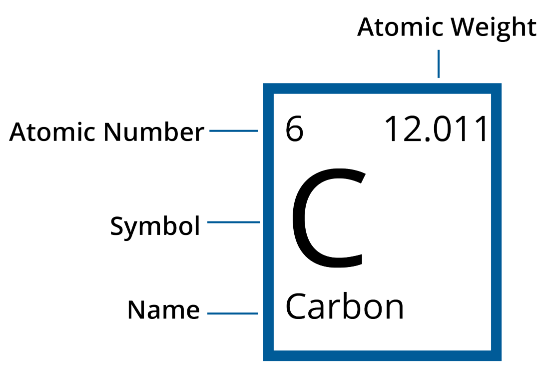
Contents

Chapter 1

Atomic and Molecular Mass

1.1 Reading a Periodic Tile

Let's look at the different information shown on a periodic tile:



The four things we learn from a periodic tile are:

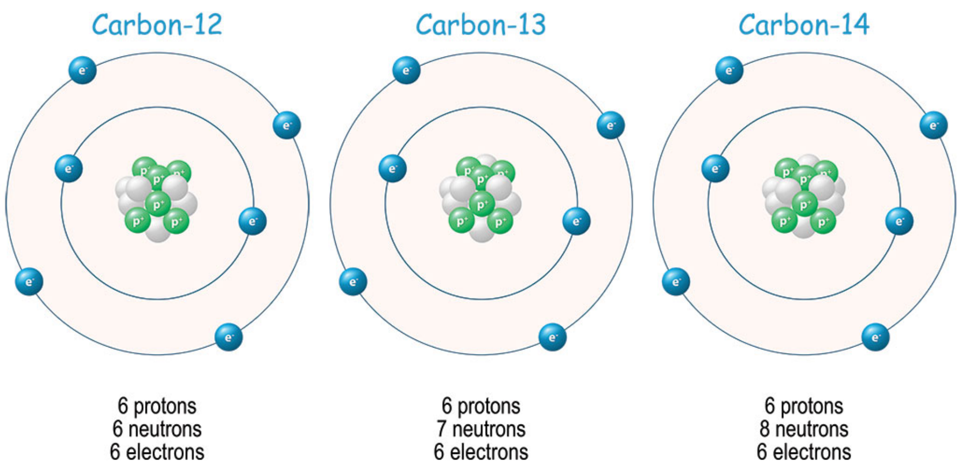
1. the symbol: as discussed in the previous chapter, each element has a unique symbol. Element symbols are used when showing the structure of a molecule and modeling chemical reactions.

2. the atomic number: this is also unique for each element. Take a look at the periodic table a few pages forward. Every tile has a unique atomic number, and the tiles are laid out in a generally increasing atomic number (you'll learn why the periodic table is arranged this way in Sequence 2).

3. the atomic weight: this is the average mass of all the atoms of that element in existence. Just like your overall grade in a class is the weighted average of all the individual grades you earned, atomic weight is the weighted average of the masses of all the individual atoms of that element. This is also sometimes referred to as atomic mass.

4. the name: not all periodic tables show the name of an element on its tile. This is why it is useful to know the symbols of common elements.

Recall from the previous chapter that we classify atoms by the number of protons they have. What this means is that if we want to know what element an atom is, we have to look at the number of protons. Take a look at the three carbon atoms below and note what is the same and what is different among them:



These different versions of carbon all have 6 protons, which is also carbon's atomic number. This isn't a coincidence: the atomic number *is* the number of protons in every atom of an element. If I tell you an atom has 4 protons, you would find atomic number 4 and see that the element is beryllium. To know how many protons an oxygen atom has, you would find its tile and see that it has atomic number 8.

Ok, so now we know atoms of the same element have the same number of protons, and that number is given by the element's atomic number. The difference between these carbon atoms explains the other number on a periodic tile: the atomic mass.

A proton and a neutron have about the same mass. An electron, on the other hand, has much less mass: One neutron weighs about the same amount as 2000 electrons. This means that the mass of any object comes mostly from the protons and neutrons in the nucleus of its atoms.

We know how many protons an atom has by what element it is, but how do we know the number of neutrons?

1.2 Mass of Atoms and Molecules

As you've seen, a periodic tile for an element tells us the average mass of an atom of that element in Daltons or amus (atomic mass units). The average mass of a carbon atom is 12.011 amu, and the average mass of an iron atom is 55.845 amu. Using the periodic table, determining the average mass of an atom is straightforward. What about molecules?

Consider water: H_2O. It is made of 2 hydrogen atoms, each with an average mass of 1.008 amu, and one oxygen atom, with an average mass of 15.999 amu. To find the mass of the molecule, called *molecular mass*, you simply add the masses of each of the atoms in the molecule. So, the molecular mass of water is 1.008 amu + 1.008 amu + 15.999 amu = 18.015 amu.

Exercise 1 Determining Molecular Mass

Find the molecular mass, in amu, of the following substances:

1. CH_4
2. $CuSO_4$
3. $C_6H_{12}O_6$

Working Space

Answer on Page 63

1.3 Mole Concept

An atomic mass unit is a very, very, very small unit; we would much rather work in grams. Grams are a large enough unit that you can develop a natural sense for how much a gram is. Additionally, while you can't see a single carbon atom with your eyes, you can see 10 grams of carbon (about enough to fill a pen cap). To convert between the very, very, very small unit of amus to the tangible unit of grams, we use *Avogadro's Number* (sometimes called *Avogadro's Constant*).

Since 1 amu is defined as $1/12^{th}$ of the mass of a carbon-12 atom, carbon-12 by definition has a mass of 12 amu. Additionally, Avogadro's number is the number of carbon atoms in 12.000 grams of pure carbon-12. This amount is called *a mole*. If you have 12 doughnuts, that's a dozen doughnuts. If you have 20 donuts, you have a score of donuts. 500 donuts: a ream of donuts. If you have $6.02214076 \times 10^{23}$ doughnuts, you have a *mole* of doughnuts.

This isn't really a practical measurement, as a mole of doughnuts would be about the size of the earth. We use moles for small things like molecules. However, a mole is a not an abbreviation for a molecule. For a better idea about how large of a number Avogadro's number is, you can watch this video: https://www.youtube.com/watch?v=TEl4jeETVmg. A mole of carbon-12 has a mass of 12.000 g, but a mole of natural carbon (which includes all the isotopes of carbon) has a mass of 12.011 g. The mole is defined such that one mole of an element is the same mass in grams as one atom is in amus. Let's say you want to know how much a mole of $NaCl$ weighs. From the periodic table, you see that Na has an atomic mass of 22.990 atomic mass units, and Cl has 35.453 atomic mass units. One atom of $NaCl$ has a mass of $22.990 + 35.453 = 58.443$ atomic mass units. This means a mole of $NaCl$ has a mass of 58.443 grams. Handy, right? This is called the *molar mass*. It is the mass of one mole of a substance, and is given in units of g/mol (grams per mole). The molar mass of NaCl is 58.443 g/mol. The molar mass of carbon is 12.011 g/mol. Using dimensional analysis and the molar mass, you can determine the mass of a given number of moles of a substance.

Example: What is the mass of 2 moles of copper?

Solution: The conversion we will use is 1 mol Cu = 63.546 g Cu.

$$\frac{2 \text{ mol Cu}}{} \times \frac{63.546 \text{ g Cu}}{1 \text{ mol Cu}} = 127.092 \text{ g Cu}$$

Therefore, 2 moles of copper has a mass of 127.092 grams.

You can also find the molar mass of a molecule, like methane. Just like with elements, a mole of a molecule has the same mass in grams as a single molecule has in amus.

Example: What is the mass of 3.5 moles of methane?

Solution: Methane (CH_4) has a molecular mass of 16.043 amu, which means 1 mole of methane has a mass of 16.043 grams.

$$\frac{3.5 \text{ mol } CH_4}{} \times \frac{16.043 g CH_4}{1 \text{ mol } CH_4} = 56.151 \text{ g } CH_4$$

You can also use the molar mass to determine how many moles of a substance there are in a given mass of that substance.

Example: A standard AAA battery contains about 7.00 g of zinc. How many moles of zinc are in a AAA battery?

Solution: Zinc's molar mass is 65.38 g/mol.

$$\frac{7.00 \text{ g Zn}}{} \times \frac{1 \text{ mol Zn}}{65.38 \text{ g Zn}} \approx 0.107 \text{ g Zn}$$

In summary, a mole of a substance contains approximately 6.02×10^{23} particles (atoms or molecules) of that substance and has a mass equal to the molecular mass in grams.

The Mole Concept

For a substance, X, with a molar mass of x g/mol,

$$1 \text{ mol X } = 6.02 \times 10^{23} \text{ particles of X } = x \text{ g of X}$$

Exercise 2 Grams, Moles, Molecules, and Atoms

Working Space

Complete the table.

Substance	num. of particles	num. of moles	grams
$NaHCO_3$			35
HCl		1.2	
KH_2PO_4	12.5×10^{24}		

Answer on Page 63

Exercise 3 Burning Methane

Working Space

Natural gas is mostly methane (CH_4). When one molecule of methane burns, two oxygen molecules (O_2) are consumed. One molecule of H_2O and one molecule of CO_2 are produced.

If you need 200 grams of water, how many grams of methane do you need to burn?

(This is how the hero in "The Martian" made water for his garden.)

Answer on Page 64

If you fill a balloon with helium, it will have two different kinds of helium atoms. Most of the helium atoms will have 2 neutrons, but a few will have only 1 neutron. We say that these are two different *isotopes* of helium. We call them helium-4 (or ^4He) and helium-3 (or ^3He). Isotopes are named for the sum of protons and neutrons the atom has: helium-3 has 2 protons and 1 neutron.

A hydrogen atom nearly always has just 1 proton and no neutrons. A helium atom nearly always has 2 protons and 2 neutrons. So, if you have a 100 hydrogen atoms and 100 helium atoms, the helium will have about 4 times more mass than the hydrogen. We say "Hydrogen is about 1 atomic mass unit (amu), and helium-4 is about 4 atomic mass units."

What, precisely, is an atomic mass unit? It is defined as 1/12 of the mass of a carbon-12 atom. Scientists have measured the mass of helium-4, and it is about 4.0026 atomic mass units. (By the way, an atomic mass unit is also called a *dalton*.)

Now you are ready to take a good look at the periodic table of elements. Here is the version from Wikipedia:

