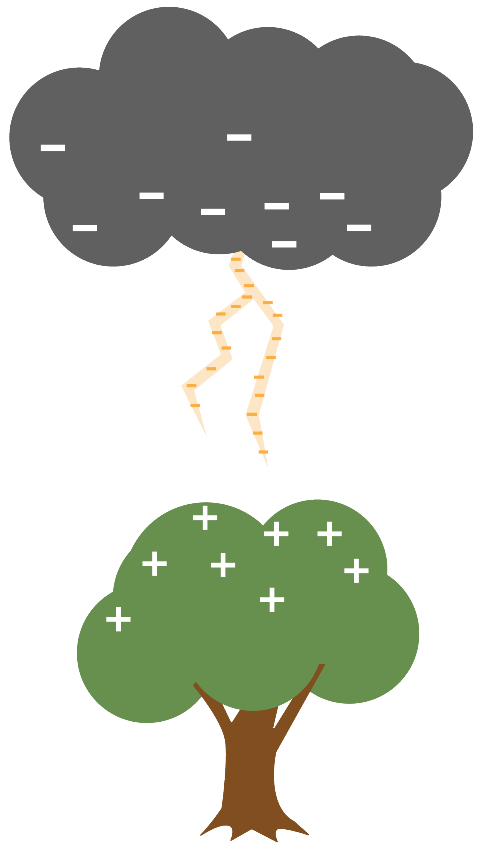
Answer on Page 47

At this point, you might ask "If the wall has zero charge, why is the balloon attracted to it?" The answer: the electrons in the wall move away from the balloon, polarizing the atoms. The negative charge on the balloon pushes electrons away from itself, so the surface of the wall gets a mild positive charge. The negative charge on the balloon is attracted more to the positive charge on the surface of the wall than the negative charge on the inside, thus the balloon sticks. There's some weirdness going on with conductors and insulators here, but we will get to that later.



5.1 Lightning

A cloud is a cluster of water droplets and ice particles. These droplets and ice particles are always moving up and down through the cloud. In this process, electrons get stripped off and end up on the water droplets at the bottom of the cloud (water droplets collect at the bottom because they are denser). The air between the droplets is a pretty good insulator, which means the electrons are reluctant to jump anywhere. However, eventually, the charge gets so strong that even the insulating properties of the air is not enough to prevent the jump, causing lightning.



A great deal of lightning moves within a cloud or between clouds. However, sometimes it jumps to the earth. These bolts of lightning vary in the amount of electrons they carry, but the average is about 15 coulombs.

Thunder occurs because the electrons heat the air they pass through, causing the air to expand suddenly. Tthe resulting shockwave is the sound we know as thunder.

5.2 But...

This idea that opposite charges attract creates some heavy questions that you do not yet have the tools to work with. So to these questions, the answer is basically "Don't ask that yet!"

However, you probably have these questions, so we will point you in the direction of the answers.

The first is "In any atom bigger than hydrogen, there are multiple protons in the nucleus. Why don't the protons push each other out of the nucleus?"

We aren't ready to talk about it, but there is a force called *the strong nuclear force*, which pulls the protons and neutrons in the nucleus of the atom toward each other. At very, very small distances, it is strong enough to overpower the repulsive force due to the protons' charges.

Another question is "Why do the electrons whiz around in a cloud so far from the nucleus of the atom? Negatively charged electrons should cling to the protons in the center, right?"

We aren't ready to talk about this either, but quantum mechanics tells us that electrons like to live in a certain specific energy level. Hugging protons isn't one of those levels.

Appendix A

Answers to Exercises

Answer to Exercise 1 (on page 4)

$$\mu = \frac{1}{6}(87 + 91 + 98 + 65 + 87 + 100) = 88$$

Answer to Exercise 2 (on page 6)

The mean of your grades is 88.

The variance, then, is

$$\sigma^2 = \frac{1}{6}\left((87-88)^2 + (91-88)^2 + (98-88)^2 + (61-88)^2 + (87-88)^2 + (100-88)^2\right) = \frac{784}{6} = 65\frac{1}{3}$$

The standard deviation is the square root of that: $\sigma = 8.083$ points.

Answer to Exercise 3 (on page 7)

In order the grades are 65, 87, 87, 91, 98, 100. The middle two are 87 and 91. The mean of those is 89. (Speed trick: The mean of two numbers is the number that is halfway between.)

Answer to Exercise 4 (on page 20)

The formula for the RMS is "=SQRT(SUMSQ(A2:A1001)/COUNT(A2:A1001))".

Answer to Exercise 5 (on page 35)

$V = IR$ so $I = \frac{V}{R} = \frac{24V}{6\Omega} = 4A$.

Answer to Exercise ?? (on page 37)

There is a total resistance of 8Ω, so your 16V will push 2A of current around the circuit.

2A going through a 5Ω resistor represents a 10V drop.

2A going through a 3Ω resitor represents a 6V drop.

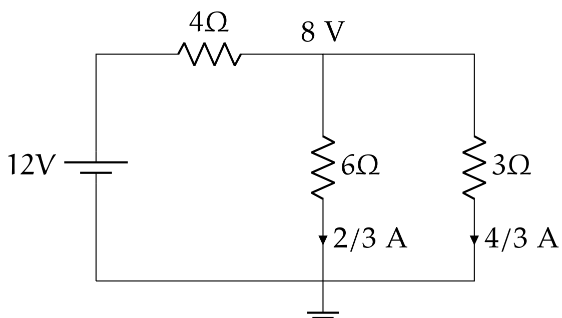
Answer to Exercise 7 (on page 39)

The effective resistance of the 6Ω and the 3Ω is 2Ω because

$$\frac{1}{R_T} = \frac{1}{6} + \frac{1}{3} == \frac{1}{2}$$

This means the battery experiences a resistance of $4\Omega + 2\Omega = 6\Omega$. A 12V will push 2A through a resistance of 6Ω.

The voltage drop across the 4Ω resistor is $2A \times 4\Omega = 8V$. Thus there will be a 4V drop across the two resistors in parallel. So 2/3 A will flow through the 6Ω resistor. 4/3 A will flow through the 3Ω resistor.



Answer to Exercise 8 (on page 42)

$$F = K\frac{|q_1 q_2|}{r^2} = (8.988 \times 10^9)\frac{(-5 \times 10^{-9})(-5 \times 10^{-9})}{0.12^2} = \frac{224.7 \times 10^{-9}}{0.0144} = 15.6 \times 10^{-6}$$

15.6 micronewtons.

Index