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Introduction to the Kontinua Sequence

You are starting a long trip. By the end, you will be a problem solver: someone who uses math, computers, and science to answer questions that nobody has answered yet.

1.1 Why Keep Going?

Problem solvers build the world. Engineers design bridges and phones. Scientists find cures for diseases. Doctors figure out what is making a patient sick. Programmers write the apps on your phone. Architects design buildings, actuaries measure risk for insurance companies, and mathematicians discover ideas no one has seen before. Many of these jobs pay well, and the world needs more people who can do them.

1.2 When It Feels Hard

Sooner or later, you will meet someone who solves problems faster than you. You might think, “I worked hard, and that person is still better. Maybe I should quit.” Do not quit.

Two ideas help. First, problem solving is a muscle. The more you use it, the stronger it gets. “I am not good at this” is really “I am not good at this *yet*.”

Second, you do not need to be the best in the world. You need to be good, and good is within reach of anyone who keeps practicing. Employers, colleges, and your future self will all be glad you did.

1.3 How This Book Works

Each chapter teaches one idea. Read the examples slowly, with a pencil in your hand. Then try the exercises *before* you peek at the answers, which are at the back of the book. The chapters build on each other, so take them in order.

1.4 Where to Start

The physicist Richard Feynman once asked: if almost all scientific knowledge were destroyed, and you could pass just one sentence to the next generation, which sentence would carry the most information in the fewest words?

His answer: “All things are made of atoms—little particles that move around in perpetual motion, attracting each other when they are a little distance apart, but repelling upon being squeezed into one another.”

That is a good place to start. Atoms are so small that even a pinch of anything holds an enormous number of them. Chemists count atoms in *moles*: one mole is about 6.02×10^{23} particles. The molar mass tells you how many grams one mole weighs.

Worked example: counting atoms in a gram of carbon. Carbon has a molar mass of about 12 g/mol. How many atoms are in 1 g of carbon?

Solution. First find the number of moles: $\frac{1 \text{ g}}{12 \text{ g/mol}} \approx 0.083 \text{ mol}$. Then multiply by the number of atoms in a mole:

$$0.083 \times 6.02 \times 10^{23} \approx 5.0 \times 10^{22} \text{ atoms}$$

That is fifty thousand billion billion atoms, in about the mass of a paperclip.

Exercise 1 Counting molecules in a glass of water

A glass holds 250 g of water. Water has a molar mass of about 18 g/mol. How many water molecules are in the glass?

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Answer on Page 63

Exercise 2 Why such big numbers?

One gram of carbon has about 5×10^{22} atoms. Why do numbers this large show up whenever we count atoms?

Working Space

Answer on Page 63

CHAPTER 2

Mass and Weight

Mass is how much stuff is in an object. Weight is how hard gravity pulls on it. Your mass is the same everywhere in the universe. Your weight is not: the Moon's gravity is about one-sixth as strong as Earth's, so on the Moon you would weigh one-sixth as much, even though you are exactly as massive.

2.1 Kilograms versus Pounds

You probably learned that 1 kilogram is about 2.2 pounds. That is only true *on Earth*. In everyday talk, people swap kilograms and pounds, or mass and weight, as if they were the same thing. Scientists do not.

Mass	Weight
measure of <i>amount</i>	measure of <i>force</i>
units include grams, kilograms, milligrams	units include pounds, newtons, dynes
a <i>scalar</i> measurement	a <i>vector</i> measurement
does <i>not</i> depend on location	does depend on location

On Earth, a 1 kg mass weighs 2.2 pounds. On the Moon, that same mass weighs about 0.36 pounds. The “conversion” you learned works everywhere on Earth, because gravity is nearly the same everywhere on Earth. Leave the planet, and it falls apart.

2.1.1 Scalars and Vectors

Mass and weight are different kinds of measurement. A *scalar* is just a size: mass, energy, temperature, and density are all scalars. A *vector* has a size *and* a direction. Force is a vector, so your weight is not just 185 pounds; it is 185 pounds *downward*. Velocity is a vector too: 35 mph east is a different trip than 35 mph west. You will see much more of vectors in the vectors chapter.

2.2 Weight Near Earth's Surface

Gravity pulls on each object in proportion to its mass. That is why we can usually get away with treating mass and weight as the same thing, as long as we stay on Earth.

The weight of an object is the force of gravity on it. When we say, “This potato weighs 1 pound,” we really mean “on Earth.” The same potato weighs about one-sixth of a pound on the Moon (see figure 2.1).