Periodic Table of Elements

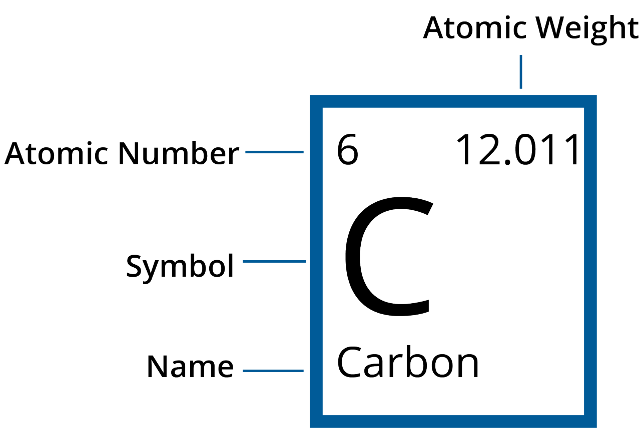
IA	IIA	IIIB	IVB	VB	VIB	VIIB	VIIIB	VIIIB	VIIIB	IB	IIB	IIIA	IVA	VA	VIA	VIIA	VIIIA
1 **H** Hydrogen 1.01																	2 **He** Helium 4.00
3 **Li** Lithium 6.94	4 **Be** Beryllium 9.01											5 **B** Boron 10.81	6 **C** Carbon 12.01	7 **N** Nitrogen 14.01	8 **O** Oxygen 16.00	9 **F** Fluorine 19.00	10 **Ne** Neon 20.18
11 **Na** Sodium 22.99	12 **Mg** Magnesium 24.31											13 **Al** Aluminum 26.98	14 **Si** Silicon 28.09	15 **P** Phosphorus 30.97	16 **S** Sulfur 32.06	17 **Cl** Chlorine 35.45	18 **Ar** Argon 39.95
19 **K** Potassium 39.10	20 **Ca** Calcium 40.08	21 **Sc** Scandium 44.96	22 **Ti** Titanium 47.87	23 **V** Vanadium 50.94	24 **Cr** Chromium 52.00	25 **Mn** Manganese 54.94	26 **Fe** Iron 55.85	27 **Co** Cobalt 58.93	28 **Ni** Nickel 58.69	29 **Cu** Copper 63.55	30 **Zn** Zinc 65.38	31 **Ga** Gallium 69.72	32 **Ge** Germanium 72.63	33 **As** Arsenic 74.92	34 **Se** Selenium 78.97	35 **Br** Bromine 79.90	36 **Kr** Krypton 83.80
37 **Rb** Rubidium 85.47	38 **Sr** Strontium 87.62	39 **Y** Yttrium 88.91	40 **Zr** Zirconium 91.22	41 **Nb** Niobium 92.91	42 **Mo** Molybdenum 95.95	43 **Tc** Technetium (98)	44 **Ru** Ruthenium 101.07	45 **Rh** Rhodium 102.91	46 **Pd** Paladium 106.42	47 **Ag** Silver 107.87	48 **Cd** Cadmium 112.41	49 **In** Indium 114.82	50 **Sn** Tin 118.71	51 **Sb** Antimony 121.76	52 **Te** Tellurium 127.60	53 **I** Iodine 126.90	54 **Xe** Xenon 131.29
55 **Cs** Cesium 132.91	56 **Ba** Barium 137.33	57 - 71 **Lanthanides**	72 **Hf** Hafnium 178.49	73 **Ta** Tantalum 180.95	74 **W** Tungsten 183.84	75 **Re** Rhenium 186.21	76 **Os** Osmium 190.23	77 **Ir** Iridium 192.22	78 **Pt** Platinum 195.08	79 **Au** Gold 196.97	80 **Hg** Mercury 200.59	81 **Tl** Thallium 204.38	82 **Pb** Lead 207.20	83 **Bi** Bismuth 208.98	84 **Po** Polonium (209)	85 **At** Astatine (210)	86 **Rn** Radon (222)
87 **Fr** Francium (223)	88 **Ra** Radium (226)	89 - 103 **Actinides**	104 **Rf** Rutherfordium (265)	105 **Db** Dubnium (268)	106 **Sg** Seaborgium (271)	107 **Bh** Bohrium (270)	108 **Hs** Hassium (277)	109 **Mt** Meitnerium (276)	110 **Ds** Darmstadtium (281)	111 **Rg** Roentgenium (280)	112 **Cn** Copernicium (285)	113 **Nh** Nihonium (284)	114 **Fl** Flerovium 289	115 **Mc** Moscovium (288)	116 **Lv** Livermorium (293)	117 **Ts** Tennessine (294)	118 **Og** Oganesson (294)

57 **La** Lanthanum 138.91	58 **Ce** Cerium 140.12	59 **Pr** Praseodymium 140.91	60 **Nd** Neodymium 144.24	61 **Pm** Promethium (145)	62 **Sm** Samarium 150.36	63 **Eu** Europium 151.96	64 **Gd** Gadolium 157.25	65 **Tb** Terbium 158.93	66 **Dy** Dysprosium 162.50	67 **Ho** Holmium 164.93	68 **Er** Erbium 167.26	69 **Tm** Thulium 168.93	70 **Yb** Ytterbium 173.05	71 **Lu** Lutetium 174.97
89 **Ac** Actinium (227)	90 **Th** Thorium 232.04	91 **Pa** Protactinium 231.04	92 **U** Uranium 238.03	93 **Np** Neptunium (237)	94 **Pu** Plutonium (244)	95 **Am** Americium (243)	96 **Cm** Curium (247)	97 **Bk** Berkelium (247)	98 **Cf** Californium (251)	99 **Es** Einsteinium (252)	100 **Fm** Fermium (257)	101 **Md** Mendelevium (258)	102 **No** Nobelium (259)	103 **Lr** Lawrencium (262)

Figure 1.1: Periodic table from Wikipedia.

There is a square for each element. In the middle, you can see the atomic symbol and the name of the element. In the upper-right corner is the atomic number — the number of protons in the atom.

In the upper-left corner is the atomic mass in atomic mass units.



Look at the atomic mass of boron. About 80% of all boron atoms have six neutrons. The other 20% have only 5 neutrons. This difference is why most boron atoms have a mass of about 11 atomic mass units, but some have a mass of about 10 atomic mass units. The atomic mass of boron is equivalent to the average mass of a boron atom: 10.811.

Exercise 4 Mass of a Water Molecule

Working Space

Using the periodic table, what is the average mass of one water molecule in atomic mass units?

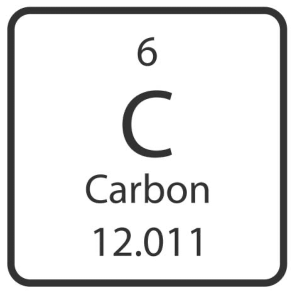
Answer on Page 64

1.4 xfer from intro chapter

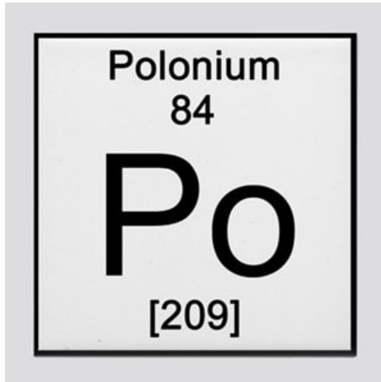
fixme integrate into this chapter

1.4.1 Reading the Periodic Table

The Periodic Table organizes what we know about the structure of different elements. Each element has its own block or tile on the Periodic Table, and the information on the tile tells us about the structure of that atom. Take a look at the tile for carbon:



The letter (or letters, as is the case for other elements) is the atomic symbol for the element. There are two key numbers: the atomic number and the average atomic mass. For carbon, the atomic number is 6 and the average atomic mass is 12.011. The atomic number tells us how many protons there are in the nucleus of any atom of carbon. Since every element has a unique number of protons, every element has a unique atomic number. All carbon atoms have 6 protons. The other number is the average atomic mass - it tells us the weighted average of the mass of all the carbons in the universe. When the average atomic mass is in a whole number, as it is for polonium, it means that the element is very unstable. As a result, the mass given is the mass of the most stable isotope (we'll talk more about stability and isotopes below). On some periodic tables, the mass number of the most stable isotope will be in parentheses or brackets. In summary, if the larger number is a whole number, it is the mass number; if it is a decimal (even if the decimal ends in .00), it is the average atomic mass, which we will discuss further below.



The Royal Society of Chemistry has a very useful interactive periodic table: periodic-table.rsc.org. We can use the periodic tile for an element to determine the number of protons, electrons, and most common number of neutrons for a neutral atom of that element (we'll explain why the periodic tile tells us the "most common number of neutrons" below).

Example: State the atomic symbol for and the number of protons, neutrons, and electrons in a neutral atom of plutonium.

Solution: The plutonium tile on your periodic table should look something like this:

94
Pu
Plutonium
(244)

[The information may be arranged differently, but you should at least see the symbol and two numbers.] As you can see, the atomic symbol for plutonium is Pu. Since its atomic number is 94, we know every atom of plutonium has 94 protons. To know the number of electrons, we will take advantage of the fact that the question is asking about a *neutral* atom. This means there are the same number of positive charges as negative charges. So, since there are 94 protons, a neutral atom of plutonium must have 94 electrons (each proton has a +1 charge and each electron has a -1 charge). Lastly, let's determine the number of neutrons. The other number, 244, is the mass number. It represents the total number of protons and neutrons in the nucleus. Since we know plutonium has 94 protons, we can find the number of neutrons by subtracting the atomic number from the mass number:

$$244 - 94 = 150$$

mass number (protons + neutrons) — atomic number (protons) = number of neutrons

Therefore, an atom of plutonium has 150 neutrons. Now let's address how to find the number of neutrons when the periodic tile shows an average atomic mass, instead of a mass number. This occurs when there is more than one ”version” of an element. In the case of plutonium, there is only one version, which is why the periodic tile shows a mass number instead of an average atomic mass. To learn about average atomic mass, we will use carbon as an example.

Have you heard of carbon-14 dating? The phrase ”carbon-14” refers to a rare type of carbon that decays radioactively. By seeing how much carbon-14 has decayed, scientists can estimate the age of organic materials, such as bone or ash. Carbon-14 is a radioactive isotope (or version) of carbon. The 14 refers to the mass number - the total amount of protons and neutrons in the nucleus (sometimes, we shorten the isotope name by just using the atomic symbol, in this case C-14). Isotopes are versions of an element with different numbers of neutrons. The atomic number is the same for them all - they all have the same number of protons. But the different number of neutrons causes different isotopes to have different masses. Examine the models of carbon-12, carbon-13, and carbon-14 below. What is different between them? What is the same?

You should have noticed that all three atoms have 6 protons and 6 electrons, while they have differing numbers of neutrons. The most common isotope of carbon is carbon-12, with 6 protons and 6 neutrons in its nucleus. Carbon-14, on the other hand, has 8 neutrons, which makes the nucleus unstable, leading to radioactive decay. The average atomic mass is the weighted average of all the carbon atoms in existence. Since the vast majority of carbon is carbon-12, the average atomic mass is very close to 12. You cannot determine the mass number of an individual atom from the periodic table; it only tells you the average of all the isotopes. However, especially for light atoms (atoms in the first two rows of the periodic table), you can usually determine the mass number of the most common isotope by rounding the average atomic mass to the nearest whole number.

Example: Germanium has atomic symbol Ge. State the number of protons, number of electrons, and most common number of neutrons in a neutral atom of germanium.

Solution: Examining the periodic table, we see that germanium has an atomic number of 32, which means a neutral atom of germanium has 32 protons and 32 electrons. The average atomic mass is 72.630, which rounds up to 73. So, the most common isotope of germanium is Ge-73, which has $73 - 32 = 41$ neutrons.

Exercise 5 Determining Numbers of Subatomic Particles

Working Space

Use a periodic table to complete the table below (assume neutral atoms):

Element Name	Atomic Symbol	Protons	Most Common Number of Neutrons	Electrons
	Fr			
				33
Erbium				
		48		

Answer on Page 64

1.5 Heavy atoms aren't stable

When you look at the periodic table, there are a surprisingly large number of elements. You might be told to "Drink milk so that you can get the calcium you need." However, no one has told you "You should eat kale so that you get enough copernicium in your diet."

Copernicium, with 112 protons and 173 neutrons, has only been observed in a lab. It is highly radioactive and unstable (meaning it decays). A copernicium atom usually lives for less than a minute before decaying.

The largest stable element is lead, which has 82 protons and between 122 and 126 neutrons. Elements with lower atomic numbers than lead, have at least one stable isotope, while elements with higher atomic numbers than lead don't.

Bismuth, with an atomic number of 83, is *almost* stable. In fact, most bismuth atoms will live for billions of years before decaying!

Chapter 2

Chemical Reactions

2.1 Chemical Reactions

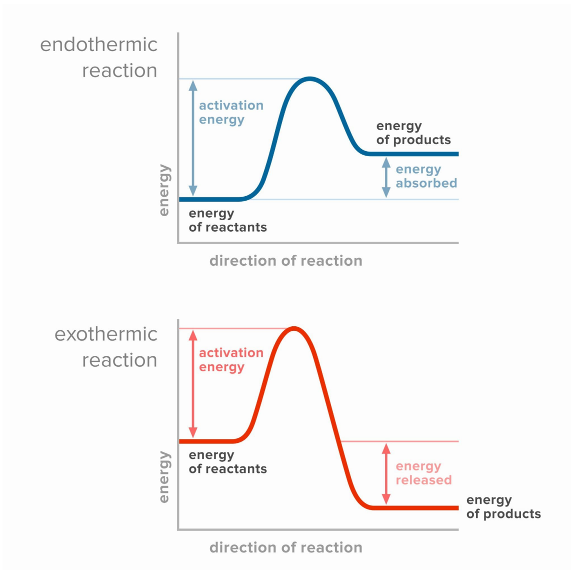


Figure 2.1

Hydrogen and oxygen have a special tendency to bond with themselves. Almost never will you find them unbonded. Oxygen is most often found as O_2 and hydrogen as H_2. If you mix these together in a 2:1 ratio of hydrogen to oxygen and light a match, they will rearrange themselves into water molecules. This is called a *chemical reaction*. In any chemical reaction, the atoms are rearranged into new molecules.

