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Atoms and Matter

1.1 What Matter Is Made Of

The universe is made of matter and energy. Current models posit that the universe is approximately 68% dark energy, 27% dark matter, and 5% ordinary matter. Everything you can see and touch is part of the small part of the universe made of ordinary matter. Most science deals with ordinary matter and its interactions; highly trained theoretical physicists are currently debating the nature and effects of dark matter and dark energy.

What is this ordinary matter made of? All things (including the air around you) are made of *atoms*. Atoms are incredibly tiny — there are more atoms in a drop of water than there are drops of water in all the oceans. Sizes like that are hard to picture. [The Scale of the Universe](#), a website by Cary and Michael Huang, lets you zoom from galaxies down to quarks. Start at $10^{-15.5}$ meters to see protons and neutrons.

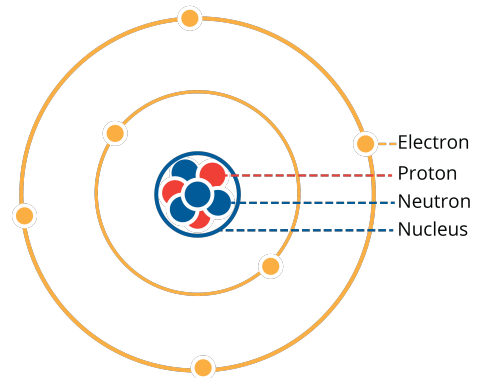


Figure 1.1

Every atom has a *nucleus* that contains *protons* and *neutrons*. Orbiting around the nucleus is a cloud of *electrons*. However, the mass of the atom comes mainly from the protons and neutrons, since they are about 2000 times as massive as an electron! These three particles, protons, neutrons, and electrons, are called *subatomic particles*. (See figure 1.1.)

Studies such as particle physics, nuclear physics, and astrophysics analyze how these particles interact.

An atom is mostly empty space. If the nucleus were a marble in the middle of a football stadium, the electrons would be specks buzzing around the outer seats. What holds them there? The *electromagnetic force*: the positive protons attract the negative electrons.

Every charged particle surrounds itself with an *electric field*, and moving charges also make a *magnetic field*. That is why an electric current, which is just electrons moving through a wire, can push a compass needle.

1.1.1 Atoms and Their Models

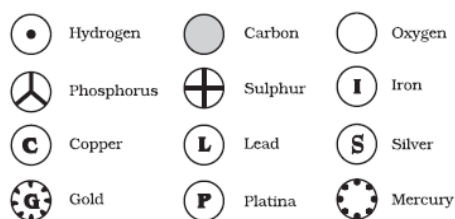


Figure 1.2

Our picture of the atom has changed a lot. Around 1800, John Dalton pictured atoms as tiny, solid, indivisible balls. Then in 1897, the British physicist J.J. Thomson discovered the electron: a light, negatively charged particle that can be pulled out of any atom. Atoms, it turned out, have parts.

In 1911, Ernest Rutherford fired tiny particles at a thin sheet of gold. Most went straight through, but a few bounced almost straight back, “as if you fired a 15-inch shell at a piece of tissue paper and it came back and hit you,” he said. The atom, he concluded, is mostly empty space, with its positive charge and almost all its mass packed into a tiny, dense *nucleus*. Subsequently, Niels Bohr explained the phenomena of spectral lines (we will discuss this further in Sequence 2) by proposing that electrons orbit at specific distances from the nucleus.



Figure 1.3

This is likely the model you are most familiar with seeing, and it is the one we will use most often in this text. The first figure shown in this chapter is a Bohr model: it shows the protons and neutrons in the nucleus, and models the electrons as moving in discrete orbits around the nucleus.

However, the Bohr model is slightly inaccurate. While it is a convenient model for thinking about atoms, in reality, electrons do not neatly orbit the nucleus. Scientists do not know exactly where an electron will be in relation to the nucleus, but they do know where it is most likely to be. They use a cloud that is thicker in the center but fades out at the edges to represent an electron’s position (see figure 1.3). This cloud represents the probability distribution of finding an electron in a particular region around the nucleus. The figure is only in two dimensions, but the actual electron cloud is three-dimensional. The shape of the cloud depends on the energy level of the electron, and different energy levels have different shapes and sizes of clouds.

While the cloud model is more accurate, we will use the Bohr model as it allows you to easily and quickly assess the number and arrangement of electrons.

Classifying Atoms

We classify atoms by the numbers of protons they have. An atom with one proton is a hydrogen atom, an atom with two protons is a helium atom, and so forth (refer to Table 2.1 on page 20)). We say that hydrogen and helium are *elements* because the classification of elements is based on the proton number. And we give each element an atomic symbol. Hydrogen gets H, helium gets He, oxygen gets O, carbon gets C, and so on. You can see an element's symbol on the periodic table.

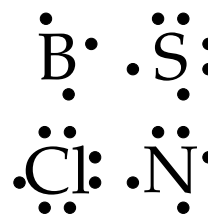


Figure 1.4

An atom will typically have the same number of electrons as proton (with an overall electric charge of 0), but when it gains or loses electrons, it becomes an *ion* with a net electric charge. **Ionization** is the addition or removal of certain particles to reach a charge imbalance. Atoms can gain or lose electrons through various processes, such as chemical reactions or ionizing radiation. An atom likes to gain or lose electrons to achieve a full outer electron shell, typically with 8 electrons (the octet rule). When an atom gains electrons, it becomes a negatively charged ion, or *anion*. When it loses electrons, it becomes a positively charged ion, or *cation*.

When Atoms Combine

When atoms of different elements combine, they make *compounds*. Compounds are substances made up of more than one element. Compounds can be *molecules* or *crystal lattices*. In the next section you will learn *why* these different structures form.

There are many kinds of compounds. You know a few:

- Table salt is crystals made of Na^+ and Cl^- ions: a sodium atom that lost an electron and a chlorine atom that gained an electron
- Baking soda, or sodium bicarbonate, is NaHCO_3 .
- O_2 is the oxygen you breathe in. (Air is a blend of gases, mostly N_2 .)
- Common quartz is SiO_2 : silicon dioxide

The subscripts indicate what quantities of the elements are present in the compound. Each number indicates the quantity of the preceding element. A molecule of water, H_2O , has two hydrogen atoms and one oxygen atom. Often times we will see these numbers used to determine the ratio of elements in a compound. To do this, we compare the quantities of each element to find their simplest whole-number ratio. This is called the *empirical formula* of the compound.

Example: What is the ratio of elements present in Epsom salt?

Solution: Epsom salt, chemical name magnesium sulfate, has the chemical formula MgSO_4 . Therefore, the ratio of Mg:S:O is 1:1:4.

Exercise 1 Ratios of Atoms in Molecules

Give the elemental ratio for each compound.

Working Space

1. methane, CH_4
2. copper (II) sulfate, CuSO_4
3. glucose, $\text{C}_6\text{H}_{12}\text{O}_6$

Answer on Page 71

1.1.2 Types of Matter

One way to classify matter is by the types of chemical bonds that hold a material's atoms together. The nature of these bonds, in turn, affects the properties of the material. For now, all you need to know is there are three types of chemical bonds: *metallic*, *covalent*, and *ionic*. Materials held together with the same type of bonds tend to have similar properties. For example, copper, bronze, iron, and steel (all containing metallic bonds) are all shiny, ductile, malleable, and good conductors of heat and electricity. On the other hand, Epsom salt and table salt form large crystals, have very high melting points, and are poor conductors of electricity in their pure form. These two substances (Epsom and table salt) both contain ionic bonds.

Covalent Compounds

Water is an example of a covalent compound: it is made of two hydrogen atoms covalently bonded to one oxygen atom (see figure 1.5). The result is a water molecule. The different atoms cluster together because they share electrons in their clouds. This is the nature of a *covalent bond*: it is formed when atoms share electrons. Sometimes, electrons are shared evenly, but in water, they are shared unevenly. Oxygen is better at attracting electrons to itself than hydrogen, and so the shared electrons spend more time on the oxygen atom than the hydrogen atoms. As a result, the oxygen side of a water molecule has a slight negative charge, while the hydrogen atoms have a slight positive charge. These slight charges are represented with a lower case Greek letter delta, δ . When electrons are shared unevenly,

we call this a *polar* covalent bond, because there are positive and negative poles at either end of the bond.

Whether electrons are shared evenly or unevenly is based on the elements' relative *electronegativities*. Electronegativity is simply a measure of how strongly an atom can attract electrons to itself. In general, elements on the right side of the periodic table are more electronegative than elements on the left side. When covalent bonds form between two elements of similar electronegativities, the electrons are shared evenly. We call this a *non-polar* covalent bond. Oil is an example of a non-polar covalent substance. Different oils have different ratios and structures, but all oils are made mainly of carbon and hydrogen, which have similar electronegativities. What happens when you try to mix oil and water? They do not mix well! This is due to the difference between their bond types. Polar substances, like water, mix best with other polar substances, while non-polar substances, like oils, mix best with non-polar substances. You will learn more about why this is in Sequence 2.

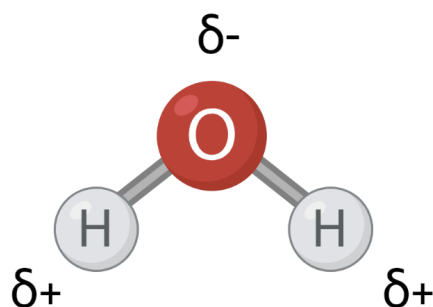
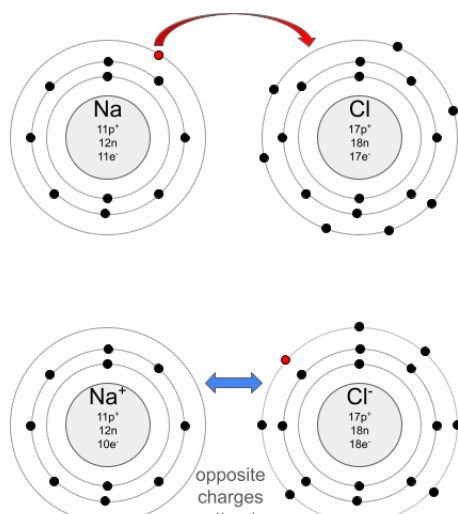


Figure 1.5

For both polar and non-polar covalent bonds, the electrons are held tightly to the nuclei, even if they are shared among atoms. Those electrons do not move to another molecule: they move around within the molecule they are already a part of. Since electrons do not flow freely in covalent substances, they are also poor conductors of electricity. Covalent compounds also tend to have lower melting and boiling points compared to ionic compounds.

Ionic Compounds



Ionic bonds are the electrical attraction between opposite-charged ions. When a neutral atom gains or loses an electron it becomes an *ion* (a charged atom), and oppositely-charged ions are attracted to each other. Which atom gets the electron and which loses it is based on their electronegativities: the more electronegative atom steals one or more electrons from the less electronegative atom.

Let us examine how a simple ionic compound is formed: sodium chloride, also known as table salt, is made up of sodium and chlorine atoms (see figure 1.6). When sodium and chlorine come in contact with each other, electrons move from the sodium to the chlorine, making a sodium *cation* (positively-charged ion) and a chloride *anion* (negatively-charged ion). Yes — *chloride* is correct! When naming an anion, the ending of the element name changes to *-ide*. Once the sodium cation and chloride anion are formed, their opposite charges attract them to each other, like north and south magnet poles.

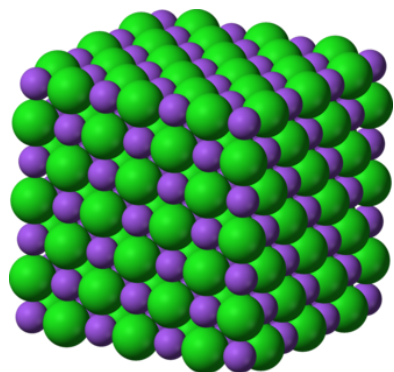


Figure 1.7

When there are many, many sodium and chloride ions around, they spontaneously arrange themselves in a pattern, giving ionic compounds their characteristic crystal structure (see figure 1.7). Because the electrons are tightly held by each ion, ionic substances do not conduct electricity well as solids. The atomic crystal lattice also determines the shape of the macroscopic crystals. Salt crystals are generally cubic, while other crystals (like quartz) form hexagonal prisms. You will learn how to predict the atomic and macroscopic crystal structure of different compounds in Sequence 2.

Modeling Ionic Compounds

You can represent ions with Lewis dot diagrams by adding or subtracting electrons from the model. Since electrons are negative, anions have gained electrons while cations have lost them. Additionally, the ion is in brackets and the overall charge is indicated outside the top right corner of the brackets.

Example: Create Lewis dot diagrams for Na^+ , F^- , O^{2-} , and Mg^{2+} .

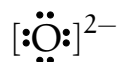
Solution: Sodium is in column 1, so a neutral sodium atom has 1 valence electron. A +1 charge means it has lost 1 electron, leaving zero.



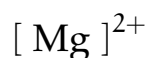
Fluorine is in column 17 and has 7 valence electrons when neutral. The anion F^- has gained one electron, for a total of 8.



Neutral oxygen has 6 valence electrons. O^{2-} has gained two, for a full set of 8.



Neutral magnesium has 2 valence electrons. Mg^{2+} has lost both, leaving none.



Notice the pattern: atoms tend to gain or lose electrons until they have a full outer shell (8) or an empty one.

To make Lewis dot diagrams of ionic compounds, you show both ions in the ratio given in the formula. For example, a Lewis dot diagram of MgCl_2 would show one magnesium cation and two chloride anions.

Metallic Compounds

You may already know that metals are excellent conductors of electricity and heat. This is a consequence of their metallic bonds. In pure metals and alloys, the outermost layer of electrons can move freely from one atom to the next. As a result, at the atomic level, metals are best characterized as a lattice of cations surrounded by a "sea of electrons" .

The free-flowing sea of electrons in pure and alloyed metal means the cation lattice can be rearranged without breaking the metallic bond. As a result, metals are ductile (able to be drawn out into a wire) and malleable (able to be hammered into a new shape without cracking or breaking). A paperclip bends; a salt crystal shatters.

Metals can be pure, like copper or iron, or *alloys*, like bronze or steel. *Alloys* are mixtures between two or more elements where at least one element is a metal. Steel is an alloy of iron and carbon; bronze is an alloy of copper and tin. Alloying changes a metal's properties because the different-sized atoms jam the lattice, making it harder for layers to slide. That is why steel is much stronger than pure iron.