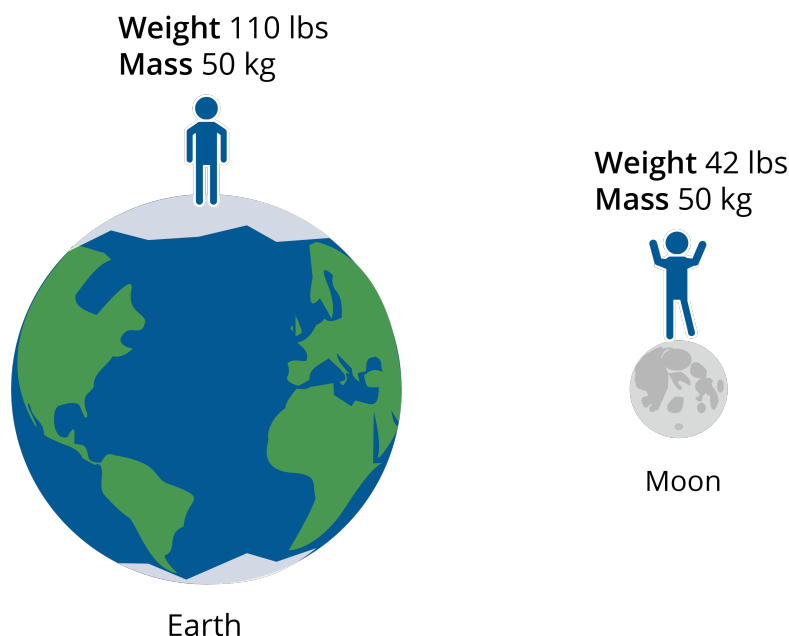


Figure 2.1: Mass is a measure of all the matter in an object. Weight is a measure of the force of gravity on that object. Mass is not location-dependent, while weight is.

Its mass, though, is 0.45 kg anywhere in the universe.

Worked example: an astronaut on the Moon. An astronaut has a mass of 60 kg. On Earth, her weight is $W = mg = (60)(9.8) = 588$ N. On the Moon, where $g = 1.62$ m/s², her weight is $W = (60)(1.62) = 97.2$ N. Her mass is 60 kg in both places.

Where does $g = 9.8$ m/s² come from? Newton’s Law of Universal Gravitation says that two masses M and m , a distance r apart, pull on each other with a force

$$F = G \frac{Mm}{r^2} \quad (2.1)$$

where G is the gravitational constant. (The next section looks at this law more closely.)

For an object near Earth’s surface, M is Earth’s mass and r is basically Earth’s radius R_E , no matter whether the object is on the floor or on a mountaintop:

$$F = G \frac{Mm}{R_E^2} \quad (2.2)$$

The only thing that changes from object to object is m , so pull it out front:

$$F = m \left(G \frac{M}{R_E^2} \right) \quad (2.3)$$

Everything in the parentheses is a constant: G , Earth's mass M , and Earth's radius R_E . We call it the gravitational field strength:

$$g = G \frac{M}{R_E^2} \quad (2.4)$$

Substituting gives the famous short version:

$$F = mg \quad (2.5)$$

where m is the object's mass and $g = 9.8 \text{ m/s}^2$. Do not mix up little g with big G : g is built from G , but they are different numbers.

Here are the steps in one place:

$$\begin{aligned} F &= G \frac{Mm}{r^2} \\ &= G \frac{Mm}{R_E^2} \\ &= m \left(G \frac{M}{R_E^2} \right) \\ g &= G \frac{M}{R_E^2} \\ F &= mg \end{aligned}$$

Equation 2.5 gives the force of gravity, F_g , on nearly everything you will meet on Earth: a thrown ball, a book on a table, a lamp hanging from a cord.¹

Exercise 3 May the Force be with You 1

A hammer is falling along the side of a building, close to the surface of the Earth. What is its acceleration?

We will not ask it yet, but think about this: why does the Earth not move as

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¹Some problems let you use 10 for g to keep the arithmetic simple. Use 9.8 unless the problem says otherwise.

much toward the hammer as the hammer moves toward the Earth?

Answer on Page 63

Exercise 4 May the Force be with You 2

A dropped coin feels a gravitational force of 0.0198 N. What is the mass of the coin?

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Answer on Page 63

2.3 Universal Gravitation

When we discuss weight, we usually mean the gravitational force between an object and the largest nearby object. My weight on Earth, a spacecraft's weight on the Moon, or a rover's weight on Mars are all examples. But planets and moons do not have some special property that makes them emit gravity. Rather, *all objects with mass are gravitationally attracted to each other*. But, compared to the mass of the Earth, the mass of all the other objects around you (a table, your family, your house or apartment complex) is very, very small.

The force of the gravitational attraction between two objects is proportional to the product of their masses, and inversely proportional to their distance squared. This means that as objects get farther away, the force decreases. If you double the distance, the force quarters. Quadruple the distance, the force is $1/16^{\text{th}}$ as much. This is why you are more attracted to the Earth than you are to distant stars, even though they have much more mass than the Earth.

Newton's Law of Universal Gravitation

Two masses (m_1 and m_2) that are a distance of r from each other are attracted toward each other with a force of magnitude:

$$F = G \frac{m_1 m_2}{r^2} \quad (2.6)$$

where G is the universal gravitational constant. If you measure the mass in kilograms and the distance in meters, G is about 6.674×10^{-11} . That will get you the force of the attraction in newtons.

Exercise 5 Gravitation between two planets

A small moon orbits a planet at a distance of 4.0×10^6 m from the planet's center. The mass of the planet is 6.0×10^{24} kg, and the mass of the moon is 8.0×10^{22} kg.

1. Using Newton's Law of Universal Gravitation, calculate the gravitational force between the planet and the moon.
2. If the moon were moved to a distance of 8.0×10^6 m from the planet's center (twice the original distance), determine the new gravitational force.

Working Space

Answer on Page 63

Exercise 6 Gravity

The Earth's mass is about 6×10^{24} kilograms.

Your spacecraft's mass is 6,800 kilograms.