When a chemical reaction happens, some bonds are broken and new bonds are formed. The breaking and forming of bonds requires energy. The amount of energy required to break bonds is different for different bonds. The amount of energy released when new bonds are formed is also different for different bonds. Some chemical reactions (like the burning of hydrogen gas described above) are *exothermic* — that is, they give off more energy than they absorb. Burning hydrogen gas happens quickly and gives off a lot of energy. If you have enough, it will make quite an explosion! Other chemical reactions are *endothermic* — they consume energy.Photosynthesis, the process by which plants consume energy from the sun to make sugar from CO_2 and H_2O requires an endothermic chemical reaction. Energy stored in chemical bonds is called *chemical energy*. which is a form of *potential energy*.

Examine the diagrams in figure 2.1. The x-axis represents time - time passes as we move from left to right across the diagram. At the far left, the energy of the reactants (the ingredients that go into the reaction) is shown. At the far right, the energy of the products (what is made in the chemical reaction) is shown. Look at the endothermic reaction diagram (the blue one). Based on the relative energies of the reactants and products, do you expect and endothermic reaction to release or absorb heat? Absorb! Since the products have more energy, they must have absorbed energy, in the form of heat, from the surroundings.

Now, the red diagram shows an exothermic reaction: the products have less energy than the reactants. Since energy is never created or destroyed, where did the energy go? It is released as heat. So, exothermic reactions release heat. What does this look and feel like in real life? If an exothermic reaction were happening in a glass beaker, you would feel warmth if you held the beaker. The heat is leaving the beaker and entering your hand, which feels warm. What about an endothermic reaction? Many students think that since an endothermic reaction absorbs heat, it must be getting hot. This is incorrect: *exothermic* reactions feel hot. If an endothermic reaction were happening in a beaker and you touched the beaker, it would feel **cold**. Why? Well, if the reaction is absorbing heat, then heat must be leaving it surroundings (your hand) and entering the reaction (this heat energy is turned into chemical energy that is stored in the new chemical bonds that are forming). So your hand feels cold.

Chapter 3

Buoyancy

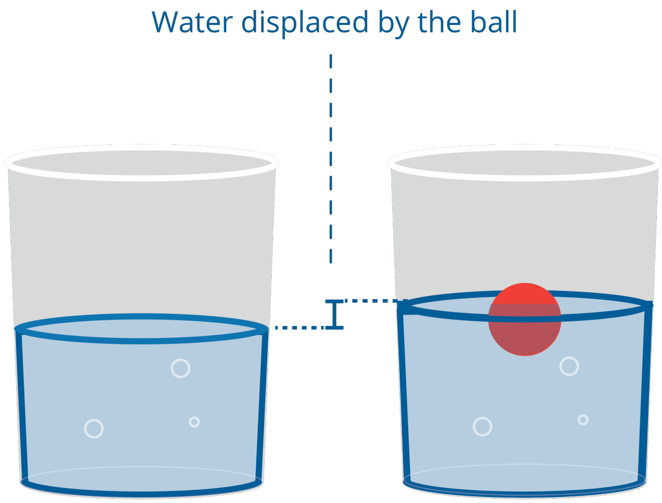
The word buoyancy probably brings to mind images of floating in water. Before we dive in, let's zoom out for a better understanding of everything buoyancy entails. You may be thinking: I want to be a computer programmer, why do I need to know about buoyancy? You might be surprised! This topic is much bigger than it seems at first glance.

Buoyancy concerns the ways in which liquids and gasses interact with gravity. The concept of buoyancy is connected to fundamental concepts about how the universe works. The *buoyant force*, as it is known in engineering, is an important concept that has wide ranging applications. A big part of engineering is moving stuff around, and understanding buoyancy helps us solve problems where we need to move things in and through fluids. Even if you don't have plans to build a robotic submarine, these are incredibly useful ideas to be familiar with. We will start exploring the topic with familiar scenarios around boats and water.

When you put a boat into water, it will sink into the water until the mass of the water it displaces is equal to the mass of the boat. We think of this in terms of forces. Gravity pulls the mass of the boat down; the *buoyant force* pushes the boat up. A boat dropped into the water will bob up and down at first before reaching an *equilibrium* where the two forces are equal.

The buoyant force pushes things up, fighting against the force of gravity. The force is equal to the weight of the fluid being replaced. For example, a cubic meter of freshwater has a mass of about 1000kg. If you submerge anything with a volume of one meter in freshwater on earth, the buoyant force will be about 9800 newtons (mass $\times$ gravity).

For some things, like a block of styrofoam, this buoyant force will be sufficient to carry it to the surface. Once it reaches the surface, it will continue to rise (displacing less water) until the mass of the water it displaces is equal to its mass. And then we say "It floats!"



For some things, like a block of lead, the buoyant force is not sufficient to lift it to the surface, and then we say "It sinks!"

This is why a helium balloon floats through the air. The air that it displaces weighs more than the balloon and the helium itself. (It is easy to forget that air has a mass, but it does.)

Exercise 6 Buoyancy

You have an aluminum box that has a heavy base, so it will always float upright. The box and its contents weigh 10 kg. Its base is 0.3 m x 0.4 m. It is 1m tall.

When you drop it into freshwater ($1000 kg/m^3$), how far will it sink before it reaches equilibrium?

Working Space

Answer on Page 64

3.1 The Mechanism of Buoyancy: Pressure

As you dive down in the ocean, you will experience greater and greater pressure from the water. And if you take a balloon with you, you will gradually see it get smaller as the water pressure compresses the air in the balloon.

Let's say you are 3 meters below the surface of the water. What is the pressure in Pascals (newtons per square meter)? You can think of the water as a column of water crushing down upon you. The pressure over a square meter is the weight of 3 cubic meters of water pressing down.

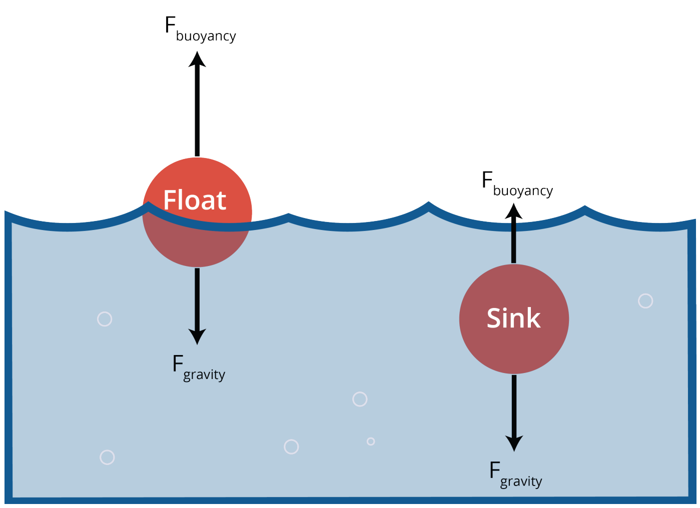
$$p = (3)(1000)(9.8) = 29,400 \text{ Pa}$$

This is called *hydrostatic pressure*. The general rule for hydrostatic pressure in Pascals p is

$p = dgh$

where d is the density of the fluid in kg per cubic meter, g is the acceleration due to gravity in m/s^2, and h is the height of the column of fluid above you.

So where does buoyant force come from? Basically, the pressure pushing up on the deepest part of the object is higher than the pressure pushing down on the shallowest part of the object. That is where bouyancy comes from.



Exercise 7 Hydrostatic Pressure

Working Space

You dive into a tank of olive oil on Mars. How much more hydrostatic pressure does your body experience at 5 meters deep than it did at the surface?

The density of olive oil is about 900 kg per square meter. The acceleration due to gravity on Mars is 3.721 m/s^2.

Answer on Page 65

3.2 The Mechanism of Buoyancy: Density

Keep in mind that although the pressure is increasing as you go deeper, the buoyant force will *not increase*, because the buoyant force is always equal to the weight of the fluid that is displaced, regardless whether that is 1 meter or 100 meters underwater.

Due to the added minerals, saltwater is denser than freshwater. This causes objects to float better in the sea than they do in a river. Lipids, like fats and oils, are less dense than water, allowing them to float on top of a glass of water. When you're facing a grease fire, you're told not to put water on it. That's because the water sinks below the grease, then boils, throwing burning grease everywhere.

Chapter 4

Heat

All mass in the universe has heat, whether you're looking at a block of dry ice (frozen CO_2, -78.5° C) or the surface of the sun ($5,600^\circ$ C). As long as the mass is above absolute zero — the coldest possible temperature in the universe — there is some amount of heat in it.

4.1 How Heat Works

As you heat up an object, you are imparting energy into it. Where does this energy go? The atoms take this energy and they begin to move, vibrating and bumping into each other, causing the heat to spread throughout. Everytime the atoms collide and bounce off of each other, they emit a tiny amount of energy as light. In most cases, that light is in the infrared spectrum, but in extreme cases can be visible, such as with molten lava or hot metal.

As objects interact, they either put heat into colder objects or take heat from warmer objects. That's what allows you to heat up anything in the first place. The hot air from a stove or bunsen burner interacts with the pan or test tube you're heating, passing the air's heat on. How could you model this?

4.2 Specific Heat Capacity

If you are heating something, the amount of energy you need to transfer to it depends on three things: the mass of the thing you are heating, the amount of temperature change you want, and the *specific heat capacity* of that substance.

Energy in Heat Transfer

The energy moved in a heat transfer is given by

$$E = mc\Delta_T$$

where m is the mass, Δ_T is the change in temperature, and c is the specific heat capacity of the substance.

(Note that this assumes there isn't a phase change. For example, this formula works nicely on warming liquid water, but it gets more complicated if you warm the water past its boiling point.)

Can we guess the specific heat capacity of a substance? It is very, very difficult to guess the specific heat of a substance, so we determine it by experimentation.

For example, it takes 0.9 joules to raise the temperature of solid aluminum one degree Celsius. So we say "The specific heat capacity of aluminum is 0.9 J/g °C."

The specific heat capacity of liquid water is about 4.2 J/g °C.

Let's say you put a 1 kg aluminum pan that is 80° C into 3 liters of water that is 20° C. Energy, in the form of heat, will be transferred from the pan to the water until they are at the same temperature. We call this "thermal equilibrium".

What will the temperature of the water be?

To answer this question, the amount of energy given off by the pan must equal the amount of energy absorbed by the water. They also need to be the same temperature at the end. Let T be the final temperature of both.

3 liters of water weighs 3,000 grams, so the change in energy in the water will be:

$$E_W = mc\Delta_T = (3000)(4.2)(T - 20) = 12600T - 252000 \text{ joules}$$

The pan weighs 1000 grams, so the change in energy in the pan will be::

$$E_P = mc\Delta_T = (1000)(0.9)(T - 80) = 900T - 72000 \text{ joules}$$

The total energy stays the same, so $E_W + E_P = 0$. This means you need to solve

$$(12600T - 252000) + (900T - 72000) = 0$$

And find that the temperature at equilibrium will be

$$T = 24°C$$

Exercise 8 Thermal Equilibrium

Working Space

Just as you put the aluminium pan in the water as described above, someone also puts a 1.2 kg block of copper cooled to 10 ° C. The specific heat of solid copper is about 0.4 J/g °C.

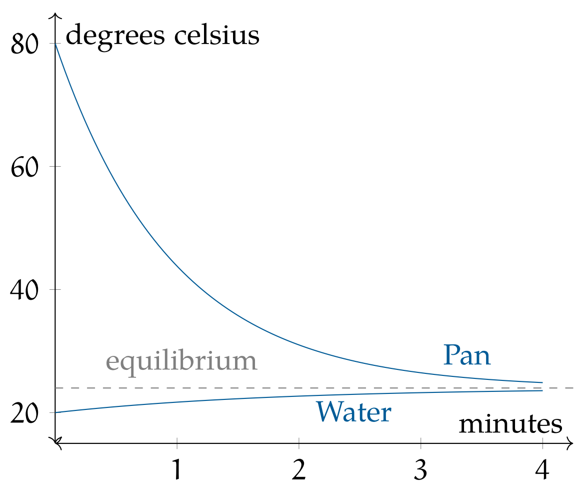
What is the new temperature at equilibrium?

Answer on Page 65

4.3 Getting to Equilibrium

When two objects with different temperatures are touching, the speed at which they exchange heat is proportional to the differences in their temperatures. As their temperatures get closer together, the heat exchange slows down.

In our example, the pan and the water will get close to equilibrium quickly, but they may never actually reach equilibrium.