1.1.3 Conclusion

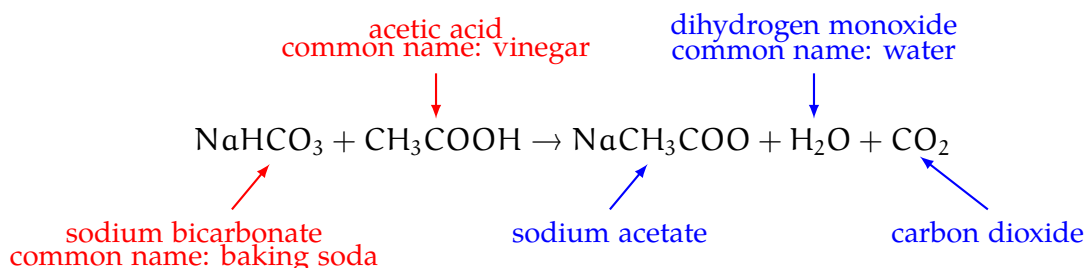
We have seen that the universe is made of dark energy, dark matter, and ordinary matter. Ordinary matter is made of atoms, which can be classified based on their number of protons. Atoms combine in different ways to make compounds, and the manner of combination (ionic, covalent, or metallic bonding) affects the macroscopic properties of the substance.

Matter has one more property that makes chemistry possible: it cannot come from nothing or vanish into nothing. That idea is the subject of the next section.

1.2 Conservation of Mass

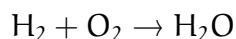
One of the most powerful laws in science says that mass cannot be created or destroyed, only moved around or rearranged. (Energy obeys a similar law, which you will meet in the Work and Energy chapter.)

Since matter cannot be created or destroyed, a chemical reaction does not change the mass of the *reactants* as they form *products*. Consider the reaction between vinegar and baking soda, which produces carbon dioxide, water, and sodium acetate:



The reactants are labeled with red, and the products with blue. This equation is *balanced*: that is, it shows the same number of each element on each side of the arrow. This shows that the atoms are not created or destroyed during a chemical reaction, they are only rearranged. Take a minute to count up each element on each side. You should find there are 5 hydrogens, 1 sodium, 3 carbons, and 5 oxygens on each side.

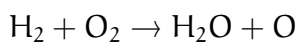
Now, let us look at an *unbalanced* chemical reaction. As you know, hydrogen and oxygen combine to form water. Additionally, hydrogen and oxygen both exist as *diatomic gases*. When we say "oxygen gas" or "hydrogen gas", we mean the diatomic molecules, O₂ and H₂, respectively. Here is an unbalanced chemical reaction between hydrogen gas and oxygen gas to form water:



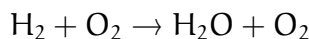
How do we know this equation is unbalanced? Count up the elements: there are two oxygen atoms on the reactant side, but only one on the product side. This violates the conservation of matter: that oxygen atom cannot just disappear!

1.2.1 Balancing Chemical Reactions

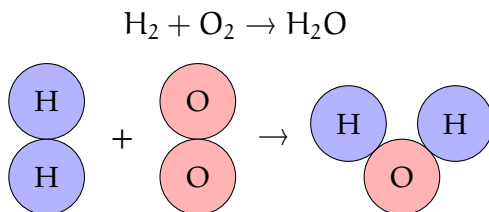
We solve this by *balancing* the chemical reaction: adjusting the number of products and reactants to comply with the Law of Conservation of Matter. You will learn strategies for balancing chemical reactions in Sequence 2, but for now we will briefly balance this chemical reaction so that it complies with the Law of Conservation of Matter. You may be tempted to simply add a lone oxygen atom to the products side:



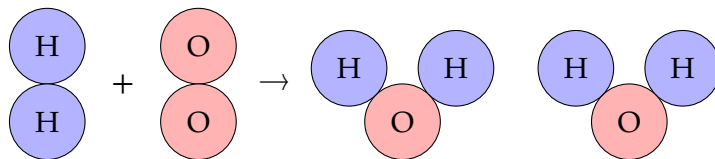
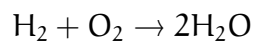
The major reason this is incorrect is that oxygen does not exist as a lone atom - as discussed above - so it does not make sense to have a lone oxygen as a product. So maybe we should add a molecule of oxygen gas to both sides?



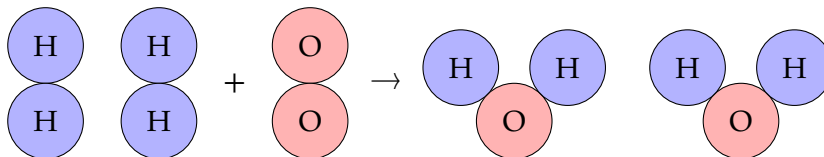
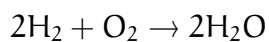
Well, now we have the same problem we started with: the oxygens are unbalanced. When balancing chemical reactions, we can only add *whole molecules* that are already in the reaction. Let us take another look at our unbalanced reaction with some molecular models for visualization:



You can clearly see we need more oxygens on the product side. Since we can only add *whole molecules*, our only option is to add another water. We do this by adding a coefficient of 2 in front of H_2O in our equation, which indicates 2 water molecules (just like $2x$ means two x's):



We have fixed our oxygen problem: now there are two oxygen atoms on both sides. But now we have a hydrogen problem: there are 2 on the reactant side and 4 on the product side. We can address this by adding another hydrogen gas molecule to the reactant side:



And now we have the same number of hydrogens and oxygens on each side! Notice we have all the same reactants and products that we started with, but now in ratios that reflect the conservation of matter.

A final note: if atoms are in parentheses followed by a subscript, the subscript applies to every atom in the parentheses. For example, zinc nitrate, $\text{Zn}(\text{NO}_3)_2$ is made of 1 zinc, 2 nitrogens, and 6 oxygens.

Exercise 2 **Balanced and Unbalanced Reactions**

Classify the following chemical reactions as balanced or unbalanced. If it is unbalanced, state what element(s) are not conserved.

Working Space

1. $\text{NiCl}_2 + 2\text{NaOH} \rightarrow \text{Ni}(\text{OH})_2 + 2\text{NaCl}$
2. $\text{HgO} \rightarrow \text{Hg} + \text{O}_2$
3. $\text{BaSO}_4 + 2\text{C} \rightarrow 2\text{BaS} + \text{CO}$
4. $\text{Cd}(\text{NO}_3)_2 + \text{H}_2\text{S} \rightarrow \text{CdS} + 2\text{HNO}_3$

Answer on Page 71

Atomic Mass and Chemical Reactions

2.1 Atomic and Molecular Mass

2.1.1 Reading a Periodic Tile

Let us look at the different information shown on a periodic tile:

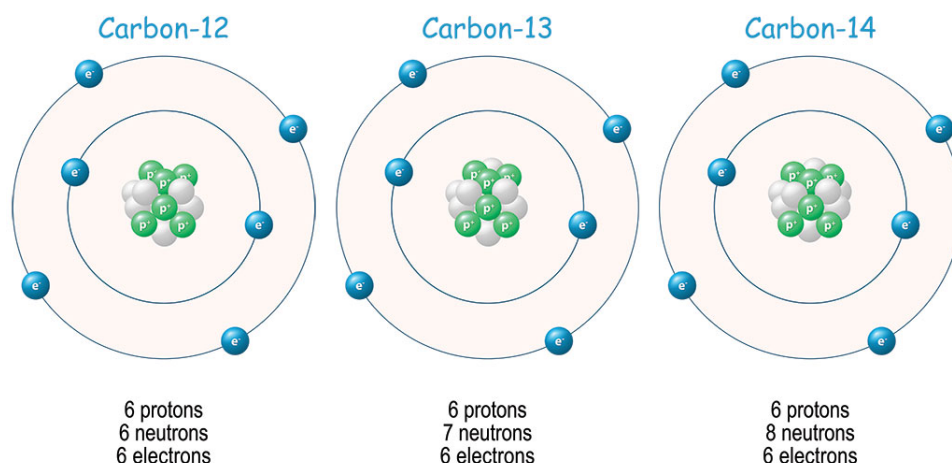
		Atomic Weight
Atomic Number	6	12.011
Symbol	C	
Name	Carbon	

The four things we learn from a periodic tile are:

1. the symbol: as discussed in the previous chapter, each element has a unique symbol. Element symbols are used when showing the structure of a molecule and modeling chemical reactions.
2. the atomic number: this is also unique for each element. Take a look at the periodic table a few pages forward. Every tile has a unique atomic number, and the tiles are laid out in a generally increasing atomic number (you will learn why the periodic table is arranged this way in Sequence 2).

3. the atomic weight: this is the average mass of all the atoms of that element in existence. Just like your overall grade in a class is the weighted average of all the individual grades you earned, atomic weight is the weighted average of the masses of all the individual atoms of that element. This is also sometimes referred to as atomic mass.
4. the name: not all periodic tables show the name of an element on its tile. This is why it is useful to know the symbols of common elements.

Recall from the previous chapter that we classify atoms by the number of protons they have. What this means is that if we want to know what element an atom is, we have to look at the number of protons. Take a look at the three carbon atoms below and note what is the same and what is different among them:



These different versions of carbon all have 6 protons, which is also carbon's atomic number. This is not a coincidence: the atomic number *is* the number of protons in every atom of an element. If I tell you an atom has 4 protons, you would find atomic number 4 and see that the element is beryllium. To know how many protons an oxygen atom has, you would find its tile and see that it has atomic number 8.

So atoms of the same element have the same number of protons, and that number is the element's atomic number. The difference between these carbon atoms explains the other number on a periodic tile: the atomic mass.

A proton and a neutron have about the same mass. An electron, on the other hand, is tiny: it takes about 1,800 electrons to match the mass of one neutron. This means that the mass of any object comes mostly from the protons and neutrons in the nucleus of its atoms.

We know how many protons an atom has by what element it is, but how do we know the number of neutrons?

2.1.2 Mass of Atoms and Molecules

As you have seen, a periodic tile for an element tells us the average mass of an atom of that element in Daltons or amu (atomic mass units). The average mass of a carbon atom is 12.011 amu, and the average mass of an iron atom is 55.845 amu. Using the periodic table, determining the average mass of an atom is straightforward. What about molecules?

Consider water: H_2O . It is made of 2 hydrogen atoms, each with an average mass of 1.008 amu, and one oxygen atom, with an average mass of 15.999 amu. To find the mass of the molecule, called *molecular mass*, you simply add the masses of each of the atoms in the molecule. So, the molecular mass of water is $1.008 \text{ amu} + 1.008 \text{ amu} + 15.999 \text{ amu} = 18.015 \text{ amu}$.

Exercise 3 Determining Molecular Mass

Find the molecular mass, in amu, of the following substances:

Working Space

1. CH_4
2. CuSO_4
3. $\text{C}_6\text{H}_{12}\text{O}_6$

Answer on Page 71

2.1.3 Mole Concept

An atomic mass unit is a very, very, very small unit; we would much rather work in grams. Grams are a large enough unit that you can develop a natural sense for how much a gram is. Additionally, while you cannot see a single carbon atom with your eyes, you can see 10 grams of carbon (about enough to fill a pen cap). To convert between the very, very, very small unit of amu to the tangible unit of grams, we use *Avogadro's Number* (sometimes called *Avogadro's Constant*).

Since 1 amu is defined as $1/12^{\text{th}}$ of the mass of a carbon-12 atom, carbon-12 by definition has a mass of 12 amu. Additionally, Avogadro's number is the number of carbon atoms in 12.000 grams of pure carbon-12. This amount is called *a mole*. A mole is just a counting word, like a dozen. Twelve doughnuts is a dozen doughnuts. Twenty is a score. Five hundred is a ream. And $6.02214076 \times 10^{23}$ doughnuts is a *mole* of doughnuts, enough

to bury the entire Earth under a layer of doughnuts more than 100 km deep. That is why we save moles for tiny things like atoms and molecules. (A mole is not short for “molecule”; the names just sound alike.) To get a feel for how huge this number is, watch this video: <https://www.youtube.com/watch?v=TEl4jeETVmg>. A mole of carbon-12 has a mass of 12.000 g, but a mole of natural carbon (which includes all the isotopes of carbon) has a mass of 12.011 g. The mole is defined such that one mole of an element is the same mass in grams as one atom is in amu. Let us say you want to know how much a mole of NaCl weighs. From the periodic table, you see that Na has an atomic mass of 22.990 atomic mass units, and Cl has 35.453 atomic mass units. One unit of NaCl (one sodium plus one chlorine) has a mass of $22.990 + 35.453 = 58.443$ atomic mass units. This means a mole of NaCl has a mass of 58.443 grams. Handy, right? This is called the *molar mass*. It is the mass of one mole of a substance, and is given in units of g/mol (grams per mole). The molar mass of NaCl is 58.443 g/mol. The molar mass of carbon is 12.011 g/mol. Using dimensional analysis and the molar mass, you can determine the mass of a given number of moles of a substance.

Example: What is the mass of 2 moles of copper?

Solution: The conversion we will use is $1 \text{ mol Cu} = 63.546 \text{ g Cu}$.

$$\frac{2 \text{ mol Cu}}{1} \times \frac{63.546 \text{ g Cu}}{1 \text{ mol Cu}} = 127.092 \text{ g Cu}$$

Therefore, 2 moles of copper has a mass of 127.092 grams.

You can also find the molar mass of a molecule, like methane. Just like with elements, a mole of a molecule has the same mass in grams as a single molecule has in amu.

Example: What is the mass of 3.5 moles of methane?

Solution: Methane (CH_4) has a molecular mass of 16.043 amu, which means 1 mole of methane has a mass of 16.043 grams.

$$\frac{3.5 \text{ mol CH}_4}{1} \times \frac{16.043 \text{ g CH}_4}{1 \text{ mol CH}_4} = 56.151 \text{ g CH}_4$$

You can also use the molar mass to determine how many moles of a substance there are in a given mass of that substance.

Example: A standard AAA battery contains about 7.00 g of zinc. How many moles of zinc are in a AAA battery?

Solution: Zinc's molar mass is 65.38 g/mol.

$$\frac{7.00 \text{ g Zn}}{65.38 \text{ g Zn}} \times \frac{1 \text{ mol Zn}}{1} \approx 0.107 \text{ mol Zn}$$

In summary, a mole of a substance contains approximately 6.02×10^{23} particles (atoms or molecules) of that substance and has a mass equal to the molecular mass in grams.

The Mole Concept

For a substance, X, with a molar mass of x g/mol,

$$1 \text{ mol X} = 6.02 \times 10^{23} \text{ particles of X} = x \text{ g of X}$$

Exercise 4 Grams, Moles, Molecules, and Atoms

Complete the table.

Substance	num. of particles	num. of moles	grams
NaHCO ₃			35
HCl		1.25	
KH ₂ PO ₄	12.5×10^{24}		

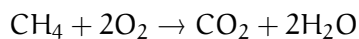
Working Space

Answer on Page 71

Exercise 5 Burning Methane

Working Space

Natural gas is mostly methane (CH₄). When one molecule of methane burns, it uses up two oxygen molecules (O₂) and makes one molecule of CO₂ and two molecules of H₂O:



If you need 200 grams of water, how many grams of methane do you need to

burn?

(The hero of *The Martian* makes water for his potato farm in a similar way, by burning hydrogen.)

[Answer on Page 72](#)

If you fill a balloon with helium, it will have two different kinds of helium atoms. Most of the helium atoms will have 2 neutrons, but a few will have only 1 neutron. We say that these are two different *isotopes* of helium. We call them helium-4 (or ${}^4\text{He}$) and helium-3 (or ${}^3\text{He}$). Isotopes are named for the sum of protons and neutrons the atom has: helium-3 has 2 protons and 1 neutron.