Your spacecraft is also about 100,000 km from the center of the Earth. (For refer-

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ence, the moon is about 400,000 km from the center of the Earth.)

What is the magnitude of the force of gravity that is pulling your spacecraft and the Earth toward each other?

Answer on Page 64

2.3.1 Gravitational Field

In Newton's law of gravity (Equation 2.6), the force grows with each mass. Double either mass, and the force doubles. That is why gravity is obvious for planets and moons, which keep each other in orbit, and invisible between you and your desk.

Now remember the hammer from Exercise 3. The Earth pulls the hammer, and the hammer pulls the Earth just as hard. So why does only the hammer seem to move? Because acceleration is force divided by mass (Newton's second law, which you will meet in the Forces and Motion chapter). The hammer's mass is about 1 kg, so $a_{\text{hammer}} = \frac{F}{m_{\text{hammer}}}$ is large. The Earth's mass is about 6×10^{24} kg, so $a_{\text{Earth}} = \frac{F}{m_{\text{Earth}}}$ is far too small to notice. Both actually fall toward their shared *center of mass*, which sits almost exactly at the center of the Earth (see the next section).

Figure 2.2 shows the direction of the pull around a planet:

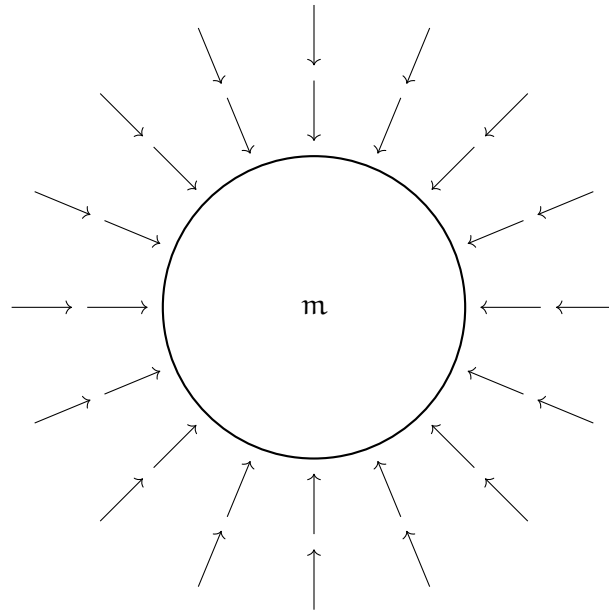


Figure 2.2: A figure of the direction of force vectors for a gravitational field.

Every arrow points toward the planet's center of mass, and the arrows are longer closer in, where the pull is stronger. Attaching an arrow to every point in space like this makes a *vector field*, an idea you will see again in vector calculus. The length of each arrow depends on the distance r from the center.

The gravitational field at a point in space is defined as the force per unit mass experienced by a small test mass placed at that point:

$$\vec{g} = \frac{\vec{F}}{m}$$

Notice that $\vec{g}$ has units of acceleration. That is no accident: the gravitational field at a point is exactly the acceleration a small object would have there if you let it go. The field gets weaker as you move away from the planet, and for a round planet it is the same at every point the same distance from the center.

2.4 Center of Mass

The *center of mass* of an object is its balance point: the spot where it acts as if all its mass were squeezed into one point. Balance a ruler on your finger, and your finger is under its center of mass. When we draw free body diagrams in a later chapter, we will treat each object as a single point at its center of mass.

If an object is heavier in some places than others, the center of mass shifts toward the heavy

part. A hammer balances near its metal head, not halfway down the wooden handle. You will see the term *center of mass* all through this sequence, though we will rarely ask you to calculate it in full.

For objects along a line, the center of mass is a weighted average of their positions:

1-Dimensional Center of Mass

$$x_{\text{CM}} = \frac{m_1x_1 + m_2x_2 + \cdots + m_nx_n}{m_1 + m_2 + \cdots + m_n}$$

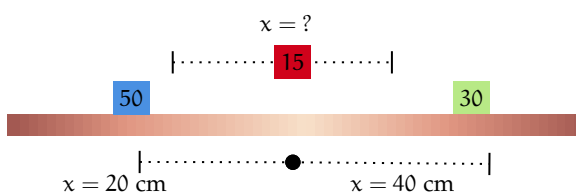
where $x_1, \dots, x_n$ and x_{CM} are measured from an origin you choose. It works just like an average, except heavier masses count more.

In summation notation:

$$x_{\text{CM}} = \frac{\sum m_i x_i}{\sum m_i}$$

Exercise 7 Balancing a seesaw

Three masses sit on a seesaw. M_1 weighs 30 lbs and sits 40 cm to the right of the pivot. M_2 weighs 50 lbs and sits 20 cm to the left of the pivot. Where should M_3 , which weighs 15 lbs, go so that the seesaw balances? Assume the plank itself is uniform.



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Answer on Page 64

For a flat or solid object whose mass is spread out unevenly, you add up tiny pieces with integrals instead of a sum. You have not learned integrals yet, so just take a peek at where this is headed:

3D Center of Mass

For a solid that fills a region V , with density $\rho(x, y, z)$ at each point, each tiny piece of volume dV has mass $dm = \rho(x, y, z) dV$. The total mass is

$$M = \iiint_V \rho(x, y, z) dV$$

and the center of mass is at

$$x_{CM} = \frac{1}{M} \iiint_V x \rho(x, y, z) dV$$

$$y_{CM} = \frac{1}{M} \iiint_V y \rho(x, y, z) dV$$

$$z_{CM} = \frac{1}{M} \iiint_V z \rho(x, y, z) dV$$

It is the same weighted average as before, with the sum replaced by an integral.