Exercise 9 Cooling Your Coffee

Working Space

You have been given a ridiculously hot cup of coffee and a small pitcher of chilled milk.

You need to start chugging your coffee in three minutes, and you want it as cool as possible at that time. When should you add the milk to the coffee?

Answer on Page 65

4.4 Specific Heat Capacity Details

For any given substance, the specific heat capacity often changes a great deal when the substance changes state. For example, ice is 2.1 J/g °C, whereas liquid water is 4.2 J/g°C.

Even within a given state, the specific heat capacity varies a bit based on the temperature and pressure. If you are trying to do these sorts of calculations with great accuracy, you will want to find the specific heat capacity that matches your situation. For example, I might look for the specific heat capacity for water at 22°C at 1 atmosphere of pressure (atm).

Chapter 5

Work and Energy

In this chapter, we are going to talk about how engineers define work and energy. It frequently takes force to get work done. Let's start with thinking about the relationship between force and energy. As we learned earlier, Force is measured in newtons, and one newton is equal to the force necessary to accelerate one kilogram at a rate of $1 m/s^2$.

When you lean on a wall, you are exerting a force on the wall, but you aren't doing any work. On the other hand, if you push a car for a mile, you are clearly doing work, as in Figure 5.1. Work, to an engineer, is the force you apply to something, as well as the distance that something moves, in the direction of the applied force. We measure work in *joules*. A joule is one newton of force applied over one meter.

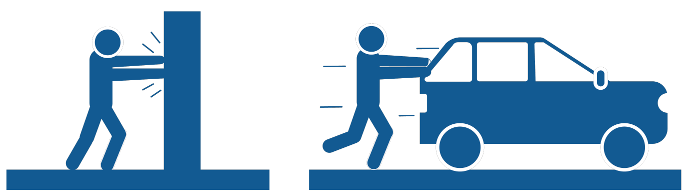


Figure 5.1: Pushing on a wall does no work while pushing a car does work! Why?

For example, if you push a car with a force of 10 newtons for 12 meters, you have done 120 joules of work.

Formula for Work

If a force applied is *constant*, the formulas for work is:

$$W = F \cdot d$$

where W is the work in joules ($N \cdot m$), F is the *force* in newtons, and d is the distance in meters.

If the force is not in the same direction as the distance, we can use the cosine of the angle between the force and the distance:

$$W = F \cdot d \cdot \cos(\theta)$$

where θ is the angle between the force and the distance. If the formula for work is *not constant*, the formula becomes:

$$W = \int \mathbf{F} \cdot \mathbf{dr}$$

where **dr** is an infinitesimal amount of displacement. An example of this kind of force is like a spring force, where the force varies with displacement.

Exercise 10 Book done by a constant force

This question is from an AP Physics C Review Book. You slowly lift a 2 kg book at a *constant velocity* through a distance of 3 m. How much work is done on the book?

Working Space

Answer on Page 65

The work-energy theorem (or work-energy principle) states that the net work done on an object is equal to the **change in its energy**. In other words, if you do work on an object, you are changing its energy. This is derived from Newton's second law of motion, covered in Chapter 4.

$$W = \Delta E$$

Work is how energy is transferred from one thing to another. When you push the car, you also burn sugars (energy of the body) in your blood. That energy is then transferred to the car after it has been pushed uphill.

Thus, we measure the energy something consumes or generates in units of work: joules, kilowatt-hours, horsepower-hours, foot-pounds, BTUs (British Thermal Unit), and calories.

Let's go over a few different forms that energy can take.

5.1 Forms of Energy

In this section we are going to learn about several different types of energy:

- Heat
- Electricity
- Chemical Energy
- Kinetic Energy
- Gravitational Potential Energy

There are also other forms of energy such as spring potential energy, which we will cover in the oscillations chapter.

5.1.1 Heat

When you heat something, you are transferring energy to it. The BTU is a common unit for heat. One BTU is the amount of heat required to raise the temperature of one pound of water by one degree. One BTU is about 1,055 joules. In fact, when you buy and sell natural gas as fuel, it is priced by the BTU.

When we talk about heat energy, we will commonly be talking about *friction*. When a force does frictional work, it is output onto as a change in heat energy within the system.

5.1.2 Electricity

Electricity is the movement of electrons. When you push electrons through a space that resists their passage (like a light bulb), energy is transferred from the power source (like a battery) into the source of the resistance.

Let's say your lightbulb consumes 60 *watts* of electricity, and you leave it on for 24 hours. We would say that you have consumed 1.44 kilowatt hours, or 3,600,000 joules. This comes into play when rating house electricity systems, lightbulbs, or electric cars. The electric energy stored in a battery of an electric car gets converted to kinetic energy and thermal energy taken out by friction.

5.1.3 Chemical Energy

As mentioned earlier, some chemical reactions consume energy and some produce energy. This means energy can be stored in the structure of a molecule. When a plant uses photosynthesis to rearrange water and carbon dioxide into a sugar molecule, it converts the energy in the sunlight (solar energy) into chemical energy. Remember that photosynthesis is a process that consumes energy. Therefore, the sugar molecule has more chemical energy than the carbon dioxide and water molecules that were used in its creation.

Recall the balanced chemical equation for photosynthesis is:

$$6CO_2 + 6H_2O + \text{energy} \rightarrow C_6H_{12}O_6 + 6O_2$$

In our diet, we measure this energy in *kilocalories*. A calorie is the energy necessary to raise one gram of water one degree Celsius, and is about 4.19 joules. This is a very small unit. An apple has about 100,000 calories (100 kilocalories), so people working with food started measuring everything in kilocalories.

(You may find it helpful that the chapter on unit conversions appears earlier in this text, so that the calorie-joule relationship can be placed in proper context.)

Here is where things get tricky: People who work with food got tired of saying “kilocalories”, so they just started using “Calorie” to mean 1,000 calories. This has created a great deal of confusion over the years. So if the C is capitalized, “Calorie” probably means kilocalorie.

5.1.4 Kinetic Energy

A mass in motion has energy. For example, if you are in a moving car and you slam on the brakes, the energy from the motion of the car is converted into heat in the brake pads and under the tires. This stored energy of motion is known as *kinetic energy*.

But how much energy does a moving object actually have?

Formula for Kinetic Energy

$$KE = \frac{1}{2}mv^2$$

where KE is the kinetic energy in joules, m is the mass in kilograms, and v is the speed in meters per second.

This equation tells us two important things:

- Kinetic energy increases *linearly* with mass. A heavier object has more energy at the same speed.
- Kinetic energy increases *quadratically* with velocity. This means that doubling the speed results in *four times* the kinetic energy.

Why does the velocity term appear squared? One way to understand this is to consider the work required to accelerate an object. To speed something up, a force must act over a distance. As the object moves faster, each additional meter of acceleration takes less time but still increases the kinetic energy significantly. The mathematical result of this relationship is the v^2 dependence. Kinetic energy ultimately tells us how powerful an object is. Real life calculations involve calculating car crash preventions, motor speeds, rocket propulsions, battery requirements, and gyroscopic forces.

Exercise 11 Kinetic and Thermal Energy

A $1,200$ kg car traveling at 22m/s comes to a stop using its brakes. Assume all the initial kinetic energy is converted into thermal energy in the brakes and tires.

(a) How much thermal energy, Q is produced?

(b) If the break system has a 15 kg and a specific heat of $c = 460(\text{J/kg}°\text{C})$, estimate the temperature increase of the brakes.

Also, you are given that the change in heat, $\Delta T = \frac{Q}{m_{brake}c}$.

Working Space

Answer on Page 65

5.1.5 Gravitational Potential Energy

When you lift something heavy onto a shelf, you are giving it *potential energy*. The amount of energy that you transferred to it is proportional to its weight and the height that you lifted it.

Formula for Gravitational Potential Energy

The formula for gravitational potentional energy is

$$E = mgh$$

where E is the energy in joules, m is the mass of the object you lifted, g is acceleration due to gravity, and h is the height that you lifted it.

On earth, then, gravitational potential energy is given by

$$E = (9.8)mh$$

since objects in free-fall near Earth's surface accelerate at $9.8 m/s^2$.

There are various kinds of potential energy. For example, when you draw a bow in order to fire an arrow, you have given that bow potential energy. When you release it, the potential energy is transferred to the arrow, which expresses it as kinetic energy. When you compress a spring, the energy put into the spring is proportional to the spring constant and distance it is compressed, $\frac{1}{2}kx^2$. We will dive deeper into this in the springs and oscillations chapter.

5.2 Conservation of Energy

The first law of thermodynamics says "Energy is neither created nor destroyed."

Energy can change forms. Your cells consume chemical energy to give gravitational potential energy to a car you push up a hill. A falling bject converts potential energy into kinetic energy, causing it to speed up. However, the total amount of energy in a closed system stays constant.

The sum of mechanical energy (usually Kinetic and Potential) is always constant:

$$W_{net,ext} = \Delta E_{sys} = \Delta K + \Delta U + \Delta E_{th}$$

Choosing a specific system or environment will benefit your problem, especially if the net external work done is 0. Problems that involve energy transformation are similar to accounting problems, the work before and after must be equal, and must equal some constant net value.

Exercise 12 The Energy of Falling

Working Space

A 5 kg cannonball falls off the top of a 3 meter ladder. As it falls, its gravitational potential energy is converted into kinetic energy. How fast is the cannonball traveling just before it hits the floor?

Answer on Page 66

Exercise 13 Frictional Losses

Working Space

A 4.0 kg block is released from rest at the top of a rough incline of height 2.5 m. By the time it reaches the bottom, its speed is 4.2 m/s.

Determine the total mechanical energy lost to thermal energy.

You may use 10 m/s^2 for g

Answer on Page 66

Exercise 14 Blast Off!

A rocket sled of mass 150 kg uses a small fuel-powered booster that releases 1.2×10^5 J of chemical energy. During the burn, 65% of the energy is converted into useful kinetic energy for the sled, the rest is lost as heat or sound. Assume the sled travels on frictionless, smooth surface.

Calculate the final speed of the rocket sled.

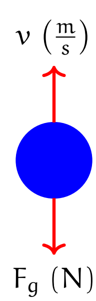
Working Space

Answer on Page 66

5.3 Work and Kinetic Energy

As stated above, the work-energy theorem tells us that the change in an object's kinetic energy is equal to the work done on that object. For now, we will only consider examples where the force and the direction of motion are parallel or perpendicular. When you learn about vectors, we will expand this to include forces that are skew to the direction of motion.

Consider what happens when you toss a ball in the air: once the ball leaves your hand, the only force acting on it is gravity. Initially, the ball is moving upwards while gravity points downwards:



Intuitively, we know that the ball will slow down (lose kinetic energy) as it moves upwards:

$$\Delta \text{KE} < 0$$

Since $W = \Delta\text{KE}$, we also know that gravity must be doing *negative work*. Whenever the direction of the force is opposite the direction of the motion, the work done by that force is negative.

Exercise 15 A ball thrown upwards

If the ball has a mass of 0.5 kg, how much kinetic energy does it lose as it moves upwards by 1 m?

Working Space

Answer on Page 67

Exercise 16 How far will you slide?

You are playing softball and have to slide into home. If you sprint at a maximum of 10 m/s and the force of friction between you and the ground is 0.3 times your weight, how far from the base can you start your slide and still reach home?