A hydrogen atom nearly always has just 1 proton and no neutrons. A helium atom nearly always has 2 protons and 2 neutrons. So, if you have 100 hydrogen atoms and 100 helium atoms, the helium will have about 4 times more mass than the hydrogen. We say “Hydrogen is about 1 atomic mass unit (amu), and helium-4 is about 4 atomic mass units.”

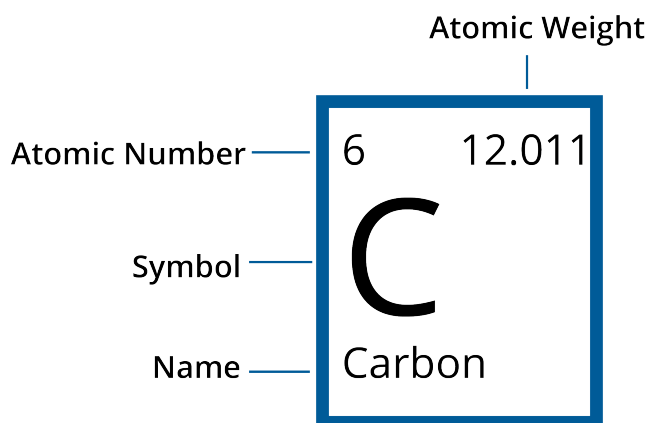
What, precisely, is an atomic mass unit? It is defined as $1/12$ of the mass of a carbon-12 atom. Scientists have measured the mass of helium-4, and it is about 4.0026 atomic mass units. (By the way, an atomic mass unit is also called a *dalton*.)

Now you are ready to take a good look at the periodic table of elements. Here is the version from Wikipedia:

Periodic Table of Elements																	VIIA																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																							
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There is a square for each element. In the middle, you can see the atomic symbol and the name of the element. In the upper-right corner is the atomic number — the number of protons in the atom.

In the upper-left corner is the atomic mass in atomic mass units.



Look at the atomic mass of boron. About 80% of all boron atoms have six neutrons. The other 20% have only 5 neutrons. This difference is why most boron atoms have a mass of about 11 atomic mass units, but some have a mass of about 10 atomic mass units. The atomic mass of boron is equivalent to the average mass of a boron atom: 10.811.

Exercise 6 Mass of a Water Molecule

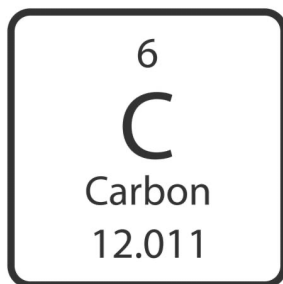
Working Space

Using the periodic table, what is the average mass of one water molecule in atomic mass units?

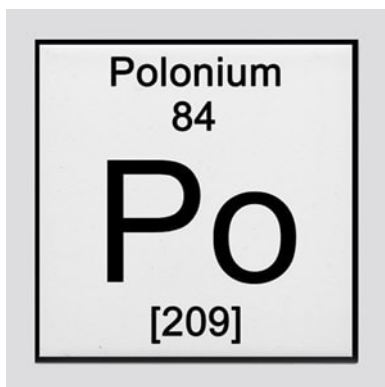
Answer on Page 72

2.1.4 Reading the Periodic Table

The Periodic Table organizes what we know about the structure of different elements. Each element has its own block or tile on the Periodic Table, and the information on the tile tells us about the structure of that atom. Take a look at the tile for carbon:



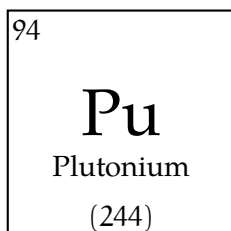
The letter (or letters, as is the case for other elements) is the atomic symbol for the element. There are two key numbers: the atomic number and the average atomic mass. For carbon, the atomic number is 6 and the average atomic mass is 12.011. The atomic number tells us how many protons there are in the nucleus of any atom of carbon. Since every element has a unique number of protons, every element has a unique atomic number. All carbon atoms have 6 protons. The other number is the average atomic mass - it tells us the weighted average of the mass of all the carbons in the universe. When the average atomic mass is in a whole number, as it is for polonium, it means that the element is very unstable. As a result, the mass given is the mass of the most stable isotope (we will talk more about stability and isotopes below). On some periodic tables, the mass number of the most stable isotope will be in parentheses or brackets. In summary, if the larger number is a whole number, it is the mass number; if it is a decimal (even if the decimal ends in .00), it is the average atomic mass, which we will discuss further below.



The Royal Society of Chemistry has a very useful interactive periodic table: periodic-table.rsc.org. We can use the periodic tile for an element to determine the number of protons, electrons, and most common number of neutrons for a neutral atom of that element (we will explain why the periodic tile tells us the “most common number of neutrons” below).

Example: State the atomic symbol for and the number of protons, neutrons, and electrons in a neutral atom of plutonium.

Solution: The plutonium tile on your periodic table should look something like this:



[The information may be arranged differently, but you should at least see the symbol and two numbers.] As you can see, the atomic symbol for plutonium is Pu. Since its atomic number is 94, we know every atom of plutonium has 94 protons. To know the number of electrons, we will take advantage of the fact that the question is asking about a *neutral* atom. This means there are the same number of positive charges as negative charges. So, since there are 94 protons, a neutral atom of plutonium must have 94 electrons (each proton has a +1 charge and each electron has a -1 charge). Lastly, let us determine the number of neutrons. The other number, 244, is the mass number. It represents the total number of protons and neutrons in the nucleus. Since we know plutonium has 94 protons, we can find the number of neutrons by subtracting the atomic number from the mass number:

$$\begin{array}{ccccc} & 244 & - & 94 & = & 150 \\ \text{mass number} & & & \text{atomic number} & & \text{number of neutrons} \\ \text{protons + neutrons} & & & \text{protons} & & \end{array}$$

Therefore, an atom of plutonium has 150 neutrons. Now let us address how to find the number of neutrons when the periodic tile shows an average atomic mass, instead of a mass number. Plutonium has no stable “versions” and barely exists in nature, so there is no natural average to report; its tile shows the mass number of its longest-lived version, Pu-244. Most elements are different: they come in a natural mix of versions, and the tile shows the average mass. Carbon is a good example.

Have you heard of carbon-14 dating? The phrase “carbon-14” refers to a rare type of

carbon that decays radioactively. By seeing how much carbon-14 has decayed, scientists can estimate the age of organic materials, such as bone or ash. Carbon-14 is a radioactive isotope (or version) of carbon. The 14 refers to the mass number - the total amount of protons and neutrons in the nucleus (sometimes, we shorten the isotope name by just using the atomic symbol, in this case C-14). Isotopes are versions of an element with different numbers of neutrons. The atomic number is the same for them all - they all have the same number of protons. But the different number of neutrons causes different isotopes to have different masses. Examine the models of carbon-12, carbon-13, and carbon-14 below. What is different between them? What is the same?

You should have noticed that all three atoms have 6 protons and 6 electrons, while they have differing numbers of neutrons. The most common isotope of carbon is carbon-12, with 6 protons and 6 neutrons in its nucleus. Carbon-14, on the other hand, has 8 neutrons, which makes the nucleus unstable, leading to radioactive decay. The average atomic mass is the weighted average of all the carbon atoms in existence. Since the vast majority of carbon is carbon-12, the average atomic mass is very close to 12. You cannot determine the mass number of an individual atom from the periodic table; it only tells you the average of all the isotopes. However, especially for light atoms (atoms in the first two rows of the periodic table), you can usually determine the mass number of the most common isotope by rounding the average atomic mass to the nearest whole number.

Example: Sodium has atomic symbol Na. State the number of protons, number of electrons, and most common number of neutrons in a neutral atom of sodium.

Solution: Sodium's atomic number is 11, so a neutral atom has 11 protons and 11 electrons. The average atomic mass is 22.990, which rounds to 23. So the most common isotope of sodium is Na-23, which has $23 - 11 = 12$ neutrons.

Be careful with heavier elements. Germanium's average mass is 72.630, which rounds to 73, but its most common isotope is actually Ge-74. When an element has several common isotopes, the average can land between them.

Exercise 7 Determining Numbers of Subatomic Particles

Use a periodic table to complete the table below (assume neutral atoms):

Working Space

Element Name	Atomic Symbol	Protons	Most Common Number of Neutrons	Electrons
	Fr			
				33
Aluminum				
		9		

Answer on Page 73

2.1.5 Heavy atoms are not stable

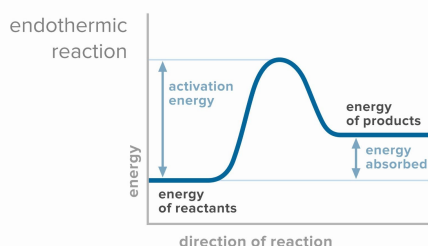
When you look at the periodic table, there are a surprisingly large number of elements. You might be told to “Drink milk so that you can get the calcium you need.” However, no one has told you “You should eat kale so that you get enough copernicium in your diet.”

Copernicium, with 112 protons and 173 neutrons, has only been observed in a lab. It is highly radioactive and unstable (meaning it decays). A copernicium atom usually lives for less than a minute before decaying. The *half-life* of a radioactive isotope is the time it takes for half of a sample to decay. Each half-life cuts the amount that is left in half again. (The Nuclear Physics and Radioactivity chapter, in Volume 12, explains why nuclei decay and how to calculate what is left.)

The largest stable element is lead, which has 82 protons and between 122 and 126 neutrons. Elements with lower atomic numbers than lead, have at least one stable isotope, while elements with higher atomic numbers than lead do not.

Bismuth, with atomic number 83, is *almost* stable. Bismuth-209 does decay, but its half-life is about 2×10^{19} years, more than a billion times the age of the universe. For anything you will ever do with it, bismuth is stable.

2.2 Chemical Reactions



Hydrogen and oxygen have a special tendency to bond with themselves. Almost never will you find them unbonded. Oxygen is most often found as O_2 and hydrogen as H_2 . If you mix these together in a 2:1 ratio of hydrogen to oxygen and light a match, they will rearrange themselves into

water molecules. This is called a *chemical reaction*. In any chemical reaction, the atoms are rearranged into new molecules.

When a chemical reaction happens, some bonds are broken and new bonds are formed. The breaking and forming of bonds requires energy. The amount of energy required to break bonds is different for different bonds. The amount of energy released when new bonds are formed is also different for different bonds. Some chemical reactions (like the burning of hydrogen gas described above) are *exothermic*

— that is, they give off more energy than they absorb. Burning hydrogen gas happens quickly and gives off a lot of energy. If you have enough, it will make quite an explosion! Other chemical reactions are *endothermic* — they consume energy. Photosynthesis, the process by which plants consume energy from the sun to make sugar from CO_2 and H_2O requires an endothermic chemical reaction. Energy stored in chemical bonds is called *chemical energy*, which is a form of *potential energy*.

Examine the diagrams in figure 2.2. The x-axis represents time - time passes as we move from left to right across the diagram. At the far left, the energy of the reactants (the ingredients that go into the reaction) is shown. At the far right, the energy of the products (what is made in the chemical reaction) is shown. Look at the endothermic reaction diagram (the blue one). Based on the relative energies of the reactants and products, do you expect an endothermic reaction to release or absorb heat? Absorb! Since the products have more energy, they must have absorbed energy, in the form of heat, from the surroundings.

Now, the red diagram shows an exothermic reaction: the products have less energy than the reactants. Since energy is never created or destroyed, where did the energy go? It is released as heat. So, exothermic reactions release heat. What does this look and feel like in real life? If an exothermic reaction were happening in a glass beaker, you would feel warmth if you held the beaker. The heat is leaving the beaker and entering your hand, which feels warm. What about an endothermic reaction? Many students think that since an endothermic reaction absorbs heat, it must be getting hot. This is incorrect: *exothermic* reactions feel hot. If an endothermic reaction were happening in a beaker and you touched the beaker, it would feel **cold**. Why? Well, if the reaction is absorbing heat, then heat must be leaving its surroundings (your hand) and entering the reaction (this heat energy is turned into chemical energy that is stored in the new chemical bonds that are forming). So your hand feels cold. You may have used both kinds: a hand warmer gets hot from an exothermic reaction, and an instant cold pack in a first-aid kit gets cold from an endothermic one.

Exercise 8 **Hot or Cold?**

For each process, say whether it is exothermic or endothermic, and whether a container holding it would feel warm or cold.

1. A campfire burning wood.
2. An instant cold pack, which dissolves a salt in water.
3. Plants making sugar from CO_2 and H_2O in sunlight.

Working Space

Answer on Page 73

Buoyancy

Why does a steel aircraft carrier, which weighs as much as 60,000 cars, float, while a steel paperclip sinks? The answer is *buoyancy*: the upward push that liquids and gases give to anything sitting in them. It explains boats, submarines, hot-air balloons, and why you feel lighter in a swimming pool. Engineers use it whenever they move things in or through a fluid. Let us start with boats.

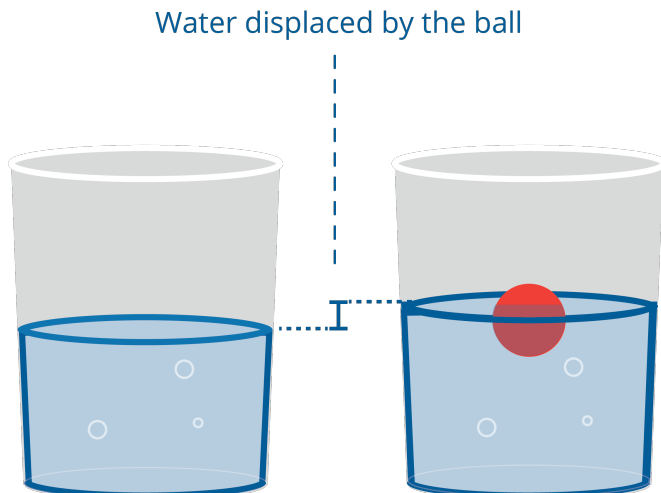
When you put a boat into water, it will sink into the water until the mass of the water it displaces is equal to the mass of the boat. We think of this in terms of forces. Gravity pulls the mass of the boat down; the *buoyant force* pushes the boat up. A boat dropped into the water will bob up and down at first before reaching an *equilibrium* where the two forces are equal.

When an object goes into a fluid, it pushes some of the fluid out of the way. The fluid that moves out of the way is called the *displaced fluid*, and its volume is the *displaced volume*. The object is the same as the displaced fluid in volume only when the object is completely underwater.

The buoyant force pushes things up, fighting against the force of gravity. This idea is called *Archimedes' principle*: the buoyant force is equal to the weight of the fluid being displaced. For example, a cubic meter of freshwater has a mass of about 1000kg. If you submerge anything with a volume of one cubic meter in freshwater on Earth, the buoyant force will be about 9800 newtons ($\text{mass} \times \text{gravity}$).