2.5 Exercises**Exercise 8 Weight on Earth and the Moon**

A person has a mass of 70 kg. Find the weight in newtons on Earth ($g = 9.8 \text{ m/s}^2$) and on the Moon ($g = 1.62 \text{ m/s}^2$).

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Answer on Page 64

Exercise 9 Center of mass

A 2 kg mass sits at $x = 0$ and a 6 kg mass sits at $x = 4 \text{ m}$. Find the center of mass.

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Answer on Page 65

Exercise 10 **Surface gravity**

Using $G = 6.674 \times 10^{-11} \text{ N m}^2/\text{kg}^2$, $M = 5.972 \times 10^{24} \text{ kg}$, and $R = 6.371 \times 10^6 \text{ m}$, find the acceleration due to gravity at Earth's surface.

Working Space

Answer on Page 65

Kinematics

3.1 Introductory Motion Concepts

3.1.1 Position, Velocity, Acceleration

To talk about motion, we need words for *where* something is, *how fast* it is going, and *how quickly* that is changing.

Position is where an object is, measured from a reference point called the *origin*. You get to choose the origin and the directions, the same way you might say “the library is two blocks north of school.” Positions come in different numbers of dimensions:

- 1D: moving along a straight line, like a train on a track (x)
- 2D: moving on a flat surface, like a hockey puck on ice (x, y)
- 3D: moving through space, like a drone (x, y, z)

Distance is how much ground you cover. *Displacement* is how far you end up from where you started, and in which direction. Walk 10 m east and then 10 m back, and you have traveled 20 m but your displacement is zero. A change in position is written Δx (horizontal) or Δy (vertical). Because displacement has a size and a direction, it is a vector. You may also see it written s .

Average *velocity*, written $\vec{v}$ or $\mathbf{v}$, is displacement divided by the time it took. Displacement is a vector, so velocity is too.

Average *speed* drops the direction: it is distance divided by time, so it is a *scalar*. Speed is the size of the velocity; velocity is speed plus a direction, such as 30 m/s north.

A car’s speedometer shows *instantaneous* speed: how fast you are going right now, not averaged over the whole trip. Think of measuring the change in position over a shorter and shorter time interval. In calculus, that limit is the *derivative* of position, $\vec{v} = \frac{ds}{dt}$.

Acceleration, $\mathbf{a}$, is how fast the velocity changes: $\mathbf{a} = \frac{d\mathbf{v}}{dt}$, which is the second derivative of position, $\frac{d^2s}{dt^2}$. Do not worry about the calculus yet; you will meet derivatives properly later. For now, remember the chain: position changes because of velocity, and velocity changes because of acceleration.

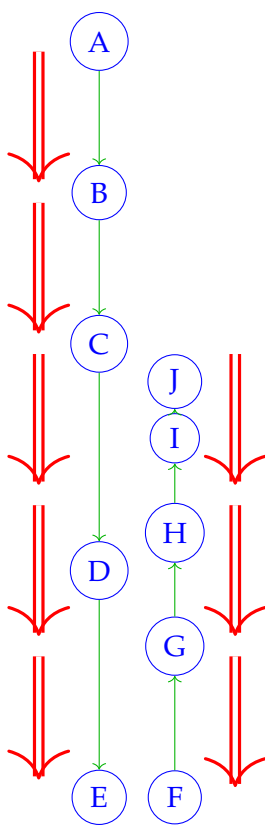
So a statement like “at 3 seconds, the ball is moving at 5 m/s and speeding up by 2 m/s every second” gives the velocity and the acceleration at one instant.

3.1.2 1D Motion Diagrams

A *motion diagram* is a strobe photo on paper. Imagine snapping a picture of a ball once every second as it drops and bounces back up. Put all the snapshots on one page, and you can see at a glance where the ball was and whether it was speeding up or slowing down. Motion diagrams usually use three symbols:

- $\cdot$ for position representation
- $\rightarrow$ for velocity vectors
- $\Rightarrow$ for acceleration vectors

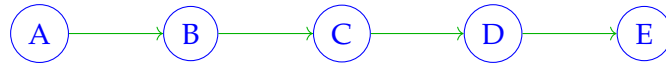
Here is the ball dropping and bouncing back up. The upward trip is shifted sideways so the two trips do not overlap. Red arrows show acceleration, green arrows show velocity, and blue dots show position at equal time steps.



Notice that in the above example, acceleration is constant throughout the motion, represented by the equally sized red arrows.

If the velocity does not change, there is no acceleration, and so no red arrows.

Here is another motion diagram, representing an object moving forwards with constant velocity.



3.2 Kinematics

Kinematics is the description of motion: where things are, how fast they go, and how that changes. A handful of equations covers any motion with constant acceleration, from a sprinter leaving the blocks to a skateboard rolling down a ramp.