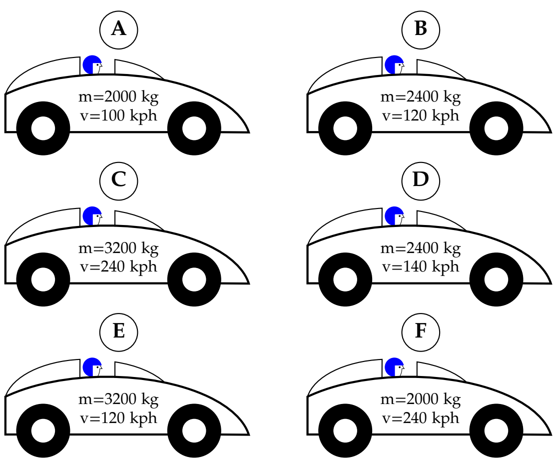
Working Space

Answer on Page 67

Exercise 17 Ranking Stopping Force

Working Space

In drag racing, cars can reach speeds of 150 miles per hour (approximately 240 kilometers per hour). In order to be able to stop quickly and safely, drag racing cars are built with parachutes that deploy at the end of the race. Consider a drag race where cars of different masses reach different maximum speeds. There is 100 meters between the finish line and the fence surrounding the race track. If all the race cars deploy their parachutes at the finish line while going their maximum speed, rank the force needed from the parachute to stop each car in the required distance from least to greatest:



Answer on Page 68

5.3.1 Forces that do no work

If the object you are pushing doesn't move. or the applied force is perpendicular to the direction of motion, that force does no work. Let's look at a few examples:

Pushing Against an Immobile Object

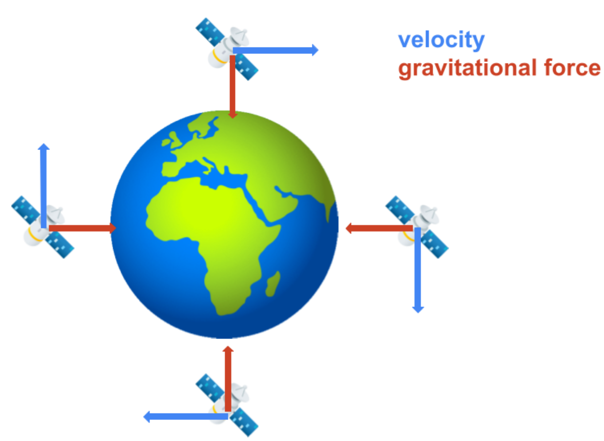
At the beginning of the chapter, we said that when you push on a wall, you don't do any work. Why is this? Well, if the wall is a good wall (that is, strong enough to not be pushed over by a person), the wall won't move while you push on it. This means the distance over which your push is applied is zero, and therefore the work done $(F \cdot d = F \cdot 0 = 0)$ is zero joules.

Walking Across a Room with a Book

Imagine holding a book flat on your hands and walking at a constant velocity. Your hand is applying an upwards force to the book, but the book is moving horizontally. This means the force and direction of motion are *perpendicular*. Recall from the beginning of the chapter than if the force and distance are not parallel, then the work is given by $W = F \cdot d \cos(\theta)$. (When the vectors are parallel, $\theta = 0$ and $\cos(\theta) = 1$, while when the vectors point in opposite directions, $\theta = 180°$ and $\cos(\theta) = -1$.) When the vectors are perpendicular, then $\theta = 90°$ and $\cos(\theta) = 0$. Therefore, $W = 0$ as well and the upward force of your hands does no net work.

Circular Motion

We will discuss circular motion further in a subsequent chapter. For now, know that constant-speed circular motion is caused by a constant-magnitude force that always points to the center of the circle the object is moving in. For example, you can take a weight on the end of a string and spin it. The tension in the string spins the weight, and the string always points from the object to your hand (the center of the weight's circular path). For a satellite, that force is gravitational attraction to the Earth.



As a result, the force changes the *direction*, but not the *magnitude* of the satellite's velocity. Let's re-examine the equation for kinetic energy:

$$\mathsf{KE} = \frac{1}{2}mv^2$$

Since the velocity is squared, the direction of motion doesn't affect the kinetic energy (a ball moving at 5 m/s upwards has the same kinetic energy as if the ball were moving at 5 m/s downwards). So, a force that causes circular motion doesn't change a circling object's kinetic energy, and therefore does no work (as expected when force and direction of motion are perpendicular)!

5.4 Efficiency and Power

Although energy is always conserved as it moves through different forms, scientists aren't always that good at controlling it.

In terms of an equation, efficiency is the ratio of the useful energy output to the total energy input. It is usually expressed as a percentage.

Formula for Efficiency

$$\text{Efficiency} = \frac{\text{Useful Energy Output}}{\text{Total Energy Input}} \times 100\%$$

where the useful energy output is the energy that is actually used to do work or complete a task, and the total energy input is the total energy consumed by the system.

A machine is considered 100% efficient only if all the input work is converted into useful output work, with no energy lost to heat, friction, or sound. 100% efficient process don't exist in real-life: every process loses some useful energy to heat.

For example, when a car engine consumes the chemical energy in gasoline, only about 20% of the energy consumed is used to turn the wheels. Most of the energy is actually lost as heat. If you run a car for a while, the engine gets very hot, as does the exhaust coming from the tailpipe.

A human is about 25% efficient. Most of the loss is in the heat produced during the chemical reactions that turns food into motion.

In general, if you are trying to increase efficiency in any system, the solution is usually easy to identify by the heat that is produced. Reduce the heat, increase the efficiency.

Light bulbs are an interesting case. To get the same amount of light of a 60 watt incandescent bulb, you can use an 8 watt LED or a 16 watt fluorescent light. This is why we say that the LED light is much more efficient. If you run both, the incandescent bulb will consume 1.44 kilowatt-hours; the LED will consume only 0.192 kilowatt-hours.

In addition to light, the incandescent bulb is producing a lot of heat. If it is inside your house, what happens to the heat? It warms your house.

In the winter, when you want light and heat, the incandescent bulb is 100% efficient!

Of course, this also means the reverse is true. In the summer, if you are running the air conditioner to cool down your house, the incandescent bulb is worse than just "inefficient at making light" — it is actually counteracting the air conditioner!

Exercise 18 Unit Costs

A machine is rated to produce 500 bolts per hour, but in actual operation it runs at only 82% efficiency due to loss and scraps. The machine requires $24 of electricity per hour to run.

(a) What is the actual output of bolts per hour?

(b) What is the unit cost (cost per bolt) of the production?

Working Space

Answer on Page 68

Another measure of Work and Energy is *Power*. Power is the rate at which work gets done (or energy gets transferred). Suppose I do work in 1000 J of work in 10 minutes but you do the same amount of work in 5 minutes. Because you did the same amount of work but in a quick amount of time, you were more *powerful*.

Formula for Power

$$\text{Power} = \frac{\text{Work}}{\text{Time}} \implies P = \frac{W}{t}$$

Power is represented by the unit watt (W), defined as one joule per second (J/s).

Since power is a rate, we can also express it using calculus:

$$P = \frac{dW}{dt}$$

Equivallently, for work done by constant forces, this simplifies to $P = \frac{F \cdot r}{t} = F \cdot v$, since

Power is measured in the unit **watt**, where one watt is one Joule per second: $1W = 1\,J/s$

Exercise 19 Hydraulic Piston Problem

A hydraulic piston pushes a crate horizontally across the floor with a constant force of 450 N. During operation, the piston moves the crate 1.8 meters in 0.75 seconds.

(a) How much work does the piston do on the crate?

(b) What is the power output of the piston during this motion?

Working Space

Answer on Page 68

Exercise 20 Spring Power

A compressed spring launches a small block. The spring delivers 120 W of power while releasing its stored energy over 0.40 seconds. The spring constant is $k = 800$ N/m.

How far was the spring compressed?

Working Space

Answer on Page 69

Exercise 21 Elevator Lift

An elevator motor lifts a 750 kg cabin vertically upward a distance of 18 m in 12 s. The motor consumes electrical energy at a rate of 18 kW.

(a) Compute the total electrical energy consumed during the lift. Note that 1 kW = 1000 watts = 1000 J/s

(b) Determine the gravitational potential energy gained by the elevator.

(c) Calculate the efficiency of the motor during this lift.

Working Space

Answer on Page 69

Chapter 6

Simple Machines

As mentioned earlier, physicists define work as the force applied times the distance over which it is applied. For example, if you push your car 100 meters with a force of 17 newtons, you have done 1700 joules of work.

Humans have long needed to move heavy objects, so many centuries ago, we developed simple machines to reduce the amount of force necessary to perform such tasks. These include:

- Levers
- Pulleys
- Inclined planes (ramps)
- Screws
- Gears
- Hydraulics
- Wedges

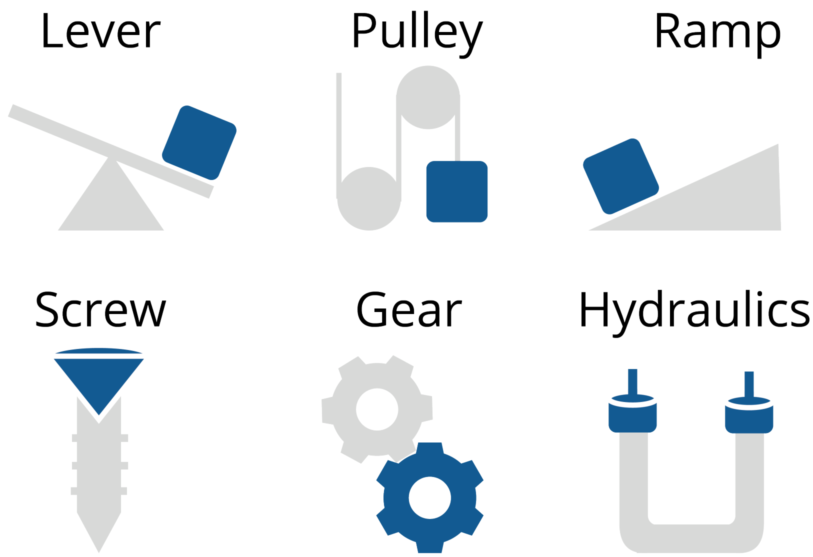


Figure 6.1: The 6 main simple machines: Levers, Pulleys, Ramps, Hydraulics, Screws, and Gears.

While these machines can reduce the force needed, they do not change the total amount of work that must be done. For instance, if the force is reduced by a factor of three, the distance over which the force must be applied *increases* by the same factor.

6.1 Mechanical Advantage

Mechanical advantage is the ratio between the force output by the machine and the force the user puts into the machine:

$$MA = \frac{F_{out}}{F_{in}} = \frac{\text{Load}}{\text{Effort}}$$

Since the input force is *applied* to the simple machine, sometimes the input force is called an applied force and abbreviated as F_a. For example, you only need to apply a relatively little force to your car's brakes in order for the hydraulic braking system to apply enough force to your tires to stop them spinning (we'll examine this further below).

6.1.1 What does it mean to work hard?

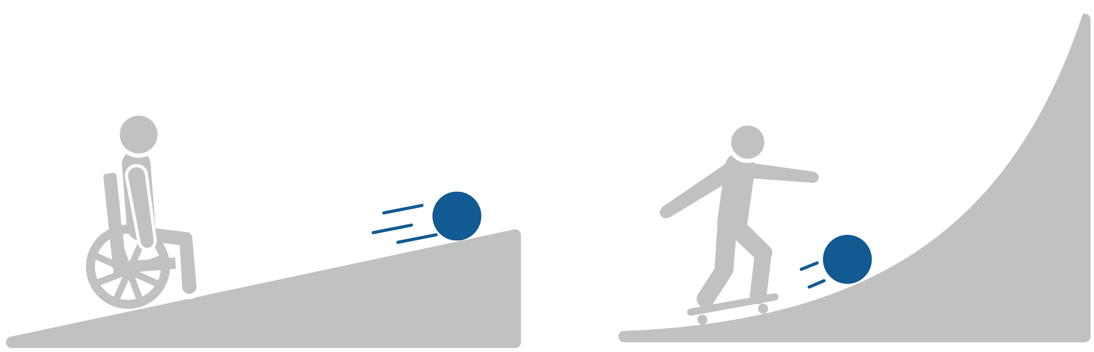
Humans use simple machines to "make work easier", but what does this mean in a physics sense? Does using a machine actually decrease the amount of work the user has to do?

When we say a task is easier, we usually mean *we have to apply less force*. You might say that it is "less work" to push something up a shallow incline than up a steep incline. But does the person pushing *actually do less work* (in a physics sense), or does that work simply require a smaller force? We'll answer this question by examining the physics of incline planes below, and the results will be true for all simple machines.

Ideally we want a higher Mechanical Advantage, which means we are putting in less input force for a greater output force.

6.2 Inclined Planes

Inclined planes, or ramps, allow you to roll or slide objects to a higher level. Steeper ramps require less mechanical advantage. For instance, it is much easier to roll a ball up a wheelchair ramp than a skateboard ramp.

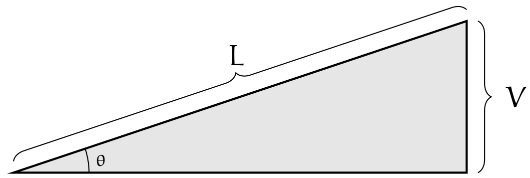


Assuming the incline has a constant steepness, the mechanical advantage is equal to the ratio of the length of the inclined plane to the height it rises.