For some things, like a block of styrofoam, this buoyant force will be sufficient to carry it to the surface. Once it reaches the surface, it will continue to rise (displacing less water) until the mass of the water it displaces is equal to its mass. And then we say “It floats!”

Worked example: the fraction of a wooden block that is underwater. A block of wood has a density of 500 kg/m^3 . When it floats, the buoyant force must equal its weight. Let f be the fraction of its volume that is underwater. Then the displaced mass is $1000f \text{ kg}$ per cubic meter, and the wood's mass is 500 kg per cubic meter, so $1000f = 500$. The fraction underwater is $f = 0.5$. The block floats with half of its volume underwater.



For some things, like a block of lead, the buoyant force is not sufficient to lift it to the surface, and then we say “It sinks!”

Gases work the same way. A helium balloon rises because the air it pushes aside weighs more than the balloon and its helium. (It is easy to forget that air has mass, but a classroom full of air has a mass of over 200 kg.)

Exercise 9 **Buoyancy**

You have an aluminum box that has a heavy base, so it will always float upright. The box and its contents have a mass of 10 kg. Its base is 0.3 m x 0.4 m. It is 1m tall.

When you drop it into freshwater ($1000\text{kg}/\text{m}^3$), how far will it sink before it reaches equilibrium?

Working Space

Answer on Page 73

3.1 The Mechanism of Buoyancy: Pressure

As you dive down in the ocean, you will experience greater and greater pressure from the water. And if you take a balloon with you, you will gradually see it get smaller as the water pressure compresses the air in the balloon.

Let us say you are 3 meters below the surface of the water. What is the pressure in Pascals (newtons per square meter)? You can think of the water as a column of water crushing down upon you. The pressure over a square meter is the weight of 3 cubic meters of water pressing down.

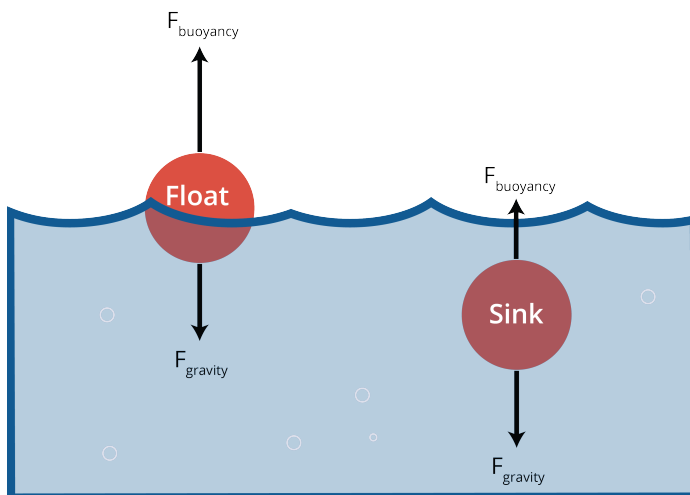
$$p = (3)(1000)(9.8) = 29,400 \text{ Pa}$$

This is called *hydrostatic pressure*. The general rule for hydrostatic pressure in Pascals p is

$$p = dgh$$

where d is the density of the fluid in kg per cubic meter, g is the acceleration due to gravity in m/s^2 , and h is the height of the column of fluid above you.

So where does the buoyant force come from? The bottom of an object is deeper than its top, so the water pushes up on the bottom harder than it pushes down on the top. That difference is the buoyant force.



Exercise 10 Hydrostatic Pressure

Working Space

You dive into a tank of olive oil on Mars. How much more hydrostatic pressure does your body experience at 5 meters deep than it did at the surface?

The density of olive oil is about 900 kg per cubic meter. The acceleration due to gravity on Mars is 3.721 m/s^2 .

Answer on Page 73

3.2 The Mechanism of Buoyancy: Density

Keep in mind that although the pressure is increasing as you go deeper, the buoyant force will *not increase*, because the buoyant force is always equal to the weight of the fluid that is displaced, regardless whether that is 1 meter or 100 meters underwater.

Due to the added minerals, saltwater is denser than freshwater. This causes objects to float better in the sea than they do in a river. Lipids, like fats and oils, are less dense than water, allowing them to float on top of a glass of water. When you are facing a grease fire, you are told not to put water on it. That is because the water sinks below the grease, then boils, throwing burning grease everywhere.

Work and Energy

Energy is what makes things happen: it moves cars, heats soup, and lights your phone screen. Work is how energy gets from one thing to another. Both ideas start with force, which, as you know, is measured in newtons.

4.1 Work

When you lean on a wall, you are exerting a force on the wall, but you are not doing any work. On the other hand, if you push a car for a mile, you are clearly doing work, as in Figure 4.1. Work, to an engineer, is the force you apply to something, as well as the distance that something moves, in the direction of the applied force. We measure work in *joules*. A joule is one newton of force applied over one meter.



Figure 4.1: Pushing on a wall does no work while pushing a car does work! Why?

For example, if you push a car with a force of 10 newtons for 12 meters, you have done 120 joules of work.

Formula for Work

If a force applied is *constant*, the formulas for work is:

$$W = F \cdot d$$

where W is the work in joules ($\text{N} \cdot \text{m}$), F is the *force* in newtons, and d is the distance in meters.

If the force is not in the same direction as the distance, we can use the cosine of the

angle between the force and the distance:

$$W = F \cdot d \cdot \cos(\theta)$$

where θ is the angle between the force and the distance. If the formula for work is *not constant*, the formula becomes:

$$W = \int \mathbf{F} \cdot d\mathbf{r}$$

where $d\mathbf{r}$ is an infinitesimal amount of displacement. An example of this kind of force is like a spring force, where the force varies with displacement.

Exercise 11 Book done by a constant force

This question is from an AP Physics C Review Book. You slowly lift a 2 kg book at a *constant velocity* through a distance of 3 m. How much work is done on the book?

Working Space

Answer on Page 73

The work-energy theorem (or work-energy principle) states that the net work done on an object is equal to the **change in its energy**. In other words, if you do work on an object, you are changing its energy. It follows from Newton's second law, which you met in the forces chapter.

$$W = \Delta E$$

Work is how energy is transferred from one thing to another. When you push the car, you also burn sugars (energy of the body) in your blood. That energy is then transferred to the car after it has been pushed uphill.

Thus, we measure the energy something consumes or generates in units of work: joules, kilowatt-hours, horsepower-hours, foot-pounds, BTUs (British Thermal Unit), and calories.

Let us go over a few different forms that energy can take.

4.2 Forms of Energy

In this section we are going to learn about several different types of energy:

- Heat
- Electricity
- Chemical Energy
- Kinetic Energy
- Gravitational Potential Energy

There are also other forms of energy such as spring potential energy, which we will cover in the oscillations chapter.

4.2.1 Heat

When you heat something, you are transferring energy to it. The BTU is a common unit for heat. One BTU is the amount of heat required to raise the temperature of one pound of water by one degree Fahrenheit. One BTU is about 1,055 joules. In fact, when you buy and sell natural gas as fuel, it is priced by the BTU.

Friction is the most common way motion turns into heat. When friction does work, that energy shows up as heat in the surfaces that rub together.

4.2.2 Electricity

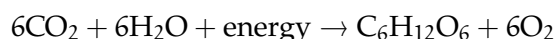
Electricity is the movement of electrons. When you push electrons through a space that resists their passage (like a light bulb), energy is transferred from the power source (like a battery) into the source of the resistance.

Let us say your lightbulb consumes 60 *watts* of electricity, and you leave it on for 24 hours. You have used $60 \times 24 = 1,440$ watt-hours, or 1.44 kilowatt-hours, which is about 5,200,000 joules. Your electric bill, a car battery, and a phone battery are all rated this way. In an electric car, the battery's energy becomes kinetic energy, and friction eventually turns it into heat.

4.2.3 Chemical Energy

As mentioned earlier, some chemical reactions consume energy and some produce energy. This means energy can be stored in the structure of a molecule. When a plant uses photosynthesis to rearrange water and carbon dioxide into a sugar molecule, it converts the energy in the sunlight (solar energy) into chemical energy. Remember that photosynthesis is a process that consumes energy. Therefore, the sugar molecule has more chemical energy than the carbon dioxide and water molecules that were used in its creation.

Recall the balanced chemical equation for photosynthesis is:



In our diet, we measure this energy in *kilocalories*. A calorie is the energy necessary to raise one gram of water one degree Celsius, and is about 4.19 joules. This is a very small unit. An apple has about 100,000 calories (100 kilocalories), so people working with food started measuring everything in kilocalories.

Here is where things get tricky: People who work with food got tired of saying “kilocalories”, so they just started using “Calorie” to mean 1,000 calories. This has created a great deal of confusion over the years. So if the C is capitalized, “Calorie” probably means kilocalorie.

4.2.4 Kinetic Energy

A mass in motion has energy. For example, if you are in a moving car and you slam on the brakes, the energy from the motion of the car is converted into heat in the brake pads and under the tires. This stored energy of motion is known as *kinetic energy*.

But how much energy does a moving object actually have?

Formula for Kinetic Energy

$$\text{KE} = \frac{1}{2}mv^2$$

where KE is the kinetic energy in joules, m is the mass in kilograms, and v is the speed in meters per second.

This equation tells us two important things:

- Kinetic energy increases *linearly* with mass. A heavier object has more energy at the same speed.

- Kinetic energy increases *quadratically* with velocity. This means that doubling the speed results in *four times* the kinetic energy.

Where does $\frac{1}{2}mv^2$ come from? Push a mass m , starting from rest, with a constant force F over a distance d . The work you do is $W = Fd = mad$. From kinematics, $v^2 = 2ad$, so $ad = \frac{1}{2}v^2$. Put those together: $W = \frac{1}{2}mv^2$. All the work you did became kinetic energy.

The v^2 explains why speed is so dangerous on the road. A car at 60 mph has four times the kinetic energy of the same car at 30 mph, so it needs four times the distance to stop with the same braking force, and it does four times the damage in a crash.

Exercise 12 Kinetic and Thermal Energy

A 1,200 kg car traveling at 22m/s comes to a stop using its brakes. Assume all the initial kinetic energy is converted into thermal energy in the brakes and tires.

Working Space

- How much thermal energy, Q is produced?
- If the brakes have a mass of 15 kg and a specific heat of $c = 460 \text{ J}/(\text{kg}^\circ\text{C})$, estimate their temperature increase.

Use the fact that the temperature change is $\Delta T = \frac{Q}{m_{\text{brake}}c}$.

Answer on Page 74

4.2.5 Gravitational Potential Energy

When you lift something heavy onto a shelf, you are giving it *potential energy*. The amount of energy that you transferred to it is proportional to its weight and the height that you lifted it.

Formula for Gravitational Potential Energy

The formula for gravitational potential energy is

$$E = mgh$$

where E is the energy in joules, m is the mass of the object you lifted, g is acceleration due to gravity, and h is the height that you lifted it.

On earth, then, gravitational potential energy is given by

$$E = (9.8)mh$$

since objects in free-fall near Earth's surface accelerate at 9.8m/s^2 .

There are various kinds of potential energy. For example, when you draw a bow in order to fire an arrow, you have given that bow potential energy. When you release it, the potential energy is transferred to the arrow, which expresses it as kinetic energy. When you compress a spring, the energy put into the spring is proportional to the spring constant and distance it is compressed, $\frac{1}{2}kx^2$. We will dive deeper into this in the springs and oscillations chapter.

4.3 Conservation of Energy

The first law of thermodynamics says “Energy is neither created nor destroyed.”

Energy can change forms. Your cells consume chemical energy to give gravitational potential energy to a car you push up a hill. A falling object converts potential energy into kinetic energy, causing it to speed up. However, the total amount of energy in a closed system stays constant.

Kinetic plus potential energy is called mechanical energy. If nothing outside the system does work on it and nothing is lost to friction, the mechanical energy stays constant. In general, outside work changes the system's kinetic energy K , potential energy U , and thermal energy E_{th} :

$$W_{\text{net,ext}} = \Delta E_{\text{sys}} = \Delta K + \Delta U + \Delta E_{\text{th}}$$

Energy problems are bookkeeping problems. Pick your system, add up the energy at the start, add up the energy at the end, and account for every joule that came in or went out.

Exercise 13 The Energy of Falling

A 5 kg cannonball falls off the top of a 3 meter ladder. As it falls, its gravitational potential energy is converted into kinetic energy. How fast is the cannonball traveling just before it hits the floor?

*Working Space**Answer on Page 74***Exercise 14 Frictional Losses**

A 4.0 kg block is released from rest at the top of a rough incline of height 2.5 m. By the time it reaches the bottom, its speed is 4.2 m/s.

Determine the total mechanical energy lost to thermal energy.

You may use 10 m/s^2 for g

*Working Space**Answer on Page 74***Exercise 15 Blast Off!**

A rocket sled of mass 150 kg uses a small fuel-powered booster that releases $1.2 \times 10^5 \text{ J}$ of chemical energy. During the burn, 65% of the energy is converted into useful kinetic energy for the sled, the rest is lost as heat or sound. Assume the sled travels on frictionless, smooth surface.

Working Space

Calculate the final speed of the rocket sled.

Answer on Page 74

4.3.1 Friction, Heat, and Energy “Loss”

Imagine rolling a ball across a flat surface: you have given the ball some kinetic energy in its motion. If the kinetic energy were conserved, the ball would keep rolling at the same speed forever, as long as it was on a flat surface. Experience tells us this is not what happens: the ball will eventually come to a stop. Why does this not violate the conservation of energy?

Friction is the force that opposes motion: whenever you slide two objects past each other, friction transforms kinetic energy into heat. Rub the palms of your hands together. You should feel warmth, a product of the friction between your hands. As the ball in the example above rolls, it also experiences friction between itself and the ground. The friction slowly transforms the kinetic energy of the ball into heat, causing the ball to lose kinetic energy. When all of the ball’s kinetic energy is transformed to heat, the ball comes to a rest. So, the kinetic energy of the ball was not destroyed and did not disappear: it became heat.

In fact, nearly all energy in the universe will eventually be transformed to heat, resulting in the inevitable “heat death” of the universe. (The Entropy and the Second Law of Thermodynamics chapter explains why.) Here is a short video about heat, entropy, and the heat death of the universe: https://www.youtube.com/watch?v=gOWt_Hq3yrE/.

4.3.2 Energy Conservation in Falling Objects

Recall that an object above the ground has gravitational potential energy $E_p = mgh$, where m is the mass in kg, g is the acceleration due to gravity (9.8m/s^2 on Earth), and h is the height above the ground in meters.

Example: What is the gravitational potential energy of a 5.0 kilogram bowling ball in the hand of a bowler (approximately 1.2 meters high)?

Solution: The mass is 5.0 kg, the height is 1.2 m, and we assume the bowler is on Earth, so g is 9.8 m/s^2 :

$$E_p = (5.0\text{kg}) (9.8\text{m/s}^2) (1.2\text{m}) = 59\text{J}$$

If the bowler were to drop that ball, it would lose potential energy and gain kinetic energy, $E_k = \frac{1}{2}mv^2$. If there is no air resistance, then as an object falls all of its potential energy is converted to kinetic energy. (If you'd like to explore what happens when friction is accounted for, you can play with this PhET simulation: <https://phet.colorado.edu/en/simulations/energy-skate-park>.)