Kinematic Equations

$$x_f = x_0 + v_0 t + \frac{1}{2} a t^2 \quad (3.1)$$

$$v_f = v_0 + a t \quad (3.2)$$

$$x_f = x_0 + \frac{1}{2} (v_f + v_0) t \quad (3.3)$$

$$v_f^2 = v_0^2 + 2a(x_f - x_0) \quad (3.4)$$

$$x_f - x_0 = v_{\text{avg}} t \quad (3.5)$$

You may also see v_i for the initial velocity v_0 (read “v-naught”).

Each equation leaves out at least one variable. That is the trick to choosing one: pick the equation that skips the quantity you do not know and do not care about.

- $x_f = x_0 + v_0 t + \frac{1}{2} a t^2$ has no v_f .
- $v_f = v_0 + a t$ has no position.
- $x_f = x_0 + \frac{1}{2} (v_f + v_0) t$ has no a .
- $v_f^2 = v_0^2 + 2a(x_f - x_0)$ has no t .
- $x_f - x_0 = v_{\text{avg}} t$ has no a and works with the average velocity.

These equations only work when the acceleration stays constant, which is called *uniformly accelerated motion*. A car braking smoothly qualifies; a car in stop-and-go traffic does not.

Example: Terri and Jerry are running a race. Terri has a maximum acceleration of 3.4 m/s^2 and a top speed of 9.2 m/s . Jerry has a maximum acceleration of 3.7 m/s^2 and a top speed of 8.7 m/s . If the race is 200 m long, who will win? Assume that each runner has maximum acceleration until they reach their top speed and that they will maintain that top speed once they reach it.

Solution: We want to know how long it takes each runner to complete the 200 m race. For each runner, we will divide their run into two sections:

1. The time they are accelerating to their top speed
2. The time they are maintaining their top speed

We can use a table to track the results of our calculations:

Runner	Terri	Jerry
Leg 1 (s)		
Leg 2 (s)		
Total (s)		

We will begin with Terri's first leg. Taking the starting line as $x = 0$, we know that:

$$x_0 = 0 \text{ m}$$

$$v_0 = 0 \frac{\text{m}}{\text{s}}$$

$$a = 3.4 \frac{\text{m}}{\text{s}^2}$$

And since we want to know how long it takes Terri to reach her top speed, we also know that $v_f = 9.2 \text{ m/s}$. Since we do not know how far Terri will run before she reaches her top speed (and we are not looking for that quantity), we need to select an equation that does not include x :

$$\begin{aligned} v_f &= v_0 + at \\ 9.2 \frac{\text{m}}{\text{s}} &= 0 \frac{\text{m}}{\text{s}} + \left(3.4 \frac{\text{m}}{\text{s}^2} \right) t \\ t &= \frac{9.2 \frac{\text{m}}{\text{s}}}{3.4 \frac{\text{m}}{\text{s}^2}} \approx 2.7 \text{ s} \end{aligned}$$

With a similar method, we can find how long it takes Jerry to reach his top speed:

$$t = \frac{v_f}{a} = \frac{8.7 \frac{\text{m}}{\text{s}}}{3.7 \frac{\text{m}}{\text{s}^2}} \approx 2.4 \text{ s}$$

Let us go ahead and record this in our table:

Runner	Terri	Jerry
Leg 1 (s)	2.7	2.4
Leg 2 (s)		
Total (s)		

Now that we know how much time it takes each runner to reach their top speed, we need to figure out how much time it takes them to complete the race from the point at which each reaches their top speed. To do this, we will first have to find *where* each runner hits their top speed. (This is because we cannot use $v_f = v_0 + at$ to find a time anymore, since from now on the runners' accelerations are zero, and all the other equations involve x .) For each runner, we know a , v_0 , v_f , x_0 , and t . You could choose any equation, but we will use this one:

$$x_f = x_0 + \frac{1}{2} (v_f + v_0) t$$

Since each runner begins on the starting line, $x_0 = 0$ m and $v_0 = 0$ m/s:

$$x_f = \frac{v_f \cdot t}{2}$$

For Terri:

$$x_f = \frac{(9.2 \frac{\text{m}}{\text{s}}) (2.7 \text{ s})}{2} \approx 12.4 \text{ m}$$

For Jerry:

$$x_f = \frac{(8.7 \frac{\text{m}}{\text{s}}) (2.4 \text{ s})}{2} \approx 10.2 \text{ m}$$

Now that we know where they reach their top speed, we can take that position as x_0 and find how long it takes each runner to reach the finish line at $x_f = 200$ m. Since $a = 0$ m/s², we can use the constant-velocity equation:

$$x_f = x_0 + v_0 t$$

Rearranging to solve for t :

$$t = \frac{x_f - x_0}{v_0}$$

For Terri:

$$t = \frac{200 \text{ m} - 12.4 \text{ m}}{9.2 \frac{\text{m}}{\text{s}}} \approx 20.4 \text{ s}$$

And for Jerry:

$$t = \frac{200 \text{ m} - 10.2 \text{ m}}{8.7 \frac{\text{m}}{\text{s}}} \approx 21.8 \text{ s}$$

Completing our table, we see that Terri will win the race by finishing in the least amount of time:

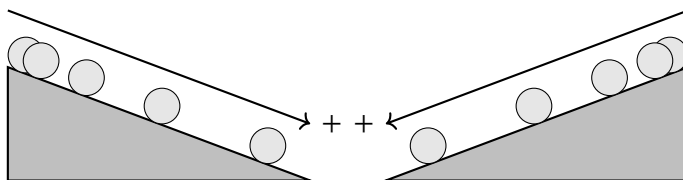
Runner	Terri	Jerry
Leg 1 (s)	2.7	2.4
Leg 2 (s)	20.4	21.8
Total (s)	23.1	24.2

3.2.1 Graphing Motion

Motion Diagrams

Example: Create a motion diagram of a ball rolling with a constant acceleration down an incline (the acceleration is in the same direction as motion).