If friction is neglected, the force required to push a weight up the inclined plane is given by:

$$F_A = \frac{V}{L} F_g \tag{6.1}$$

where F_A is the applied force, L is the length of the inclined plane, V is the vertical rise, and F_g is the gravitational force acting on the mass.



(We will discuss sine function later, but in case you're familiar with it, note that:

$$\frac{V}{L} = \sin\theta \tag{6.2}$$

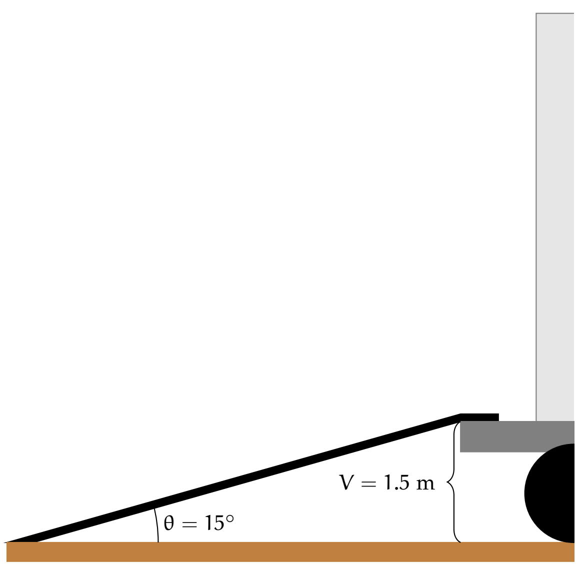
where θ is the angle between the inclined plane and the horizontal surface.)

Let's compare the force needed and work done when pushing a load up a ramp versus just lifting it vertically. Consider a family on moving day: there's a hand trolley loaded with 200 N (about 45 pounds) of boxes. If the bed of the moving truck is 1.5 m high, how much work would it take to lift the boxes straight up into the truck? What about with a ramp?

First, let's look at how much force and work is needed if you were to lift the entire 200 N load straight up into the air. You'd need to apply 200 N of force upwards for a distance of 1.5 m:

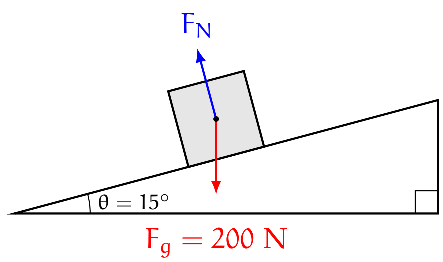
$$W = F \cdot d = (200\ \text{N})\,(1.5\ \text{m}) = 300\ \text{J}$$

So, without a ramp, you would have to apply 200 N and do 300 J of work. Suppose your moving truck comes with a ramp that has an incline of 15 degrees:

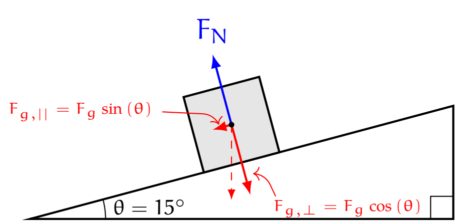


Since $\sin(\theta) = V/L$, we know that $L = V/\sin(\theta)$. You can use a calculator or search engine to find that the sine of $15° \approx 0.26$. Therefore, the length of the ramp is approximately 5.8 meters. How much force does it take to move the load of boxes up the ramp? Intuitively, we know it is less force. We can use a *free body diagram* to determine the minimum force needed to push the box up the ramp. (A free body diagram is a simplified model showing all the forces acting on an object. You'll learn to create and use your own free body diagrams in a later chapter. For now, just follow along.)

Before you push it, there are two forces acting on the loaded hand trolley: its weight (F_g) and the normal force between the trolley and the ramp (F_N):

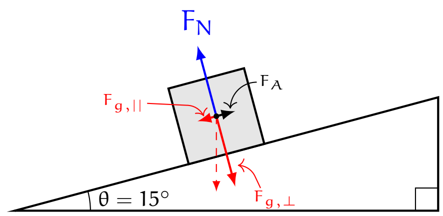


Notice that the normal force is perpendicular to the ramp! We want to know how much force it takes to push the load up the ramp, so we will "split" the weight force vector into two parts: one part parallel to the ramp ($F_{g,||}$) and one part perpendicular ($F_{g,\perp}$):



We did this by treating the weight vector as the hypotenuse of a right triangle with legs perpendicular and parallel to the ramp. You'll learn how to do this and why it works in the chapter on vectors. For now, just trust that the part of the hand trolley's weight that is perpendicular to the ramp is $F_g \cos(\theta)$ and the part that is parallel to the ramp is $F_g \sin(\theta)$.

What force do you need to overcome to push the hand trolley up the ramp? Just the part of the weight that is parallel to the ramp! You'll need to apply an equal force in the opposite direction (up the ramp) to move the hand trolley:



So, we know that you are pushing with an applied force of $F_A = F_{g,||} = F_g \sin(\theta)$. Therefore, the work you would do pushing the hand trolley up the ramp is:

$$F_A \cdot L = F_g \sin(\theta) \cdot \left(\frac{V}{\sin(\theta)}\right) = F_g \cdot V = 300 \text{ J}$$

Therefore, when using a ramp, you still perform the same amount of work! This is a key property of simple machines: *the work done doesn't change*.

So what makes it "easier" to use a ramp to lift the hand trolley? The fact that you need to apply less *force* to move the hand trolley ($F_A < F_g$). Now, let's look at the mechanical advantage of the ramp. In this case, the mechanical advantage is given by:

$$MA = \frac{F_g}{F_A}$$

Substituting for F_A, we see that:

$$MA = \frac{F_g}{F_g \sin(\theta)} = \frac{1}{\frac{V}{L}} = \frac{L}{V}$$

So for a ramp whose length is L and vertical rise is V, the mechanical advantage is equal to the length divided by the rise.

Ramps

For a ramp, the mechanical advantage is equal to $\frac{L}{V}$ and the force needed to push an object with weight W up the ramp is given by $W \cdot \frac{V}{L} = W \cdot \sin(\theta)$, where L is the length of the ramp, V is the vertical rise of the ramp, and θ is the angle the ramp forms with the (horizontal) ground.

We have a whole inclined planes chapter coming up, but this was just a brief overview of their function as a mechanic

Exercise 22 Ramp

You need to lift a barrel of oil with a mass of 136 kilograms. You can apply a force of up to 300 newtons. You need to get the barrel onto a platform that is 2 meters high. What is the shortest length of inclined plane you can use?

Working Space

Answer on Page 69

6.3 Levers

A lever pivots on a fulcrum. To decrease the necessary force, the load is placed closer to the fulcrum than where the force is applied.

Physicists also discuss the concept of *torque* created by a force. When you apply force to a lever, the torque is the product of the force you exert and the distance from the point of rotation.

Torque is typically measured in newton-meters (N·m).

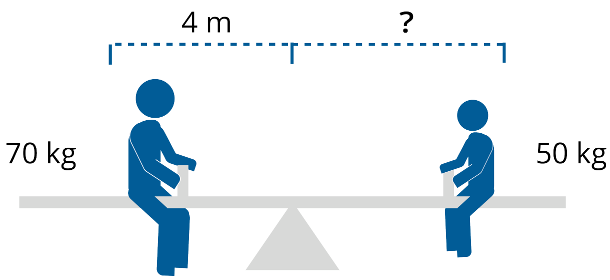
To balance two torques, the products of force and distance must be equal. Thus, assuming the forces are applied in the correct direction, the equation becomes:

$$R_L F_g = R_A F_A \tag{6.3}$$

where R_L and R_A represent the distances from the fulcrum to where the load's weight and the applied force are exerted, respectively, and F_g and F_A are the magnitudes of the forces.

Exercise 23 **Lever**

Paul, whose mass is 70 kilograms, sits on a see-saw 4 meters from the fulcrum. Jan, whose mass is 50 kilograms, wishes to balance the see-saw. How far should Jan sit from the fulcrum?



Working Space

Answer on Page 70

6.4 Gears

Gears are rotating parts of machines that transmit torque or other types of rotational motion through intertwined teeth. Often gears are meshed together; two or more meshed gears are refered to as a gear train.

Gears have teeth that mesh with each other. When you apply torque to one gear, it transfers torque to the other. The resulting torque is increased or decreased depending on the ratio of the number of teeth on the gears. We will go in depth on torque in a future chapter, but know that torque depends on the radius of the gear, and the radius of a gear is proportional to the number of teeth, the tooth count directly controls the torque.

6.4.1 Involute Gears

Gear teeth have a very unique shape. The vast majority of modern gears use what is known as an *involute gear profile*. An involute gear is one whose tooth shape is derived from the *involute of a circle*. This curve is generated by imagining a fixed string being unwrapped from the circumference of a circle. The path traced by the end of the string forms the involute.

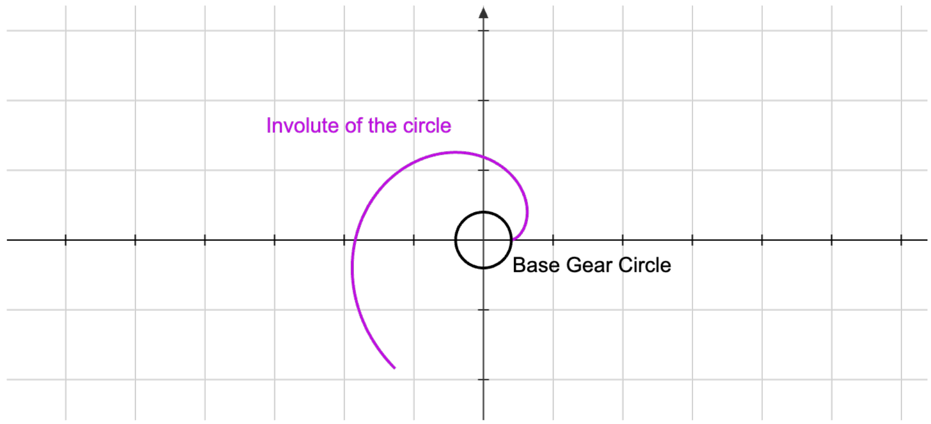


Figure 6.2: The involute of a circle shown. The purple line is revolved for 4π radians or 2 revolutions (for clarity).

Although the involute curve resembles a spiral, it is not an Archimedean spiral. Instead, its geometry is defined by the constant unwrapping of the string, which gives the involute several important mechanical properties.

The primary advantage of the involute profile shape is that it ensures smooth and reliable torque transfer between two or more meshing gears. When two involute gears engage, their teeth make contact at a single point that moves along a straight line known as the **line of action**. This line is tangent to the base circles of both gears. As the gears rotate, the point of contact travels along this line while maintaining a constant pressure angle, which allows the gears to transmit motion at a constant speed ratio even if the center distance between them varies slightly.

Some more gear construction terminology:

The pressure angle The acute angle between the line of action and a normal to the line connecting the gear centers. May vary depending on the involute shape, but most commonly 20°.

Addendum The difference between the pitch circle and its tooth tip circle.

Dedendum The difference between the radius of the pitch circle of a gear and its root circle

Tooth Height The distance between its root circle and the tip are called the tooth height (h).

Total Height of Tooth The total height of the gear is the sum of the addendum (ha = 1.00m) and the dedendum (hf = 1.25m).

FIXME Possible gear diagram in tikz wth labeled everything

Figure 6.3: Two gears meshing together diagram (not directly conjoined for clarity).

For part of the motion, Gear One pushes Gear Two down. Once the contact point passes the midpoint of the line of action, Gear One pushes up on Gear Two. This causes a constant transfer of torque.

FIXME Diagram of this

A parametric function, defined by $x(\theta)$ and $y(\theta)$ is given by

$$\begin{bmatrix} x(\theta) \\ y(\theta) \end{bmatrix} = r \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \end{bmatrix} + r\theta \begin{bmatrix} \sin(\theta) \\ -\cos(\theta) \end{bmatrix}$$

For gears, a large torque makes it easier to start rotating an object or to continue rotating it against resistance, such as climbing a hill or accelerating a heavy load.

Power, on the other hand, measures how quickly work is being done. In rotational motion, power depends on both torque and rotational speed. A system can produce high torque at low speed, or lower torque at high speed, and still deliver the same power.

Gears allow a machine to adjust this balance between torque and speed without significantly changing the power being transmitted (neglecting losses). When gears increase torque, they reduce rotational speed, and visa versa.