Example: If the bowling ball were dropped, what would its speed be right before it hits the ground? (Neglect air resistance.)

Solution: Ignoring air resistance, according to the Law of Conservation of Energy, the kinetic energy of the ball right before it hits the ground must be equal to the gravitational potential energy of the ball right before its release. Therefore:

$$mgh = \frac{1}{2}mv^2$$

We can eliminate an m from both sides and rearrange to solve for v :

$$v = \sqrt{2gh} = \sqrt{2(9.8\text{m/s}^2)(1.2\text{m})} = \sqrt{23.52\frac{\text{m}^2}{\text{s}^2}} = 4.8\frac{\text{m}}{\text{s}}$$

The ball will have a speed of 4.8 m/s just before it hits the ground.

Exercise 16 Kinetic and Potential Energy

Working Space

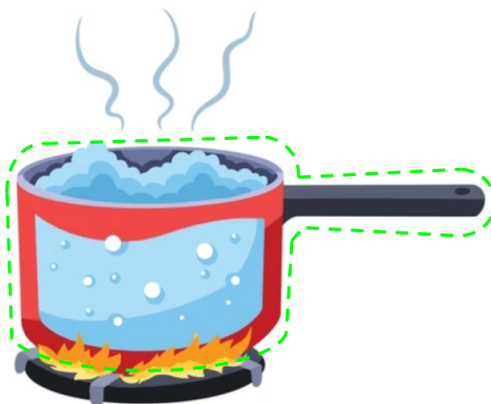
1. How fast should you toss a ball straight up if you want it to reach your friend on the second floor (3.7 meters above you)?
2. Your little brother is teasing you from a treehouse 5.0 meters off the ground. If your slingshot can shoot a pebble at 15 m/s, can you hit your little brother?
3. A 63-kg roller-blader rolls down a hill. If the hill is 8.0 meters high and

they reach the bottom of the hill with a speed of 11.2 m/s, how much energy was converted to heat through friction?

Answer on Page 75

4.4 Types of Systems

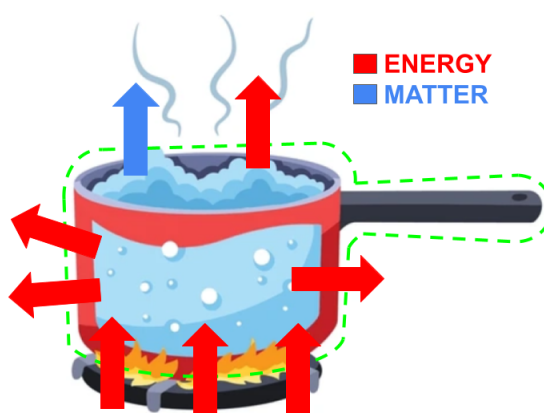
We classify systems based on the flow of matter and energy. A *system* is a set of interconnected elements. Your body is a system, as is a television or a boiling pot of water. Scientists define systems by separating the parts of the system from the rest of the universe, usually called "the surroundings". You can represent this separation with a dashed line. Here is a diagram defining a pot of boiling water as a system:



Everything inside the dashed line is the system: the pot and the water boiling in it. Everything outside the dashed line is the surroundings: the stove, the air around the pot, etc. Defining a system is *arbitrary*: there is not one hard and fast definition of a system. Think of your school: you could look at the system of a classroom, the system of a hallway and all the classes connected to it, or the entire school building. How you define a system depends on what you are studying.

4.4.1 Open Systems

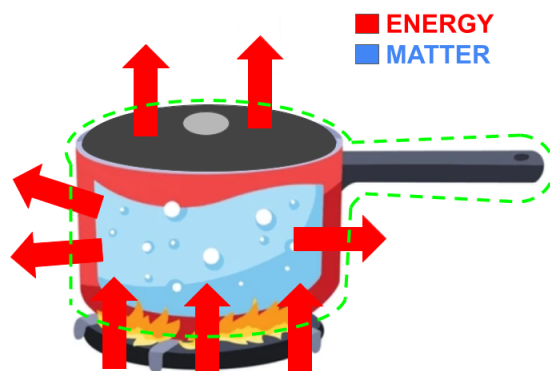
An *open system* allows for matter and energy to cross the imaginary boundary between the system and its surroundings. The uncovered pot of boiling water is an example of an open system. Energy enters the system as heat from the stove, and leaves the system as heat in the steam rising from the pot. Notice that steam rising: see how it crosses the imaginary boundary? The steam is matter *leaving the system*. Since matter and energy can cross the boundary between the system and its surroundings, the uncovered boiling pot is an open system.



The system also loses energy through the sides of the pot: if you touched the pot, it would feel hot, which means heat energy can also leave the system through the sides of the pot. This is due to the collision between air particles (the surroundings) and the outside of the pot (the system). With every collision, a little heat is transferred from the pot to the air. This is why your hot drink gets cold if you leave it out, even if you do not add any ice.

4.4.2 Closed Systems

A *closed system* allows for the transfer of energy but not the transfer of matter. If we put a lid on this pot, it would become a closed system.



Notice that the difference between an open and closed system is the flow of *matter*: open systems allow for the movement of matter, while closed systems do not. However, energy can still enter and leave closed systems. Sealed containers that are not insulated are good examples of closed systems - a car with the windows up and doors closed.

4.4.3 Isolated Systems

An *isolated system* does not allow for the flow of matter *or* energy in or out of the system. There is no such thing as a truly isolated system: in reality, a small amount of energy can be transferred even through the best thermal insulators. However, for well-insulated systems, it can be a good approximation to model that system as isolated. A simple example would be a sealed, well-insulated coffee thermos. The transfer of heat energy between the coffee in the thermos and the thermos' surroundings is so slow that we can ignore that small amount of transfer and approximate the thermos as an isolated system.

4.4.4 Classifying Systems

To quickly categorize a system as open, closed, or isolated, ask yourself two questions:

1. Can matter enter or leave the system?
2. Can energy enter or leave the system?

If your answer to the first question is yes, you know automatically the system is open. If no, move to the second question. If energy can enter or leave, the system is closed. If not, the system is isolated. Sometimes, textbooks and exams will describe a system as "well-insulated". This is directing you to assume any transfer of energy between the system and surroundings is negligible, and that you should treat the system as an isolated system.

Exercise 17 **Open, Closed, and Isolated Systems**

Classify each system as open, closed, or isolated. Justify your answer.

Working Space

1. The human body
2. Earth
3. Your cell phone
4. A well-insulated cooler with the lid sealed
5. A well-insulated cooler with the lid open
6. A bottle of soda before it is opened to be drunk

Answer on Page 75

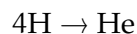
4.5 Mass-Energy Equivalence

You have probably seen the equation

$$E = mc^2$$

E is energy, m is mass, and c is the speed of light in a vacuum (about $3 \times 10^8 \frac{\text{m}}{\text{s}}$). In the Atoms and Matter chapter, mass was conserved; in this chapter, energy is conserved. This equation shows that the two are connected: matter can be converted to energy, and vice versa. This is the source of the light energy emitted by the sun.

The Sun is mostly hydrogen. At the very center, it is so hot and dense that the nuclei of hydrogen atoms are fused together to form helium atoms, a process called the *proton-proton chain reaction*. The actual reaction involves several steps and is more complicated, but the overall process can be summarized as:



If every hydrogen atom has a mass of approximately $1.6735575 \times 10^{-27}$ kg and every helium atom has a mass of approximately $6.6464731 \times 10^{-27}$ kg, how much energy is released when one atom of helium is created? First, notice that one helium has less mass than 4 hydrogen atoms:

$$4 \times (1.6735575 \times 10^{-27}) - 6.6464731 \times 10^{-27} = 4.77569 \times 10^{-29}$$

Now, we can use $E = mc^2$ to find out how much energy is equivalent to 4.77569×10^{-29} kg:

$$E = (4.77569 \times 10^{-29} \text{ kg}) \left(2.99792458 \times 10^8 \frac{\text{m}}{\text{s}} \right)^2$$
$$E \approx 4.292 \times 10^{-12} \text{ joules}$$

That is a tiny number, so let us scale up. Suppose you fused 1 kilogram of hydrogen (as a gas, enough to fill a small bedroom). The lost mass is $\frac{4.77569 \times 10^{-29}}{4 \times 1.6735575 \times 10^{-27}} \approx 0.71\%$ of the hydrogen's mass, so about 0.00713 kg disappears, roughly the mass of three pennies.

$$E = (0.00713 \text{ kg}) \left(2.99792458 \times 10^8 \frac{\text{m}}{\text{s}} \right)^2 \approx 6.4 \times 10^{14} \text{ joules}$$

That is about 180,000,000 kWh (kilowatt-hours). An average US home uses about 30 kWh per day, so one kilogram of hydrogen could power a home for *over 16,000 years!* That is why scientists keep working on nuclear fusion power. The nuclear power plants currently running around the world today rely on *nuclear fission*: the splitting apart of atoms, the opposite process of *nuclear fusion*. Fission releases several times less energy per kilogram of fuel than fusion, and thus stable, affordable nuclear fusion power plants remain a “holy grail” of scientific research. The Nuclear Physics and Radioactivity chapter explains fission and fusion in detail.

The next chapter, Work, Kinetic Energy, and Power, connects work directly to an object's speed, and asks how efficiently and how quickly energy gets a job done.

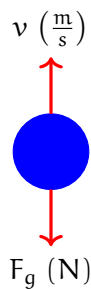
Work, Kinetic Energy, and Power

How much faster does a skateboard go if you push twice as hard? How far does a car skid after the brakes lock? Why does a car engine waste most of its fuel? In the Work and Energy chapter, you learned that energy changes form but is never created or destroyed. This chapter makes that bookkeeping precise for moving objects, with the work-energy theorem, and then asks how quickly, and how efficiently, energy does a job.

5.1 Work and Kinetic Energy

The work-energy theorem, which you met in the Work and Energy chapter, says that the change in an object's kinetic energy is equal to the work done on that object. For now, we will only consider examples where the force and the direction of motion are parallel or perpendicular. When you learn about vectors, we will expand this to include forces that are skew to the direction of motion.

Consider what happens when you toss a ball in the air: once the ball leaves your hand, the only force acting on it is gravity. Initially, the ball is moving upwards while gravity points downwards:



Intuitively, we know that the ball will slow down (lose kinetic energy) as it moves upwards:

$$\Delta KE < 0$$

Since $W = \Delta KE$, we also know that gravity must be doing *negative work*. Whenever the direction of the force is opposite the direction of the motion, the work done by that force is negative.

Exercise 18 A ball thrown upwards

If the ball has a mass of 0.5 kg, how much kinetic energy does it lose as it moves upwards by 1 m?

Working Space

Answer on Page 76

Exercise 19 How far will you slide?

You are playing softball and have to slide into home. If you sprint at a maximum of 10 m/s and the force of friction between you and the ground is 0.3 times your weight, how far from the base can you start your slide and still reach home?

Working Space

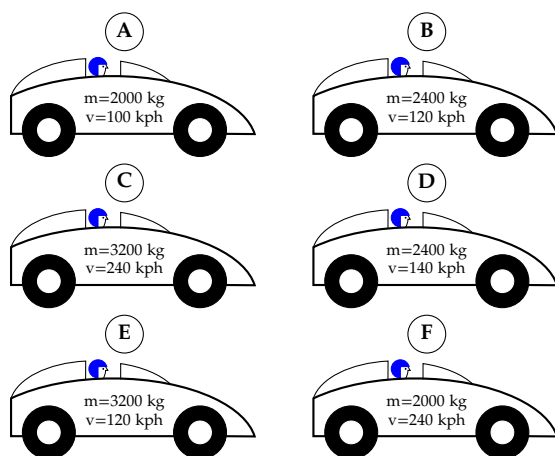
Answer on Page 76

Exercise 20 Ranking Stopping Force

In drag racing, cars can reach speeds of 150 miles per hour (approximately 240 kilometers per hour). In order to be able to stop quickly and safely, drag racing cars are built with parachutes that deploy at the end of the race. Consider a drag race where cars of different masses reach different maximum speeds. There is 100 meters between the finish line and the fence surrounding the race track. If all the race cars deploy their parachutes

Working Space

at the finish line while going their maximum speed, rank the force needed from the parachute to stop each car in the required distance from least to greatest:



Answer on Page 77

5.1.1 Forces that do no work

If the object you are pushing does not move, or the applied force is perpendicular to the direction of motion, that force does no work. Let us look at a few examples:

5.1.2 Pushing Against an Immobile Object

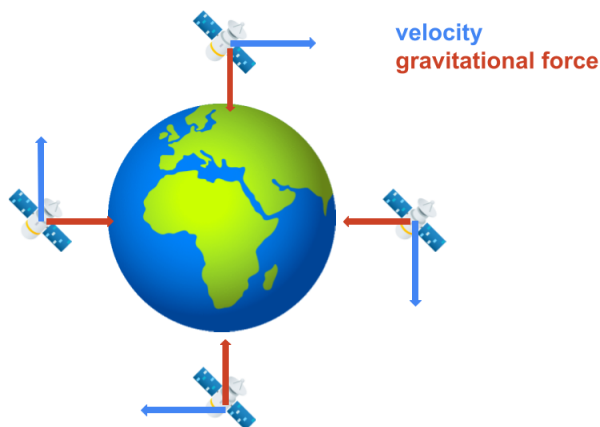
At the beginning of the chapter, we said that when you push on a wall, you do not do any work. Why is this? Well, if the wall is a good wall (that is, strong enough to not be pushed over by a person), the wall will not move while you push on it. This means the distance over which your push is applied is zero, and therefore the work done ($F \cdot d = F \cdot 0 = 0$) is zero joules.

5.1.3 Walking Across a Room with a Book

Imagine holding a book flat on your hands and walking at a constant velocity. Your hand is applying an upwards force to the book, but the book is moving horizontally. This means the force and direction of motion are *perpendicular*. Recall from the beginning of the chapter that if the force and distance are not parallel, then the work is given by $W = F \cdot d \cos(\theta)$. (When the vectors are parallel, $\theta = 0$ and $\cos(\theta) = 1$, while when the vectors point in opposite directions, $\theta = 180^\circ$ and $\cos(\theta) = -1$.) When the vectors are perpendicular, then $\theta = 90^\circ$ and $\cos(\theta) = 0$. Therefore, $W = 0$ as well and the upward force of your hands does no net work.

5.1.4 Circular Motion

We will discuss circular motion further in a subsequent chapter. For now, know that constant-speed circular motion is caused by a constant-magnitude force that always points to the center of the circle the object is moving in. For example, you can take a weight on the end of a string and spin it. The tension in the string spins the weight, and the string always points from the object to your hand (the center of the weight's circular path). For a satellite, that force is gravitational attraction to the Earth.