Solution: The ball is accelerating down the ramp, so it will cover more distance each second. You could draw your ramp going left or right, as long as the distance covered each time interval increases as time passes.



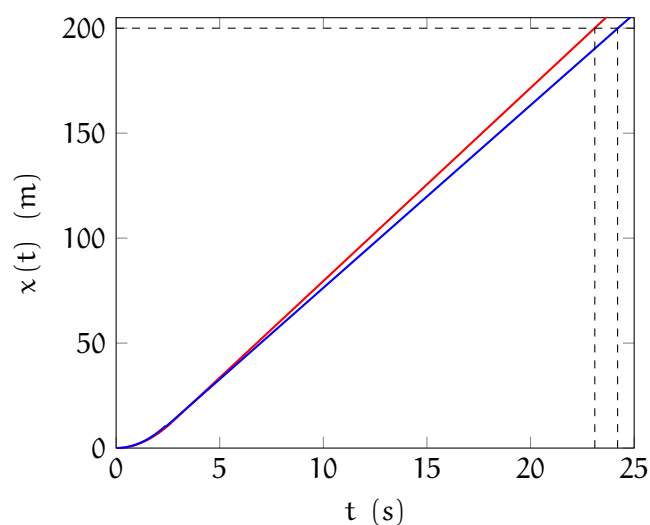
Position-Time and Velocity-Time Graphs

In the example problem above, graphs of each runner's motion would allow us to immediately see who would win. We can describe Terri's and Jerry's runs with piecewise functions:

$$x_{\text{Terri}}(t) = \begin{cases} (1.7 \frac{\text{m}}{\text{s}^2}) t^2 & \text{if } 0 \leq t < 2.7 \text{ s} \\ 12.4 \text{ m} + (9.2 \frac{\text{m}}{\text{s}}) (t - 2.7 \text{ s}) & \text{if } t \geq 2.7 \text{ s} \end{cases}$$

$$x_{\text{Jerry}}(t) = \begin{cases} (1.85 \frac{\text{m}}{\text{s}^2}) t^2 & \text{if } 0 \leq t < 2.4 \text{ s} \\ 10.2 \text{ m} + (8.7 \frac{\text{m}}{\text{s}}) (t - 2.4 \text{ s}) & \text{if } t \geq 2.4 \text{ s} \end{cases}$$

Graphing Terri in red and Jerry in blue:

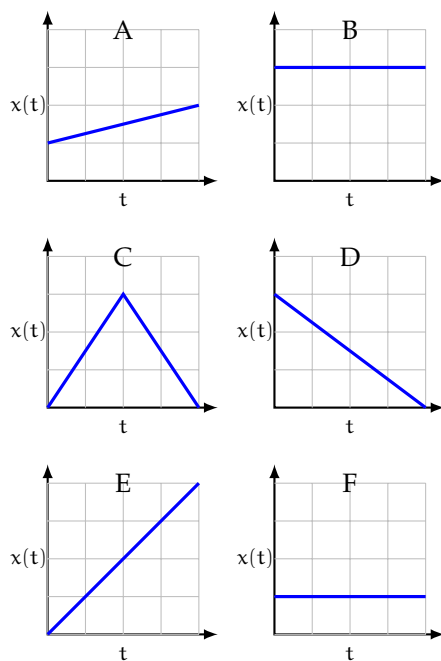


Since the red function (Terri) crosses $x(t) = 200$ m first, we see that Terri will win.

Exercise 11

The following graphs show the position of an object from $t = 0$ s to $t = 10$ s. The scales on the y-axes are all the same.

Working Space



Rank the objects from least to greatest

(all negative values are lower than all positive values) in terms of:

1. displacement from $t = 0$ s to $t = 10$ s.
2. instantaneous velocity at $t = 7.5$ s.
3. distance traveled from $t = 0$ s to $t = 10$ s