A familiar example is riding a bicycle up a hill. When climbing, it becomes difficult to pedal in a high gear because the torque required at the pedals is large. Shifting to a lower gear increases the torque applied to the rear wheel for the same pedaling effort, making it easier to climb. However, the pedals must turn more times for each rotation of the wheel, so the bicycle moves more slowly. The rider's power output remains roughly the same, but it is delivered as higher torque at lower speed.

The same principle applies to the gears in a car. When starting from rest or climbing a steep incline, a low gear is used to provide high torque at the wheels, allowing the vehicle to overcome inertia and gravity. At highway speeds, higher gears are used to reduce torque and increase speed, improving efficiency and reducing engine wear. In both cases, the engine produces power, and the transmission adjusts how that power is distributed between torque and rotational speed.

In mechanical systems, gears therefore do not create power; instead, they shape how power is delivered.

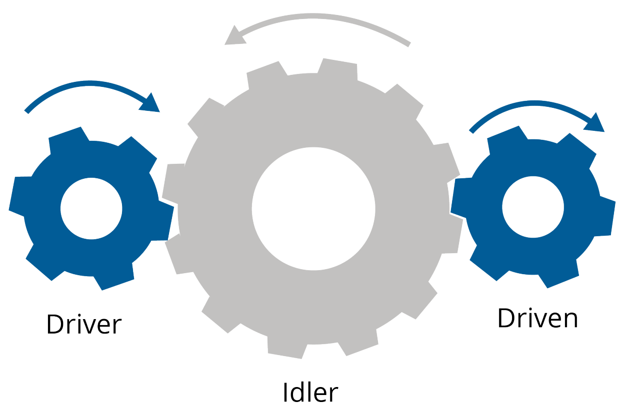


Figure 6.4: .

6.4.2 Teeth

If N_A is the number of teeth on the gear you are turning with a torque of T_A, and N_L is the number of teeth on the gear it is turning, the resulting torque is:

$$T_L = \frac{N_A}{N_L} T_A$$

Exercise 24 Gears

In a bicycle, the goal is not always to gain mechanical advantage, but to spin the pedals slower while applying more force.

You like to pedal your bike at 70 revolutions per minute. The chainring connected to your pedals has 53 teeth. The circumference of your tire is 2.2 meters. You want to ride at 583 meters per minute.

How many teeth should the rear sprocket have?

Working Space

Answer on Page 70

6.5 Hydraulics

In a hydraulic system, such as a car's braking system, you exert force on a piston filled with fluid. The fluid transmits this pressure into another cylinder, where it pushes yet another piston that moves the load. The pressure at each end of the hydraulic system must be the same.

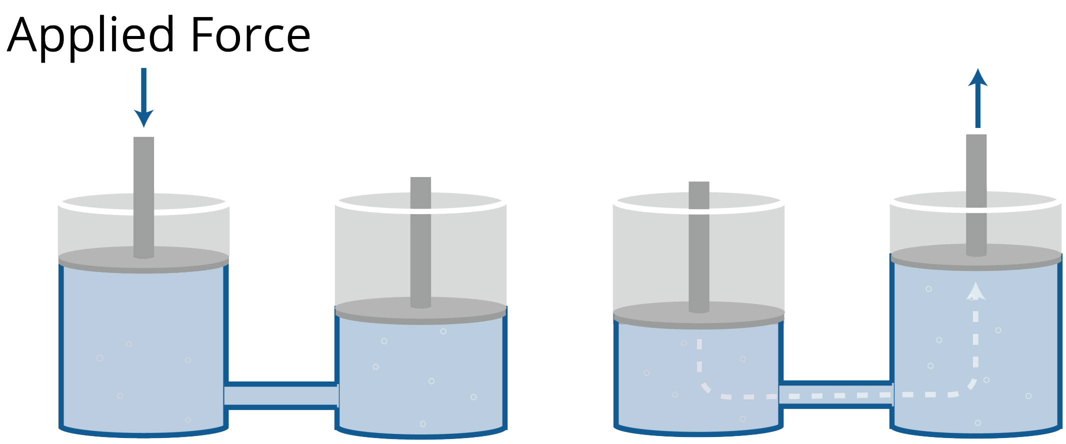


Figure 6.5: A diagram showing the transfer of fluid from one container to the other.

Pressure is force applied to an area; it is calculated by dividing the force by the area. The pressure in the fluid is typically measured in pascals (Pa), which is equivalent to N/m^2. We will use pascals for this calculation.

To calculate the pressure you create, divide the force applied F_a by the area of the piston head A. To determine the force on the other piston, multiply the pressure by the area of the second piston.

$$P = \frac{F_{a_1}}{A_1} = \frac{F_{a_2}}{A_2} \tag{6.4}$$

Exercise 25 Hydraulics

Your car has disc brakes. When you apply 2,500,000 pascals of pressure to the brake fluid, the car stops quickly. As the car designer, you want this to require only 12 newtons of force from the driver's foot.

What should the radius of the master cylinder (the piston the driver pushes) be?

Working Space

Answer on Page 70

6.6 Pulleys

Pulleys are anything that changes the direction of a force, typically by using a wheel and a rope. A single pulley can make things easier by allowing you to pull down instead of pushing up. This lets you use your body weight to help you pull rather than just your arm strength.

By attaching multiple pulleys, you can do more and actually reduce the force needed to lift a load. For example, if you use two pulleys, you can reduce the force needed to lift a load by half. However, you will have to pull twice as much rope to lift the load the same distance.

Each additional pulley you add to the system adds another segment of rope that supports the load. The mechanical advantage of a pulley system is equal to the number of rope segments supporting the load.

$$MA = N$$

where N is the number of rope segments supporting the load.

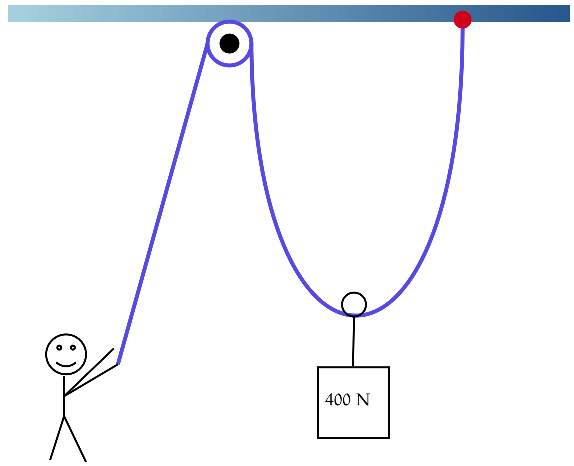


Figure 6.6: A 400 N weight being distributed by one pulley.

Here, a rope is carrying a 400 N weight. Attached to the weight is a movable pulley, while the top pulley is fixed to the ceiling. Assuming an ideal (massless) rope and frictionless pulleys, the tension is the same everywhere in the rope. The movable pulley is supported by two rope segments, so the upward force on the load is 2T. For the load to be held or lifted at constant speed,

$$2T = 400 \text{ N} \quad \Rightarrow \quad T = 200 \text{ N}$$

Assuming an ideal (massless) rope and frictionless pulleys, the tension is the same everywhere in the rope, so there is 200 N throughout the rope. Since there are two ropes supporting the load, the Mechanical Advantage is 2.

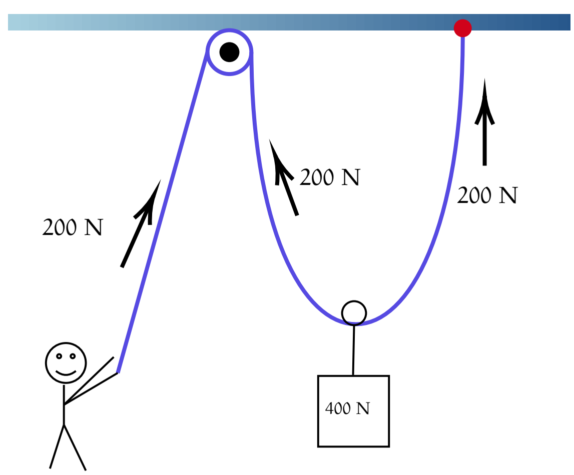


Figure 6.7: The tension in the rope is 200 N.

However, this does come at a cost! The weight of the load is 400 N so the pulley system must provide a total upward force of 400 N on the load. However, both sections of rope (although technically the same rope) pull at 200 N for a total of 400 N.

The work done, however, is still the same! When the person pulls the rope down by 2 m, each supporting rope segment shortens by 1 m, so the load rises by 1 m. This is all ignoring friction and assuming constant speed. Work done by the person:

$$W = F \times d = 200\,\text{N} \times 2\,\text{m} = 400\,\text{J}$$

Work gained by the load:

$$W = 400\,\text{N} \times 1\,\text{m} = 400\,\text{J}$$

This satisfies our work equation

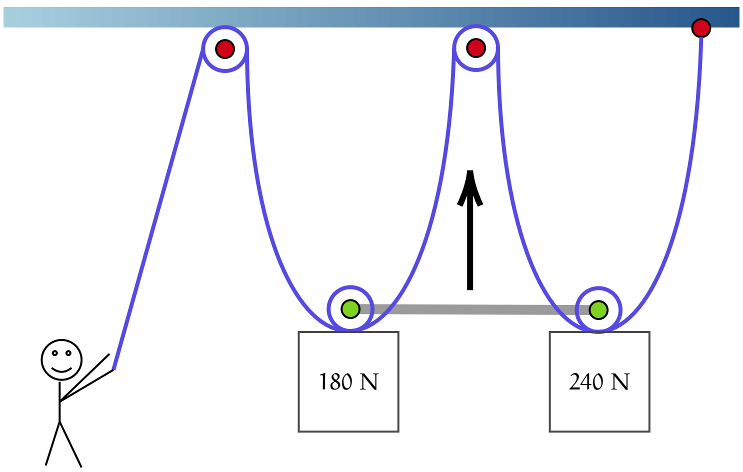
$$\text{Work}_{\text{in}} = \text{Work}_{\text{out}} \tag{6.5}$$

Note that we can then rewrite our mechanical advantage as a proportion of distance:

$$\text{MA} = \frac{d_{\text{effort}}}{d_{\text{load}}} \tag{6.6}$$

Exercise 26 Pulleys

Working Space



A person is pulling 2 weights tensioned by 4 pulleys. There are 2 fixed pulleys (shown in red) and 2 movable pulleys (shown in green). The two movable pulleys are fixed such that they move together vertically. Calculate the input force that is needed to begin lifting the weights.

Answer on Page 71

6.7 Wedges

A wedge is a triangular shaped simple machine that converts forces from one direction to another. Often, it is from a vertical direction to a horizontal one.

Similar to an inclined plane, wedges have steep angles to manipulate the properties of triangles. The force applied to the top portion of a wedge is transfered acting perpendicular to the sloped sides of the wedge.

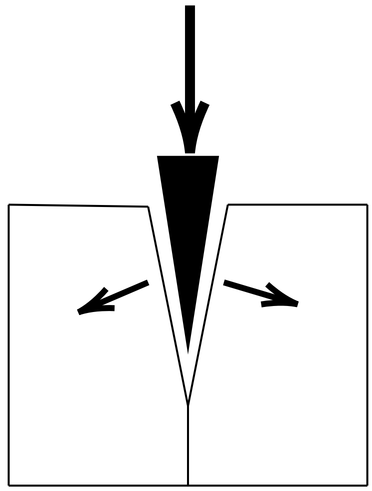


Figure 6.8: A wedge splitting a block of wood, with the forces acting perpendicular.

Similar to the inclined plane, the Mechanical Advantage is the proportion of the vertical rise or width, and the length of the sloped portion of the triangle.

$$MA = \frac{L}{V} \tag{6.7}$$

The smaller the angle of a wedge, the greater the ratio of the length of its slope to its width, and the more mechanical advantage it will yield. This is a concept that comes directly from trigonometry.

Exercise 27 Wedges 1

An axe is used to split wood. The length of the wedge is 12 cm and the thickness is 3 cm. Calculate the mechanical advantage.

Working Space

Answer on Page 71

Exercise 28 Wedges 2

Explain why a flathead-screwdriver tip is a poor option for cutting a steak compared to a knife.