As a result, the force changes the *direction*, but not the *magnitude* of the satellite's velocity. Let us re-examine the equation for kinetic energy:

$$KE = \frac{1}{2}mv^2$$

Since the velocity is squared, the direction of motion does not affect the kinetic energy (a ball moving at 5 m/s upwards has the same kinetic energy as if the ball were moving at 5 m/s downwards). So, a force that causes circular motion does not change a circling

object's kinetic energy, and therefore does no work (as expected when force and direction of motion are perpendicular)!

5.2 Efficiency and Power

Energy is never destroyed, but it does leak away into forms we cannot use, usually heat. Efficiency measures how much of the energy you put in does the job you wanted.

In terms of an equation, efficiency is the ratio of the useful energy output to the total energy input. It is usually expressed as a percentage.

Formula for Efficiency

$$\text{Efficiency} = \frac{\text{Useful Energy Output}}{\text{Total Energy Input}} \times 100\%$$

where the useful energy output is the energy that is actually used to do work or complete a task, and the total energy input is the total energy consumed by the system.

A machine is considered 100% efficient only if all the input work is converted into useful output work, with no energy lost to heat, friction, or sound. 100% efficient processes do not exist in real life: every process loses some useful energy to heat.

For example, when a car engine consumes the chemical energy in gasoline, only about 20% of the energy consumed is used to turn the wheels. Most of the energy is actually lost as heat. If you run a car for a while, the engine gets very hot, as does the exhaust coming from the tailpipe.

A human is about 25% efficient. Most of the loss is in the heat produced during the chemical reactions that turns food into motion.

So if you want to make a machine more efficient, follow the heat. Wherever it gets hot, energy is leaking away.

Light bulbs are an interesting case. To get the same amount of light of a 60 watt incandescent bulb, you can use an 8 watt LED or a 16 watt fluorescent light. This is why we say that the LED light is much more efficient. If you run both, the incandescent bulb will consume 1.44 kilowatt-hours; the LED will consume only 0.192 kilowatt-hours.

In addition to light, the incandescent bulb is producing a lot of heat. If it is inside your house, what happens to the heat? It warms your house.

In the winter, when you want light and heat, the incandescent bulb is 100% efficient!

Of course, this also means the reverse is true. In the summer, if you are running the air conditioner to cool down your house, the incandescent bulb is worse than just “inefficient at making light” — it is actually counteracting the air conditioner!

Exercise 21 Unit Costs

A machine is rated to produce 500 bolts per hour, but in actual operation it runs at only 82% efficiency due to loss and scraps. The machine requires \$24 of electricity per hour to run.

Working Space

- (a) What is the actual output of bolts per hour?
- (b) What is the unit cost (cost per bolt) of the production?

Answer on Page 77

Power is how fast work gets done. Suppose we each carry the same box up the stairs, doing 1000 J of work. I take 10 minutes; you take 5. We did the same work, but you did it twice as fast, so you were twice as *powerful*.

Formula for Power

$$\text{Power} = \frac{\text{Work}}{\text{Time}} \Rightarrow P = \frac{W}{t}$$

Power is represented by the unit watt (W), defined as one joule per second (J/s). Since power is a rate, we can also express it using calculus:

$$P = \frac{dW}{dt}$$

For a constant force moving an object at speed v in the direction of the force, $P = \frac{Fd}{t} = Fv$, since $v = \frac{d}{t}$.

Exercise 22 Hydraulic Piston Problem

A hydraulic piston pushes a crate hori-

Working Space

zontally across the floor with a constant force of 450 N. During operation, the piston moves the crate 1.8 meters in 0.75 seconds.

- (a) How much work does the piston do on the crate?
- (b) What is the power output of the piston during this motion?

Answer on Page 77

Exercise 23 Spring Power

A compressed spring launches a small block. The spring delivers 120 W of power while releasing its stored energy over 0.40 seconds. The spring constant is $k = 800 \text{ N/m}$.

How far was the spring compressed?

Working Space

Answer on Page 78

Exercise 24 Elevator Lift

An elevator motor lifts a 750 kg cabin vertically upward a distance of 18 m in 12 s. The motor consumes electrical energy at a rate of 18 kW.

- (a) Compute the total electrical energy consumed during the lift. Note that $1 \text{ kW} = 1000 \text{ watts} = 1000 \text{ J/s}$
- (b) Determine the gravitational poten-

Working Space

tial energy gained by the elevator.

- (c) Calculate the efficiency of the motor during this lift.

Answer on Page 78

Simple Machines

How did ancient Egyptians lift 2-ton stone blocks without engines? With simple machines. A simple machine lets you trade distance for force: push more gently, but over a longer distance. Recall that work is force times distance; push your car 100 meters with a force of 17 newtons, and you have done 1700 joules of work. Simple machines include:

- Levers
- Pulleys
- Inclined planes (ramps)
- Screws
- Gears
- Hydraulics
- Wedges

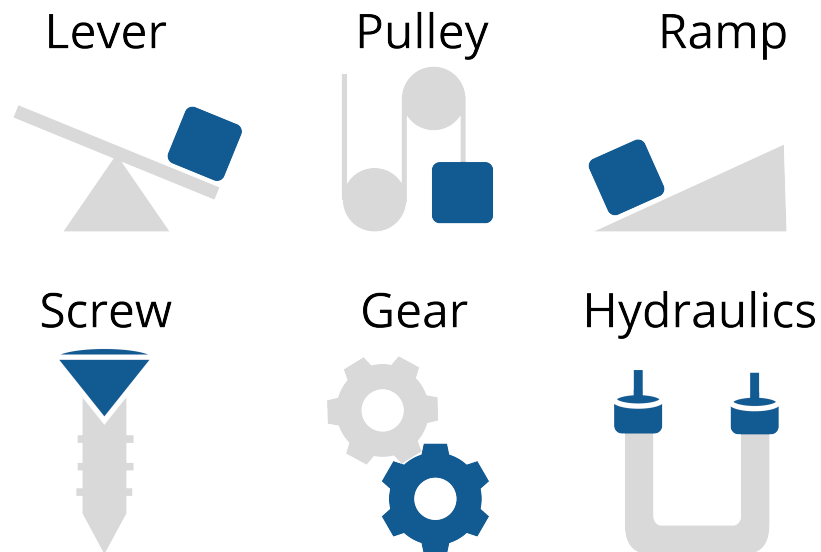


Figure 6.1: The 6 main simple machines: Levers, Pulleys, Ramps, Hydraulics, Screws, and Gears.

Here is the catch: no machine reduces the *work*. If a machine cuts the force to one third, you must push three times as far. How much the machine multiplies your force is called its mechanical advantage.

6.1 Mechanical Advantage

Mechanical advantage is the ratio between the force output by the machine and the force the user puts into the machine:

$$MA = \frac{F_{\text{out}}}{F_{\text{in}}} = \frac{\text{Load}}{\text{Effort}}$$

Since the input force is *applied* to the simple machine, sometimes the input force is called an applied force and abbreviated as F_a . For example, you only need to apply a relatively little force to your car's brakes in order for the hydraulic braking system to apply enough force to your tires to stop them spinning (we will examine this further below).

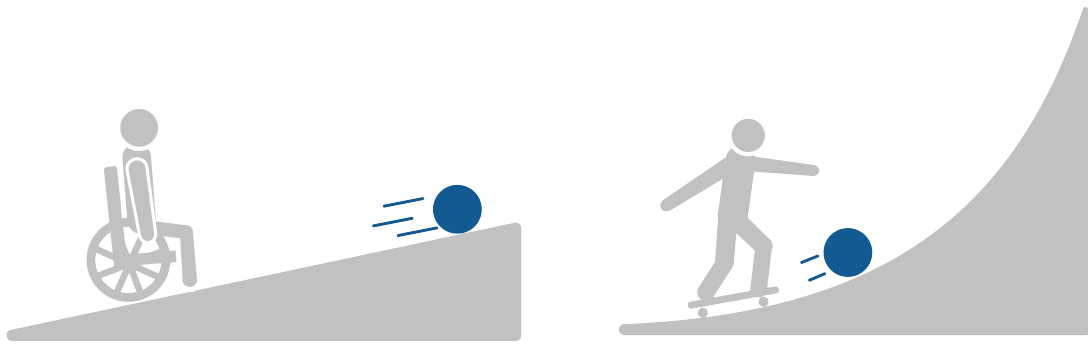
6.1.1 What does it mean to work hard?

People say machines “make work easier,” but in physics they do not reduce work at all. A shallow ramp feels easier than a steep one, yet, as you will see with the moving-truck ramp below, you do exactly the same work either way. “Easier” really means *less force*, spread over a longer distance. That is true of every simple machine.

Ideally we want a higher mechanical advantage, which means we put in less input force for a greater output force.

6.2 Inclined Planes

Inclined planes, or ramps, allow you to roll or slide objects to a higher level. A gentler ramp gives a larger mechanical advantage. You push with less force, but you push over a longer distance. A wheelchair ramp is gentle, so it is easier to roll a chair up it than to climb a steep set of stairs.



Assuming the incline has a constant steepness, the mechanical advantage is equal to the ratio of the length of the inclined plane to the height it rises.

If friction is neglected, the force required to push a weight up the inclined plane is given by:

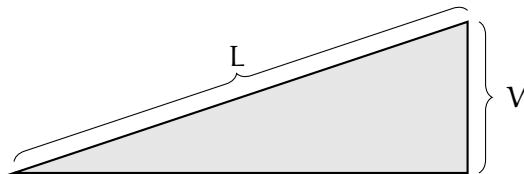
$$F_A = \frac{V}{L} F_g \quad (6.1)$$

where F_A is the applied force, L is the length of the inclined plane, V is the vertical rise, and F_g is the gravitational force acting on the mass.

Worked example: pushing a crate up a ramp. A crate weighs 200 N. The ramp is 5 m long and rises 1 m. Its mechanical advantage is $\frac{L}{V} = \frac{5}{1} = 5$. Without friction, the force needed is

$$F_A = \frac{V}{L} F_g = \frac{1}{5} (200 \text{ N}) = 40 \text{ N}$$

You push with one fifth of the crate's weight, but you push over five times the distance.



Why is the force $\frac{V}{L} F_g$? Because the work cannot change. Lifting a load of weight F_g straight up by a height V takes work $F_g V$. Pushing it up a frictionless ramp of length L with force F_A takes work $F_A L$. These must be equal, so $F_A L = F_g V$, which gives $F_A = \frac{V}{L} F_g$.

Let us check this with numbers. Consider a family on moving day: a hand trolley is loaded with 200 N (about 45 pounds) of boxes, and the bed of the moving truck is 1.5 m high. Lifting the boxes straight up takes 200 N of force over 1.5 m:

$$W = F \cdot d = (200 \text{ N}) (1.5 \text{ m}) = 300 \text{ J}$$

Now suppose the truck has a ramp 6 m long. The force needed to push the trolley up it is

$$F_A = \frac{V}{L} F_g = \frac{1.5}{6} (200 \text{ N}) = 50 \text{ N}$$

and the work is $(50 \text{ N})(6 \text{ m}) = 300 \text{ J}$, exactly the same. The ramp does not save work. It lets you push with a quarter of the force, over four times the distance: its mechanical advantage is $\frac{F_g}{F_A} = \frac{200}{50} = 4 = \frac{L}{V}$.

Ramps

For a frictionless ramp of length L and vertical rise V , the mechanical advantage is $\frac{L}{V}$, and the force needed to push an object of weight W up the ramp is $W \cdot \frac{V}{L}$.

Real ramps have friction, and real problems usually give the ramp's angle instead of its length. Handling those takes trigonometry, so we return to ramps in the Inclined Planes chapter in Volume 7.

Exercise 25 Ramp

You need to lift a barrel of oil with a mass of 136 kilograms. You can apply a force of up to 300 newtons. You need to get the barrel onto a platform that is 2 meters high. What is the shortest length of inclined plane you can use?

Working Space

Answer on Page 78

6.3 Levers

A lever pivots on a fulcrum. To decrease the necessary force, the load is placed closer to the fulcrum than where the force is applied.

Physicists also discuss the concept of *torque* created by a force. When you apply force to a lever, the torque is the product of the force you exert and the distance from the point of rotation.

Torque is typically measured in newton-meters (N·m).

To balance two torques, the products of force and distance must be equal. Thus, assuming the forces are applied in the correct direction, the equation becomes:

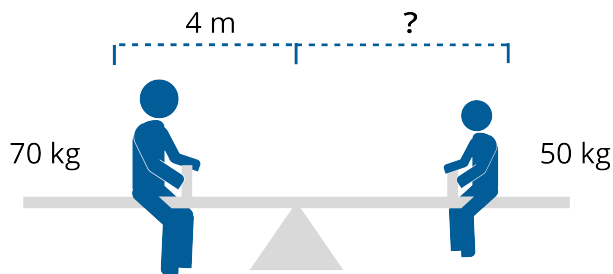
$$R_L F_g = R_A F_A \quad (6.2)$$

where R_L and R_A represent the distances from the fulcrum to where the load's weight and the applied force are exerted, respectively, and F_g and F_A are the magnitudes of the forces.

Exercise 26 Lever

Paul, whose mass is 70 kilograms, sits on a see-saw 4 meters from the fulcrum. Jan, whose mass is 50 kilograms, wishes to balance the see-saw. How far should Jan sit from the fulcrum?

Working Space



Answer on Page 79

6.4 Pulleys

A pulley is a wheel with a rope running over it. A single fixed pulley, like the one at the top of a flagpole, just changes direction: you pull down, and the load goes up. That alone helps, because you can use your body weight instead of only your arms.

By attaching multiple pulleys, you can do more and actually reduce the force needed to lift a load. For example, if you use two pulleys, you can reduce the force needed to lift a load by half. However, you will have to pull twice as much rope to lift the load the same distance.

Each additional pulley you add to the system adds another segment of rope that supports the load. The mechanical advantage of a pulley system is equal to the number of rope segments supporting the load.

$$MA = N$$

where N is the number of rope segments supporting the load.

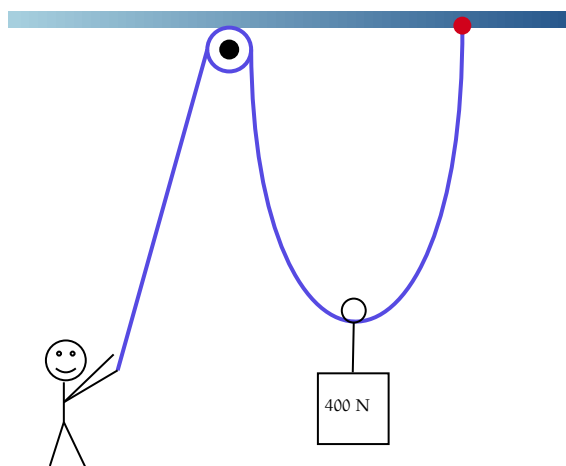


Figure 6.2: A 400 N weight being distributed by one pulley.

Here, a rope is carrying a 400 N weight. Attached to the weight is a movable pulley, while the top pulley is fixed to the ceiling. Assuming an ideal (massless) rope and frictionless pulleys, the tension is the same everywhere in the rope. The movable pulley is supported by two rope segments, so the upward force on the load is $2T$. For the load to be held or lifted at constant speed,

$$2T = 400 \text{ N} \Rightarrow T = 200 \text{ N}$$

So you only pull with 200 N to hold up 400 N. Two rope segments support the load, so the mechanical advantage is 2.