Answer on Page 65

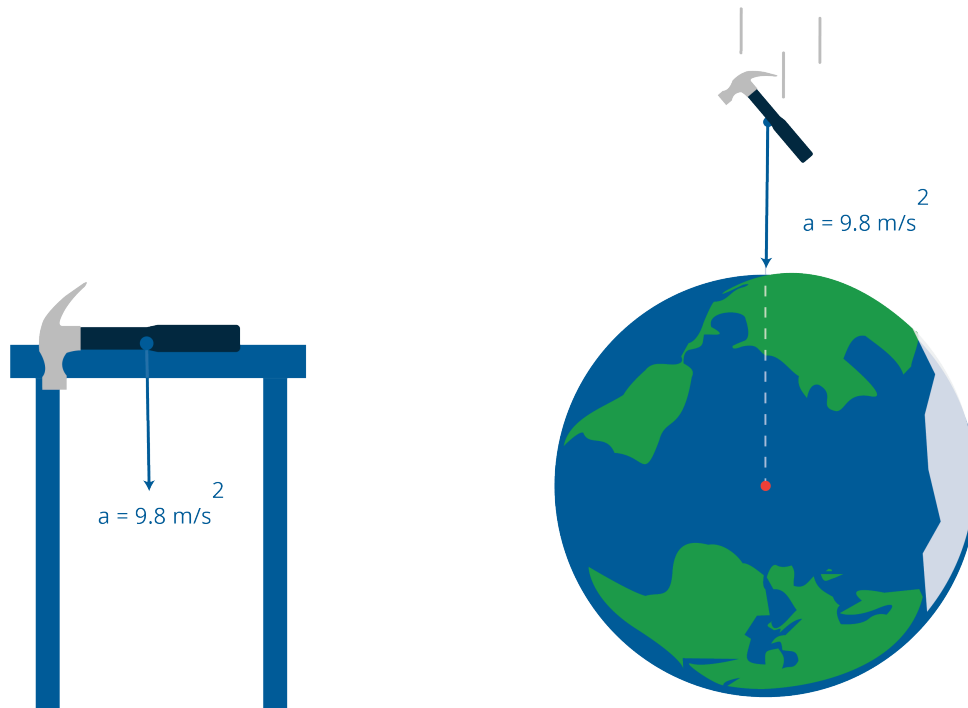
3.2.2 Separation of Components

So far we have described motion along a line. In a later chapter, you will describe motion in two dimensions, and here is the surprise: you can treat the horizontal and vertical motions separately! Try this puzzle. A cannon on a cliff fires a cannonball straight out, horizontally. At the same instant, someone drops an identical cannonball from the same height. Which one hits the ground first?

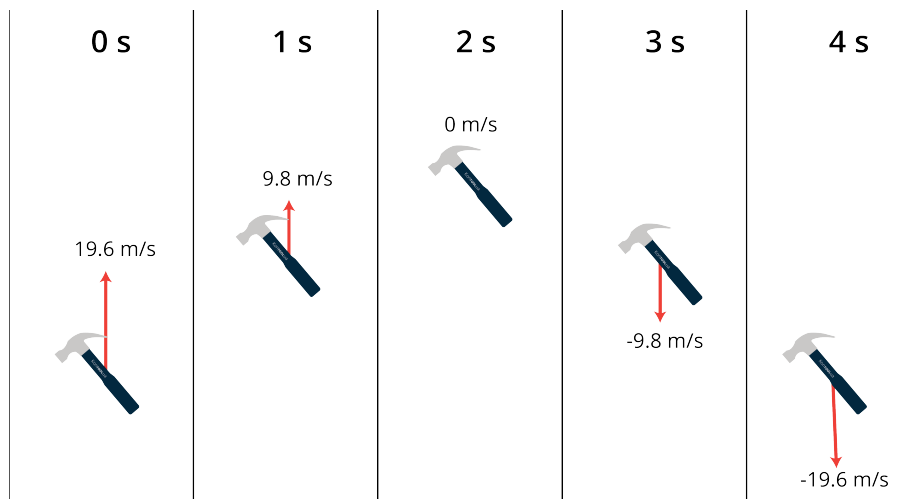
Go ahead and jot down what you think would happen: would the dropped ball hit first, the launched ball hit first, or would they both reach the ground below at the same time? Then, take a look at this video: <https://www.youtube.com/watch?v=zMF4CD7i3hg>. In it, two balls are released from the same height at the same time. One is dropped from rest while the other is launched horizontally (that is, its initial velocity is entirely in the x -direction). Based on the video, was your cannonball prediction correct? You will learn to explain this in a later chapter.

3.3 Falling Bodies

Toss a hammer straight up. From the moment it leaves your hand until it lands, gravity pulls it toward the center of the earth with a constant acceleration, on the way up, at the top, and on the way down.



Acceleration can be defined as change in velocity. If the hammer leaves your hand with a velocity of 12 meters per second upward, one second later, it will be rising, and its velocity will have slowed to 2.2 meters per second. One second after that, the hammer will be falling at a rate of 7.6 meters per second. Every second the hammer's velocity is changing by 9.8 meters per second, and that change is always toward the center of the earth. When the hammer is going up, gravity is slowing it down by 9.8 meters per second, each second it is in the air. When the hammer is coming down, gravity is increasing the speed of its descent by 9.8 meters per second.



Acceleration due to gravity on earth is a constant negative 9.8 meters per second per second:

$$a = -9.8 \frac{\text{m}}{\text{s}^2}$$

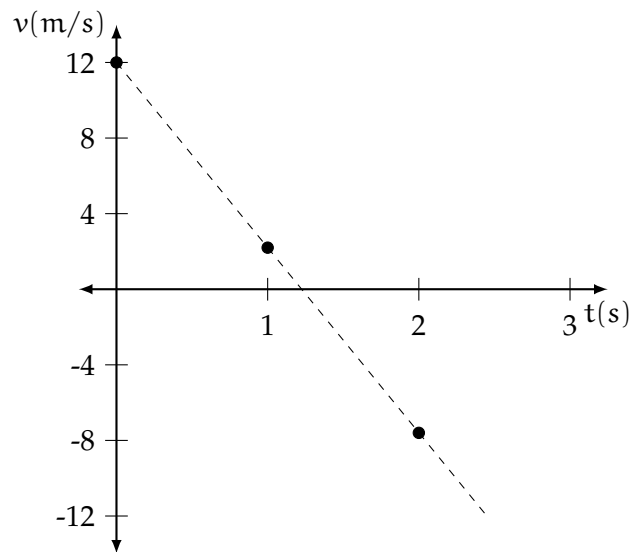
(Why is it negative? We are talking about height, which is generally considered to be increasing as you go away from the center of the earth. Since “up” is the positive direction, we take “down” as a negative direction. Therefore, since gravity pulls things “down”, the acceleration due to gravity is considered negative.)