Working Space

Answer on Page 71

Appendix A

Answers to Exercises

Answer to Exercise 1 (on page 5)

1. $12.011 \text{ amu} + 4(1.008 \text{ amu}) = 16.043 \text{ amu}$
2. $63.546 \text{ amu} + 32.06 \text{ amu} + 4(15.999 \text{ amu}) = 159.602 \text{ amu}$
3. $6(12.011 \text{ amu}) + 12(1.008 \text{ amu}) + 6(15.999) \text{ amu} = 180.156 \text{ amu}$

Answer to Exercise 2 (on page 7)

Substance	num. of particles	num. of moles	grams
$NaHCO_3$	2.509×10^{23}	0.4166	35.00
HCl	7.53×10^{23}	1.25	45.58
KH_2PO_4	12.5×10^{24}	20.8	2820

$$\frac{35.00 \text{ g NaHCO}_3}{} \times \frac{1 \text{ mol NaHCO}_3}{84.007 \text{ g NaHCO}_3} = 0.4166 \text{ mol NaHCO}_3$$

$$\frac{0.4166 \text{ mol NaHCO}_3}{} \times \frac{6.02214076 \times 10^{23} \text{ molec NaHCO}_3}{1 \text{ mol NaHCO}_3} = 2.509 \times 10^{23} \text{ molec NaHCO}_3$$

$$\frac{1.25 \text{ mol HCl}}{} \times \frac{36.46 \text{ g HCl}}{1 \text{ mol HCl}} = 45.58 \text{ g HCl}$$

$$\frac{1.25 \text{ mol HCl}}{} \times \frac{6.02214076 \times 10^{23} \text{ molec HCl}}{1 \text{ mol HCl}} = 7.53 \times 10^{23} \text{ molec HCl}$$

$$\frac{12.5 \times 10^{24} \text{ molec KH}_2\text{PO}_4}{} \times \frac{1 \text{ mol KH}_2\text{PO}_4}{6.02214076 \times 10^{23} \text{ molec KH}_2\text{PO}_4} = 20.8 \text{ mol KH}_2\text{PO}_4$$

$$\frac{20.8 \text{ mol } KH_2PO_4}{} \times \frac{136.086 \text{ g } KH_2PO_4}{1 \text{ mol } KH_2PO_4} = 2820 \text{ g } KH_2PO_4$$

Answer to Exercise 3 (on page 8)

From the last exercise, you know that 1 mole of water weighs 18.01528 grams, meaning 200 grams of water is about 11.1 moles. So you need to burn 11.1 moles of methane.

What does one mole of methane weigh? Using the periodic table: $12.0107 + 4 \times 1.00794 = 16.04246$ grams.

$16.0424 \times 11.10 = 178.1$ grams of methane.

Answer to Exercise 4 (on page 12)

The average hydrogen atom has a mass of 1.00794 atomic mass units.

The average oxygen atom has a mass of 15.9994.

$2 \times 1.00794 + 15.9994 = 18.01528$ atomic mass units.

Answer to Exercise 5 (on page 16)

Element Name	Atomic Symbol	Protons	Most Common Number of Neutrons	Electrons
Francium	Fr	87	136	87
Arsenic	As	33	42	33
Erbium	Er	68	99	68
Cadmium	Cd	48	64	48

Answer to Exercise 6 (on page 20)

Equilibrium will be achieved when the box has displaced 10 kg of water. In other words, when it has displaced 0.01 cubic meters.

The area of the base of the box is 0.12 square meters. So if the box sinks x meters into the water it will displace $0.12x$ cubic meters.

Thus at equilibrium $x = \frac{0.01}{0.12} \approx 0.083$ m. So the box will sink 8.3 cm into the water before reaching equilibrium.

Answer to Exercise 7 (on page 22)

$$p = dgh = (900)(3.721)(5) = 16,744.5 \text{ Pa}$$

Answer to Exercise 8 (on page 25)

$$E_C = (1200)(0.4)(T - 10) = 480T - 4800$$

Total energy stays constant:

$$0 = (12600T - 252000) + (900T - 72000) + (480T - 4800)$$

Solving for T gets you $T = 23.52°$ C.

Answer to Exercise 9 (on page 26)

During the 3 minutes, you want the coffee to give off as much of its heat as possible, so you want to maximize the difference between the temperature of the coffee and the temperature of the room around it.

You wait until the last moment to put the milk in.

Answer to Exercise 10 (on page 28)

In the case, the force you exert must balance the weight of the book, so $F = mg = 2(9.8) = 19.6$N. Since this force is straight upward and the displacement is in the same direction, $W = F \cdot d = 19.6 \cdot 3 = 59.8$ J.

Answer to Exercise 11 (on page 31)

(a) Since all kinetic energy is transferred into thermal energy, we can say:

$$Q = KE = \frac{1}{2}(1200)(22)^2 = 290,400 \text{ J}$$

(b) The rise in heat is given by $\Delta T = \frac{Q}{mc} = \frac{290,400}{15 \times 460} \approx 42^\circ$ C

Answer to Exercise 12 (on page 33)

At the top of the ladder, the cannonball has $(9.8)(5)(3) = 147$ J of potential energy.

At the bottom, the kinetic energy $\frac{1}{2}(5)v^2$ must be equal to 147 joules. So $v^2 = \frac{294}{5}$. This means it is going about 7.7 m/s.

(You may be wondering about air resistance. Yes, a tiny amount of energy is lost to air resistance, but for a dense object moving at these relatively slow speeds, this energy is negligible.)

Answer to Exercise 13 (on page 33)

The initial total gravitational potential energy is $mgh = 4(10)(2.5) = 100$ J. The finial kinetic energy is $\frac{1}{2}mv^2 = \frac{1}{2}(4)(4.2)^2 = 35.28$. So, the rest of the energy must be lost due to friction: $100 - 35.28 = 64.72$ J.

Answer to Exercise 14 (on page 34)

The total useful energy can be calculated as $0.65(1.2 \times 10^5) = 7.8 \times 10^4$ J. Then, we can set the final KE $\frac{1}{2}mv^2 = 7.8 \times 10^4$ and solve for v

$$\begin{aligned}
\frac{1}{2}(150)v^2 &= 7.8 \times 10^4 \\
(150)v^2 &= 15.6 \times 10^4 \\
v^2 &= 1,040 \\
v &\approx 32.249 \text{ m/s}
\end{aligned}$$

Answer to Exercise 15 (on page 35)

The force acting on the ball is its weight, $F_w = mg$, and we will designate this as negative since weight points downwards. Using the work-energy theorem,

$$\Delta KE = F \cdot d = (mg) \cdot d = (0.5kg)\left(-9.8\frac{m}{s^2}\right)(1m) = -4.9J$$

Therefore, the ball loses 4.9 joules of kinetic energy for every 1 meter it moves upwards (the fact it is *losing kinetic energy* is represented by the result being negative). Note that this is also the formula for potential energy, so it gained 4.9 J of potential energy.

Answer to Exercise 16 (on page 35)

$$F_f \cdot d = \Delta KE = KE_f - KE_i$$

You'll reach the maximum distance you can slide when you stop moving, so we will use a final velocity of zero, which means a final kinetic energy of zero:

$$F_f \cdot d = -KE_i = \frac{1}{2}mv^2$$

Since the force of friction is 0.3 times your weight, we know that:

$$F_f = 0.3F_w = 0.3mg$$

Substituting and canceling the mass:

$$(0.3mg) \cdot d = \frac{1}{2}mv^2$$

$$0.3g \cdot d = \frac{1}{2}v^2$$

Since we know g and v, we can solve for d:

$$d = \frac{v^2}{0.6g} = \frac{\left(10\frac{m}{s}\right)^2}{0.6\left(9.8\frac{m}{s^2}\right)} \approx 1.7m$$

So, if you want to reach home base, you should start your slide no more than 1.7 m from home.

Answer to Exercise 17 (on page 36)

Since all the cars need to stop in the same distance, the cars with the greatest kinetic energy will take the most force to stop. Calculating the kinetic energies (we won't change the units from kilometers per hour to meters per second, since we're just comparing the values):

Car	Mass [kg]	Max speed [kph]	KE [kg (kph)^2]
A	2000	100	1×10^7
B	2400	120	1.728×10^7
C	3200	240	9.216×10^7
D	2400	140	2.352×10^7
E	3200	120	2.304×10^7
F	2000	240	5.67×10^7

The correct ranking is A, B, E, D, F, C.

Answer to Exercise 18 (on page 39)

(a) $500 \times 0.82 = 410$ bolts

(b) $\frac{\$24}{410} \approx \$0.06$

Answer to Exercise 19 (on page 40)

(a)

$$W = F \cdot d = 450 \cdot 1.8 = 810 \text{ J}$$

(b)

$$P = \frac{W}{t} = \frac{810}{.75} = 1080 \text{ W}$$

Alternatively,

$$v = \frac{d}{t} = \frac{1.8}{0.75} = 2.4 \text{ m/s}$$
$$P = F \cdot v = 450 \cdot 2.4 = 1080 \text{ W}$$

Answer to Exercise 20 (on page 40)

$$P = \frac{E}{t}$$
$$E = P \cdot t = 120(0.4) = 48 \text{ J} \qquad = 48\text{J}$$

So 48 J of energy is stored into the spring. We can find the spring's PE converted to distance by:

$$E = \frac{1}{2}kx^2$$
$$48 = \frac{1}{2}(800)x^2$$
$$x^2 = \frac{48}{400}$$
$$x = \sqrt{0.12} \approx 0.346 \text{ m}$$

So the spring had to have been compressed 34.6 cm.

Answer to Exercise 21 (on page 41)

(a) Electrical energy input can be calculated as $E = Pt = 18 \text{ kW} \times 1000 \text{ J/s} \times 12 \text{ sec} = 216,000 \text{ J}$.

(b) The elevator travels 18 m upwards, so the total gravitational potential energy is $mgh = 750(9.8)(18) = 132,300 \text{ J}$.

(c) The efficiency can be calculated as $\frac{\text{Useful Output}}{\text{Total Input}} = \frac{132,000}{216,000} \approx 61.1\%$

Answer to Exercise 22 (on page 49)

The weight of the barrel is 136 kg $\times$ 9.8 $\frac{m}{s^2}$ = 1332.8 N.

Let L be the length of the inclined plane. The force needed to push the barrel up is related by:

$$300 \text{ N} = \frac{2 \text{ m}}{L} \times 1332.8 \text{ N}$$

Solving for L, we find $L = \frac{2 \text{ m} \times 1332.8}{300} \approx 8.885$ m.

Answer to Exercise 23 (on page 50)

Paul exerts a force of $70 \text{ kg} \times 9.8 \frac{\text{m}}{\text{s}^2} = 686$ N at a distance of 4 meters from the fulcrum, creating a torque of $686 \text{ N} \times 4 \text{ m} = 2744 \text{ N} \cdot \text{m}$. Jan exerts a force of $50 \text{ kg} \times 9.8 \frac{\text{m}}{\text{s}^2} = 490$ N.

Let r be the distance from the fulcrum to Jan's seat. To balance the torques:

$$490 \text{ N} \times r = 2744 \text{ N} \cdot \text{m}$$

Solving for r, we find $r = \frac{2744}{490} \approx 5.6$ meters.

Answer to Exercise 24 (on page 54)

The equation relating these quantities is:

$$583 = 70 \times 2.2 \times \frac{53}{n}$$

Solving for n, we find $n = 14$ teeth.

Answer to Exercise 25 (on page 56)

We are solving for the radius r of the piston. The area of the piston is πr^2, so the pressure is:

$$\text{Pressure} = \frac{12}{\pi r^2}$$

Setting the pressure equal to 2,500,000 pascals:

$$2,500,000 = \frac{12}{\pi r^2}$$

Solving for r, we find:

$$r = \sqrt{\frac{12}{\pi \times 2.5 \times 10^6}} \approx 0.00124 \text{ meters.}$$

Answer to Exercise 26 (on page 59)

There are 4 segments supporting the net weight total of 420 N. Each segment, then, splits the weight into $\frac{420 \text{ N}}{4} = 105$ N. The input force required to move both weights is 105 N, which moves it 4 m. The output work matches as well: 420 N is moved 1 m.

Answer to Exercise 27 (on page 60)

The mechanical advantage of the wedge is $\frac{12 \text{ cm}}{3 \text{ cm}} = 4$.

Answer to Exercise 28 (on page 61)

A knife cuts better than a screwdriver tip because its edge is much thinner, giving it a greater mechanical advantage. This allows the applied force to be concentrated over a very small area, producing much higher pressure. As a result, less force is needed to cut the material. A screwdriver, on the other hand, has a very thick edge (in comparison), so its pressure and mechanical advantage are much less.

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