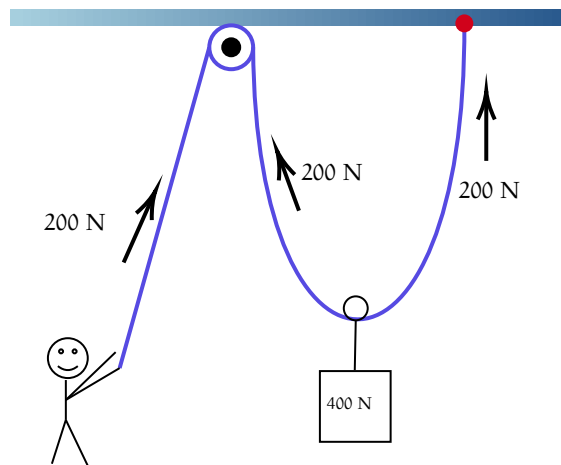


Figure 6.3: The tension in the rope is 200 N.

There is a cost, of course. The two rope segments each pull up with 200 N, together supporting the 400 N load, but you pay for that in distance.

The work done, however, is still the same! When the person pulls the rope down by 2 m, each supporting rope segment shortens by 1 m, so the load rises by 1 m. This is all ignoring friction and assuming constant speed. Work done by the person:

$$W = F \times d = 200 \text{ N} \times 2 \text{ m} = 400 \text{ J}$$

Work gained by the load:

$$W = 400 \text{ N} \times 1 \text{ m} = 400 \text{ J}$$

This satisfies our work equation

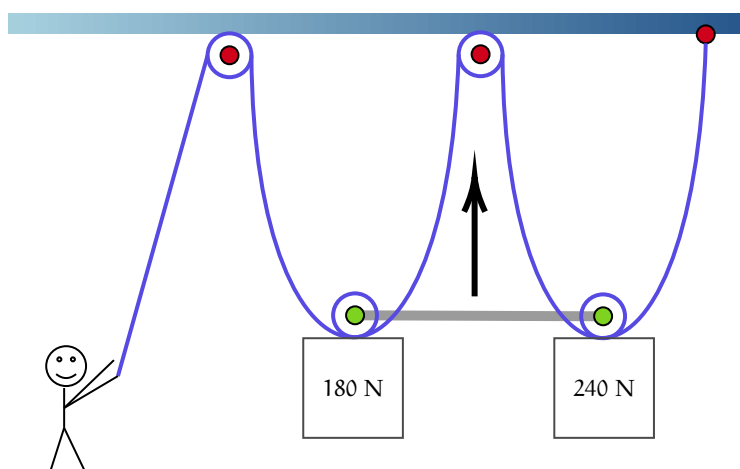
$$\text{Work}_{\text{in}} = \text{Work}_{\text{out}} \quad (6.3)$$

Note that we can then rewrite our mechanical advantage as a proportion of distance:

$$\text{MA} = \frac{d_{\text{effort}}}{d_{\text{load}}} \quad (6.4)$$

Exercise 27 Pulleys

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A person is pulling 2 weights tensioned by 4 pulleys. There are 2 fixed pulleys (shown in red) and 2 movable pulleys (shown in green). The two movable pulleys are fixed such that they move together vertically. Calculate the input force that is needed to begin lifting the weights.

Answer on Page 79

6.5 Wedges

A wedge is a moving ramp. Hit the back of an axe head downward, and its sloped sides push the wood apart sideways, with forces perpendicular to the sloped faces. Knives, chisels, doorstops, and even your front teeth are wedges.

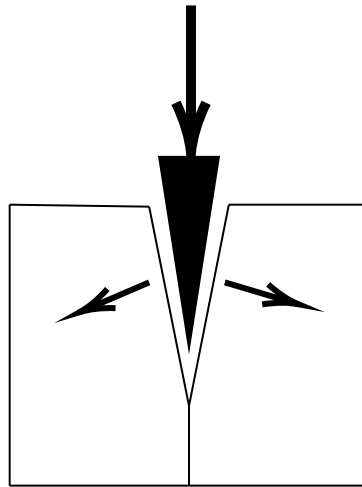


Figure 6.4: A wedge splitting a block of wood, with the forces acting perpendicular.

Like a ramp, a wedge's mechanical advantage is the length of its sloped side, L , divided by its thickness, V :

$$MA = \frac{L}{V} \quad (6.5)$$

The thinner and longer the wedge, the bigger its mechanical advantage. That is why a sharp knife slides through a tomato and a dull one squashes it.

Exercise 28 **Wedges 1**

An axe is used to split wood. The length of the wedge is 12 cm and the thickness is 3 cm. Calculate the mechanical advantage.

Working Space

Answer on Page 79

Exercise 29 **Wedges 2**

Explain why a flathead-screwdriver tip is a poor option for cutting a steak com-

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pared to a knife.

Answer on Page 79

6.6 Gears

Gears are rotating parts of machines that transmit torque or other types of rotational motion through intertwined teeth. Often gears are meshed together; two or more meshed gears are referred to as a gear train.

Gears have teeth that mesh with each other. When you apply torque to one gear, it transfers torque to the other. The resulting torque is increased or decreased depending on the ratio of the number of teeth on the gears. We will go in depth on torque in a future chapter. For now, know that torque depends on the radius of the gear, and the radius is proportional to the number of teeth, so the tooth count controls the torque.

6.6.1 Involute Gears

Gear teeth have an unusual shape. The vast majority of modern gears use what is known as an *involute gear profile*. An involute gear is one whose tooth shape is derived from the *involute of a circle*. This curve is generated by imagining a fixed string being unwrapped from the circumference of a circle. The path traced by the end of the string forms the involute.

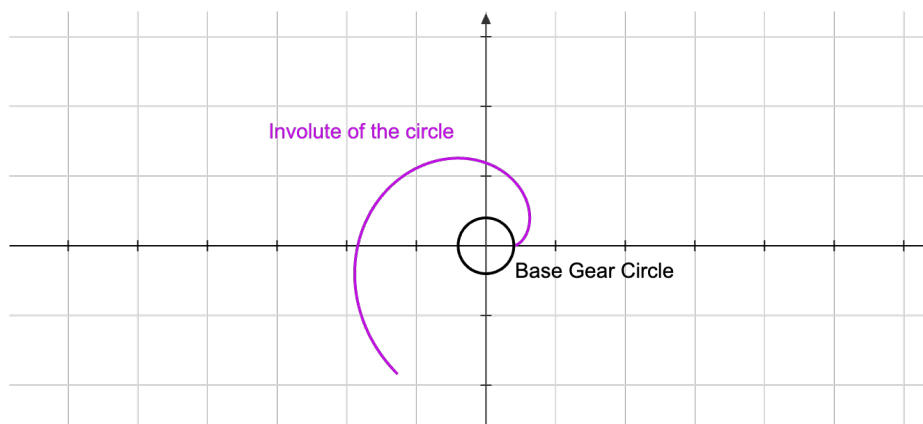


Figure 6.5: The involute of a circle shown. The purple line is revolved for 4π radians or 2 revolutions (for clarity).

Although the involute curve resembles a spiral, it is not an Archimedean spiral. Instead, its geometry is defined by the constant unwrapping of the string, which gives the involute several important mechanical properties.

The primary advantage of the involute profile shape is that it ensures smooth and reliable torque transfer between two or more meshing gears. When two involute gears engage, their teeth make contact at a single point that moves along a straight line known as the **line of action**. This line is tangent to the base circles of both gears. As the gears rotate, the point of contact travels along this line while maintaining a constant pressure angle, which allows the gears to transmit motion at a constant speed ratio even if the center distance between them varies slightly.

Some more gear construction terminology:

The pressure angle The acute angle between the line of action and a normal to the line connecting the gear centers. May vary depending on the involute shape, but most commonly 20° .

Addendum The difference between the pitch circle and its tooth tip circle.

Dedendum The difference between the radius of the pitch circle of a gear and its root circle

Tooth Height The distance between its root circle and the tip are called the tooth height (h).

Total Height of Tooth The total height of the gear is the sum of the addendum ($h_a = 1.00$ module) and the dedendum ($h_f = 1.25$ module). The module is a size unit for gear teeth, not a length in meters.



Figure 6.6: Two gears meshing together diagram (not directly conjoined for clarity).

For part of the motion, Gear One pushes Gear Two down. Once the contact point passes the midpoint of the line of action, Gear One pushes up on Gear Two. This causes a constant transfer of torque.

If you are curious, here is the involute of a circle of radius r as a parametric curve, where θ is how far the string has unwound:

$$\begin{bmatrix} x(\theta) \\ y(\theta) \end{bmatrix} = r \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \end{bmatrix} + r\theta \begin{bmatrix} \sin(\theta) \\ -\cos(\theta) \end{bmatrix}$$

For gears, a large torque makes it easier to start rotating an object or to continue rotating it against resistance, such as climbing a hill or accelerating a heavy load.

Power, on the other hand, measures how quickly work is being done. In rotational motion, power depends on both torque and rotational speed. A system can produce high torque at low speed, or lower torque at high speed, and still deliver the same power.

Gears allow a machine to adjust this balance between torque and speed without significantly changing the power being transmitted (neglecting losses). When gears increase torque, they reduce rotational speed, and vice versa.

A familiar example is riding a bicycle up a hill. When climbing, it becomes difficult to pedal in a high gear because the torque required at the pedals is large. Shifting to a lower gear increases the torque applied to the rear wheel for the same pedaling effort, making it easier to climb. However, the pedals must turn more times for each rotation of the wheel, so the bicycle moves more slowly. The rider's power output remains roughly the same, but it is delivered as higher torque at lower speed.

The same principle applies to the gears in a car. When starting from rest or climbing a steep incline, a low gear is used to provide high torque at the wheels, allowing the vehicle to overcome inertia and gravity. At highway speeds, higher gears are used to reduce torque and increase speed, improving efficiency and reducing engine wear. In both cases, the engine produces power, and the transmission adjusts how that power is distributed between torque and rotational speed.

In mechanical systems, gears therefore do not create power; instead, they shape how power is delivered.

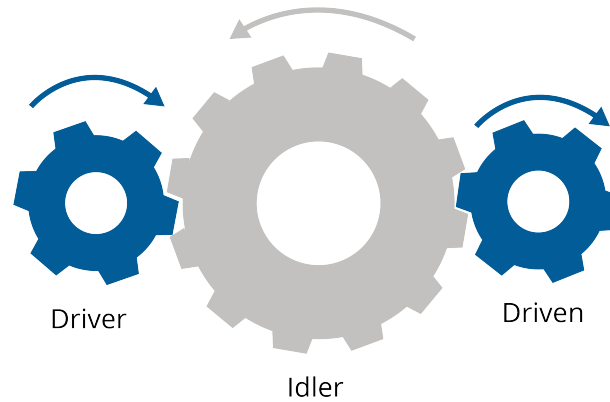


Figure 6.7: A gear train with an idler. The idler between the driver and the driven gear makes the driven gear turn the same direction as the driver; it does not change the overall gear ratio.

6.6.2 Teeth

If N_A is the number of teeth on the gear you are turning with a torque of T_A , and N_L is the number of teeth on the gear it is turning, the resulting torque is:

$$T_L = \frac{N_L}{N_A} T_A$$

Worked example: a gear train. Gear A has $N_A = 20$ teeth and turns with torque $T_A = 10$ N·m. It drives gear L, which has $N_L = 60$ teeth. The torque on gear L is

$$T_L = \frac{60}{20}(10 \text{ N} \cdot \text{m}) = 30 \text{ N} \cdot \text{m}$$

Gear L turns at one third the speed of gear A, because it has three times as many teeth. Torque goes up by a factor of 3, and speed goes down by a factor of 3.

Exercise 30 Gears

Bike gears let you choose how fast your

Working Space

legs spin, no matter how fast you are going.

You like to pedal your bike at 70 revolutions per minute. The chainring connected to your pedals has 53 teeth. The circumference of your tire is 2.2 meters. You want to ride at 583 meters per minute.

How many teeth should the rear sprocket have?

Answer on Page 80

6.7 Hydraulics

In a hydraulic system, such as a car's braking system, you exert force on a piston filled with fluid. The fluid transmits this pressure into another cylinder, where it pushes yet another piston that moves the load. The pressure at each end of the hydraulic system must be the same.

Applied Force

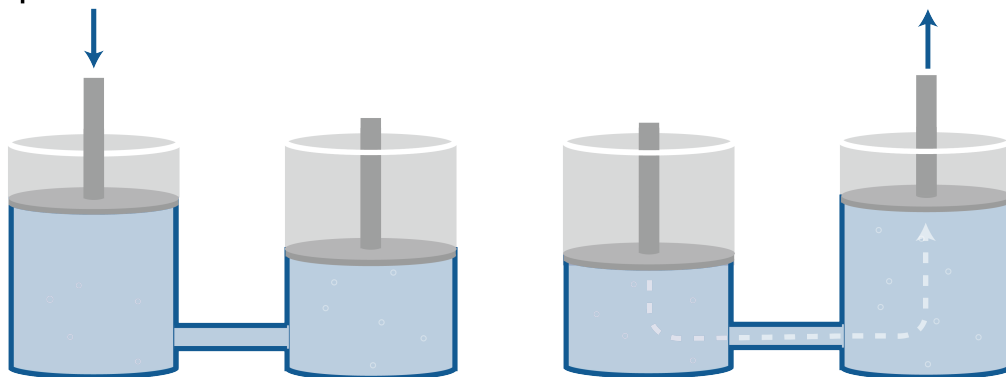


Figure 6.8: A diagram showing the transfer of fluid from one container to the other.

Pressure is force applied to an area; it is calculated by dividing the force by the area. The pressure in the fluid is typically measured in pascals (Pa), which is equivalent to N/m^2 . We will use pascals for this calculation.

To calculate the pressure you create, divide the force applied F_a by the area of the piston head A . To determine the force on the other piston, multiply the pressure by the area of the second piston.

$$P = \frac{F_{a1}}{A_1} = \frac{F_{a2}}{A_2} \quad (6.6)$$

Exercise 31 Hydraulics

Your car has disc brakes. When you apply 2,500,000 pascals of pressure to the brake fluid, the car stops quickly. As the car designer, you want this to require only 12 newtons of force from the driver's foot.

What should the radius of the master cylinder (the piston the driver pushes) be?

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Answer on Page 80

Answers to Exercises

Answer to Exercise 1 (on page 6)

1. C:H = 1:4
2. Cu:S:O = 1:1:4
3. C:H:O = 6:12:6 = 1:2:1

Answer to Exercise 2 (on page 12)

1. balanced
2. unbalanced; oxygen
3. unbalanced; barium, sulfur, oxygen, and carbon
4. balanced

Answer to Exercise 3 (on page 15)

1. $12.011 \text{ amu} + 4(1.008 \text{ amu}) = 16.043 \text{ amu}$
2. $63.546 \text{ amu} + 32.06 \text{ amu} + 4(15.999 \text{ amu}) = 159.602 \text{ amu}$
3. $6(12.011 \text{ amu}) + 12(1.008 \text{ amu}) + 6(15.999 \text{ amu}) = 180.156 \text{ amu}$

Answer to Exercise 4 (on page 17)

Substance	num. of particles	num. of moles	grams
NaHCO ₃	2.509×10^{23}	0.4166	35.00
HCl	7.53×10^{23}	1.25	45.58
KH ₂ PO ₄	12.5×10^{24}	20.8	2820

$$\frac{35.00 \text{ g NaHCO}_3}{84.007 \text{ g NaHCO}_3} \times \frac{1 \text{ mol NaHCO}_3}{1 \text{ mol NaHCO}_3} = 0.4166 \text{ mol NaHCO}_3$$

$$\frac{0.4166 \text{ mol NaHCO}_3}{1 \text{ mol NaHCO}_3} \times \frac{6.02214076 \times 10^{23} \text{ molec NaHCO}_3}{1 \text{ mol NaHCO}_3} = 2.509 \times 10^{23} \text{ molec NaHCO}_3$$

$$\frac{1.25 \text{ mol HCl}}{1 \text{ mol HCl}} \times \frac{36.46 \text{ g HCl}}{1 \text{ mol HCl}} = 45.58 \text{ g HCl}$$

$$\frac{1.25 \text{ mol HCl}}{1 \text{ mol HCl}} \times \frac{6.02214076 \times 10^{23} \text{ molec HCl}}{1 \text{ mol HCl}} = 7.53 \times 10^{23} \text{ molec HCl}$$

$$\frac{12.5 \times 10^{24} \text{ molec KH}_2\text{PO}_4}{6.02214076 \times 10^{23} \text{ molec KH}_2\text{PO}_4} \times \frac{1 \text{ mol KH}_2\text{PO}_4}{1 \text{ mol KH}_2\text{PO}_4} = 20.8 \text{ mol KH}_2\text{PO}_4$$

$$\frac{20.8 \text{ mol KH}_2\text{PO}_4}{1 \text{ mol KH}_2\text{PO}_4} \times \frac{136.086 \text{ g KH}_2\text{PO}_4}{1 \text{ mol KH}_2\text{PO}_4} = 2820 \text{ g KH}_2\text{PO}_4$$

Answer to Exercise 5 (on page 17)

One mole of water has a mass of about 18.015 grams, so 200 grams of water is about $200/18.015 \approx 11.1$ moles. Each methane molecule makes *two* water molecules, so you need half as many moles of methane: about 5.55 moles.

One mole of methane has a mass of $12.011 + 4 \times 1.008 = 16.043$ grams.

$16.043 \times 5.55 \approx 89.0$ grams of methane.

Answer to Exercise 6 (on page 21)

The average hydrogen atom has a mass of 1.00794 atomic mass units.

The average oxygen atom has a mass of 15.9994.

$2 \times 1.00794 + 15.9994 = 18.01528$ atomic mass units.

Answer to Exercise 7 (on page 24)

Element Name	Atomic Symbol	Protons	Most Common Number of Neutrons	Electrons
Francium	Fr	87	136	87
Arsenic	As	33	42	33
Aluminum	Al	13	14	13
Fluorine	F	9	10	9

Answer to Exercise 8 (on page 27)

1. Exothermic. It releases heat, so it feels warm (very warm!).
2. Endothermic. It absorbs heat from its surroundings, so it feels cold.
3. Endothermic. Photosynthesis stores energy from sunlight in the sugar's chemical bonds.

Answer to Exercise 9 (on page 30)

Equilibrium will be achieved when the box has displaced 10 kg of water. In other words, when it has displaced 0.01 cubic meters.

The area of the base of the box is 0.12 square meters. So if the box sinks x meters into the water it will displace $0.12x$ cubic meters.

Thus at equilibrium $x = \frac{0.01}{0.12} \approx 0.083$ m. So the box will sink 8.3 cm into the water before reaching equilibrium.

Answer to Exercise 10 (on page 31)

$$p = dgh = (900)(3.721)(5) = 16,744.5 \text{ Pa}$$

Answer to Exercise 11 (on page 34)

In the case, the force you exert must balance the weight of the book, so $F = mg = 2(9.8) = 19.6\text{N}$. Since this force is straight upward and the displacement is in the same direction,

$$W = F \cdot d = 19.6 \cdot 3 = 58.8 \text{ J.}$$

Answer to Exercise 12 (on page 37)

(a) Since all kinetic energy is transferred into thermal energy, we can say:

$$Q = KE = \frac{1}{2}(1200)(22)^2 = 290,400 \text{ J}$$

(b) The rise in heat is given by $\Delta T = \frac{Q}{mc} = \frac{290,400}{15 \times 460} \approx 42^\circ \text{ C}$

Answer to Exercise 13 (on page 39)

At the top of the ladder, the cannonball has $(9.8)(5)(3) = 147 \text{ J}$ of potential energy.

At the bottom, the kinetic energy $\frac{1}{2}(5)v^2$ must be equal to 147 joules. So $v^2 = \frac{294}{5}$. This means it is going about 7.7 m/s.

(You may be wondering about air resistance. Yes, a tiny amount of energy is lost to air resistance, but for a dense object moving at these relatively slow speeds, this energy is negligible.)

Answer to Exercise 14 (on page 39)

The initial total gravitational potential energy is $mgh = 4(10)(2.5) = 100 \text{ J}$. The final kinetic energy is $\frac{1}{2}mv^2 = \frac{1}{2}(4)(4.2)^2 = 35.28$. So, the rest of the energy must be lost due to friction: $100 - 35.28 = 64.72 \text{ J}$.

Answer to Exercise 15 (on page 39)

The total useful energy can be calculated as $0.65(1.2 \times 10^5) = 7.8 \times 10^4 \text{ J}$. Then, we can set

the final KE $\frac{1}{2}mv^2 = 7.8 \times 10^4$ and solve for v

$$\begin{aligned}\frac{1}{2}(150)v^2 &= 7.8 \times 10^4 \\ (150)v^2 &= 15.6 \times 10^4 \\ v^2 &= 1,040 \\ v &\approx 32.249 \text{ m/s}\end{aligned}$$

Answer to Exercise 16 (on page 41)

1. $v = \sqrt{2gh} = \sqrt{2(9.8\text{m/s}^2)(3.7\text{m})} = 8.5\text{m/s}$
2. $h = \frac{v^2}{2g} = \frac{(15\frac{\text{m}}{\text{s}})^2}{2(9.8\frac{\text{m}}{\text{s}^2})} = 11.5\text{m}$. You can hit your little brother, since you could shoot the pebble as high as 11.5 m and he is only 5.0 m above you.
3. We know that $E_{p,\text{initial}} = E_{k,\text{final}} + E_{\text{lost to friction}}$. Therefore, $E_{\text{lost to friction}} = E_{p,\text{initial}} - E_{k,\text{final}} = mgh - \frac{1}{2}mv^2 = (63\text{kg})(9.8\frac{\text{m}}{\text{s}^2})(8\text{m}) - \frac{1}{2}(63\text{kg})(11.2\frac{\text{m}}{\text{s}})^2 = 4939.2\text{J} - 3951.4\text{J} = 988\text{J}$.

Answer to Exercise 17 (on page 45)

1. The human body is an open system because matter can enter (food, water, oxygen) and leave (waste, carbon dioxide, sweat) your body.
2. The Earth is an open system because matter can enter (asteroids falling, spaceships returning) and leave (space vehicles and astronauts). On the other hand, the Earth can be well-approximated as a closed system. Before the 20th Century, humans had no way to deliberately expel matter from the Earth, and the mass of asteroids that are pulled in by the Earth's gravity is negligible compared to the Earth. Therefore, in the right circumstances, it would be appropriate to model the Earth as a closed system. (It is closed because energy in the form of sunlight is constantly entering the system.)
3. A cell phone is a closed system - you do not put any matter in or take it out of your phone, but it constantly uses battery and then is recharged, showing that energy enters and leaves your phone.
4. Since the cooler is described as well-insulated and the lid is closed, it can be approximated as an isolated system. Scientific equipment, like bomb calorimeters, rely on this approximation.
5. With the lid open, matter can enter and leave and therefore the cooler is an open system

(yes, even though it is well-insulated).

6. A sealed bottle of soda is a closed system - the soda and carbon dioxide cannot escape, but energy in the form of heat can be transferred in and out of the system (the contents of the bottle will lose heat if you put it in the fridge and gain heat if you leave it in the sun).

Answer to Exercise 18 (on page 48)

The force acting on the ball is its weight, $F_w = mg$, and we will designate this as negative since weight points downwards. Using the work-energy theorem,

$$\Delta KE = F \cdot d = (mg) \cdot d = (0.5\text{kg}) \left(-9.8 \frac{\text{m}}{\text{s}^2}\right) (1\text{m}) = -4.9\text{J}$$

Therefore, the ball loses 4.9 joules of kinetic energy for every 1 meter it moves upwards (the fact it is *losing kinetic energy* is represented by the result being negative). Note that this is also the formula for potential energy, so it gained 4.9 J of potential energy.

Answer to Exercise 19 (on page 48)

$$F_f \cdot d = \Delta KE = KE_f - KE_i$$

You will reach the maximum distance you can slide when you stop moving, so we will use a final velocity of zero, which means a final kinetic energy of zero:

$$F_f \cdot d = -KE_i = \frac{1}{2}mv^2$$

Since the force of friction is 0.3 times your weight, we know that:

$$F_f = 0.3F_w = 0.3mg$$

Substituting and canceling the mass:

$$(0.3mg) \cdot d = \frac{1}{2}mv^2$$

$$0.3g \cdot d = \frac{1}{2}v^2$$

Since we know g and v , we can solve for d :

$$d = \frac{v^2}{0.6g} = \frac{\left(10\frac{\text{m}}{\text{s}}\right)^2}{0.6\left(9.8\frac{\text{m}}{\text{s}^2}\right)} \approx 17\text{m}$$

So you can start your slide as far as 17 m from home.

Answer to Exercise 20 (on page 48)

Since all the cars need to stop in the same distance, the cars with the greatest kinetic energy will take the most force to stop. Calculating the kinetic energies (we will not change the units from kilometers per hour to meters per second, since we are just comparing the values):

Car	Mass [kg]	Max speed [kph]	KE [kg (kph) ²]
A	2000	100	1×10^7
B	2400	120	1.728×10^7
C	3200	240	9.216×10^7
D	2400	140	2.352×10^7
E	3200	120	2.304×10^7
F	2000	240	5.76×10^7

The correct ranking is A, B, E, D, F, C.

Answer to Exercise 21 (on page 52)

(a) $500 \times 0.82 = 410$ bolts

(b) $\frac{\$24}{410} \approx \$0.06$

Answer to Exercise 22 (on page 52)

(a)

$$W = F \cdot d = 450 \cdot 1.8 = 810 \text{ J}$$

(b)

$$P = \frac{W}{t} = \frac{810}{.75} = 1080 \text{ W}$$

Alternatively,

$$v = \frac{d}{t} = \frac{1.8}{0.75} = 2.4 \text{ m/s}$$
$$P = F \cdot v = 450 \cdot 2.4 = 1080 \text{ W}$$

Answer to Exercise 23 (on page 53)

$$P = \frac{E}{t}$$
$$E = P \cdot t = 120(0.4) = 48 \text{ J}$$

So 48 J of energy is stored into the spring. We can find the spring's PE converted to distance by:

$$E = \frac{1}{2}kx^2$$
$$48 = \frac{1}{2}(800)x^2$$
$$x^2 = \frac{48}{400}$$
$$x = \sqrt{0.12} \approx 0.346 \text{ m}$$

So the spring had to have been compressed 34.6 cm.

Answer to Exercise 24 (on page 53)

- (a) Electrical energy input can be calculated as $E = Pt = 18 \text{ kW} \times 1000 \text{ J/s} \times 12 \text{ sec} = 216,000 \text{ J}$.
- (b) The elevator travels 18 m upwards, so the total gravitational potential energy is $mgh = 750(9.8)(18) = 132,300 \text{ J}$.
- (c) The efficiency can be calculated as $\frac{\text{Useful Output}}{\text{Total Input}} = \frac{132,300}{216,000} \approx 61.3\%$

Answer to Exercise 25 (on page 58)

The weight of the barrel is $136 \text{ kg} \times 9.8 \frac{\text{m}}{\text{s}^2} = 1332.8 \text{ N}$.

Let L be the length of the inclined plane. The force needed to push the barrel up is related

by:

$$300 \text{ N} = \frac{2 \text{ m}}{L} \times 1332.8 \text{ N}$$

Solving for L, we find $L = \frac{2 \text{ m} \times 1332.8}{300} \approx 8.885 \text{ m}$.

Answer to Exercise 26 (on page 59)

Paul exerts a force of $70 \text{ kg} \times 9.8 \frac{\text{m}}{\text{s}^2} = 686 \text{ N}$ at a distance of 4 meters from the fulcrum, creating a torque of $686 \text{ N} \times 4 \text{ m} = 2744 \text{ N} \cdot \text{m}$. Jan exerts a force of $50 \text{ kg} \times 9.8 \frac{\text{m}}{\text{s}^2} = 490 \text{ N}$.

Let r be the distance from the fulcrum to Jan's seat. To balance the torques:

$$490 \text{ N} \times r = 2744 \text{ N} \cdot \text{m}$$

Solving for r , we find $r = \frac{2744}{490} \approx 5.6 \text{ meters}$.

Answer to Exercise 27 (on page 61)

There are 4 segments supporting the net weight total of 420 N. Each segment, then, splits the weight into $\frac{420 \text{ N}}{4} = 105 \text{ N}$. The input force required to move both weights is 105 N, which moves it 4 m. The output work matches as well: 420 N is moved 1 m.

Answer to Exercise 28 (on page 63)

The mechanical advantage of the wedge is $\frac{12 \text{ cm}}{3 \text{ cm}} = 4$.

Answer to Exercise 29 (on page 63)

A knife cuts better than a screwdriver tip because its edge is much thinner, giving it a greater mechanical advantage. This allows the applied force to be concentrated over a very small area, producing much higher pressure. As a result, less force is needed to cut the material. A screwdriver, on the other hand, has a very thick edge (in comparison), so its pressure and mechanical advantage are much less.

Answer to Exercise 30 (on page 67)

The equation relating these quantities is:

$$583 = 70 \times 2.2 \times \frac{53}{n}$$

Solving for n , we find $n = 14$ teeth.

Answer to Exercise 31 (on page 69)

We are solving for the radius r of the piston. The area of the piston is πr^2 , so the pressure is:

$$\text{Pressure} = \frac{12}{\pi r^2}$$

Setting the pressure equal to 2,500,000 pascals:

$$2,500,000 = \frac{12}{\pi r^2}$$

Solving for r , we find:

$$r = \sqrt{\frac{12}{\pi \times 2.5 \times 10^6}} \approx 0.00124 \text{ meters.}$$



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