3.3.1 Calculating the Velocity

Given that the acceleration is constant, it makes sense that the velocity is a straight line. Assuming once again that the hammer leaves your hand at 12 meters per second, let us create a quick data table and graph of the hammer’s velocity. Every second, we will subtract 9.8 m/s from the previous velocity.

Time elapsed (s)	Velocity (m/s)
0	12
1	2.2
2	-7.6

Here is a plot of these data with a dashed fit line:



We can find an equation for the line: $v = 12 - 9.8t$. What is this saying? It says that at any time, t , after $t = 0$ s, the velocity of the hammer is its starting velocity (12 m/s) plus the acceleration (-9.8 m/s^2) multiplied by the time elapsed (t). (Why did we say “plus” instead of “minus”? Well, since the acceleration is *negative*, $v_0 - 9.8t$ is the same as $v_0 + (-9.8t)$, and we are really adding the negative acceleration. When the acceleration is positive, we will see a plus sign!)

Exercise 12 When is the apex of flight?

Given the hammer’s velocity is calculated as $12 - 9.8t$, at what time (in seconds) does it stop rising and begin to fall?

Working Space

Answer on Page 65

We can generalize this to other falling bodies. For any falling object with initial velocity v_0 and constant acceleration a , its velocity is given by:

$$v(t) = v_0 + a \cdot t$$

Example: The acceleration due to gravity on the Moon is approximately 1.63 m/s^2 . If a hammer tossed upwards on the Moon takes 3.4 seconds to reach its apex, what was the hammer’s initial velocity?

Solution: We know that $a = -1.63\text{m/s}^2$ and $t = 3.4\text{s}$. Substituting and solving the velocity equation:

$$0 \frac{\text{m}}{\text{s}} = v_0 + \left(-1.63 \frac{\text{m}}{\text{s}^2}\right) (3.4\text{s})$$

$$v_0 = (1.63 \cdot 3.4) \frac{\text{m}}{\text{s}} \approx 5.5 \frac{\text{m}}{\text{s}}$$

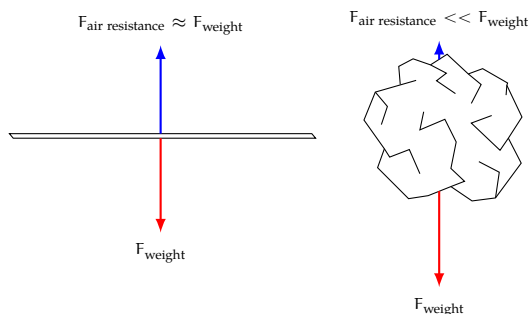
This hammer must have been tossed upwards with an initial velocity of approximately 5.5 m/s.

Air Resistance

We have been ignoring something: the air. As the hammer moves, the air pushes back on it. This force is called *air resistance*. For a big, fast object like an airplane, it is huge. For a dense, slow object like a hammer tossed by hand, it is small enough to ignore.

Consider two otherwise identical pieces of paper: one completely flat and unfolded, the other balled up as tightly as possible. If you dropped them both from the same height, which would hit the ground first? Intuitively, you know the balled-up paper would: but *why*? Since the papers have the same mass, you know that the force of gravity is the same on both papers! However, if you give this a try (go ahead, all you need are two sheets of printer paper and a safe place to drop them from), the acceleration of the two papers is different.

We can explain this with Newton's Second Law, which you will study in the Forces and Motion chapter: the net force on an object equals its mass times its acceleration, $F_{\text{net}} = ma$. So the more the forces on an object cancel, the smaller its acceleration. First, let us draw diagrams to represent the forces acting on the papers:



For the crumpled paper, the decrease in surface area results in very little air resistance. As a result, the net force on the crumpled paper is approximately equal to the paper's weight.

$$F_{\text{net}}^{\text{crumpled}} = F_{\text{air resistance}} + F_{\text{weight}} \approx F_{\text{weight}}$$

$$F_{\text{net}}^{\text{crumpled}} = ma_{\text{crumpled}} \approx F_{\text{weight}} = mg$$

$$a_{\text{crumpled}} \approx g$$

Thus, you observe the crumpled paper to fall at nearly -9.8 m/s^2 . The flat paper, on the other hand, has a much larger surface area, and therefore experiences significantly more air resistance. Thus, the observed acceleration of the flat paper is much slower:

$$F_{\text{net}}^{\text{flat}} = F_{\text{air resistance}} + F_{\text{weight}}$$

$$ma_{\text{flat}} = F_{\text{air resistance}} + mg$$

$$a_{\text{flat}} = \frac{F_{\text{air resistance}}}{m} + g$$

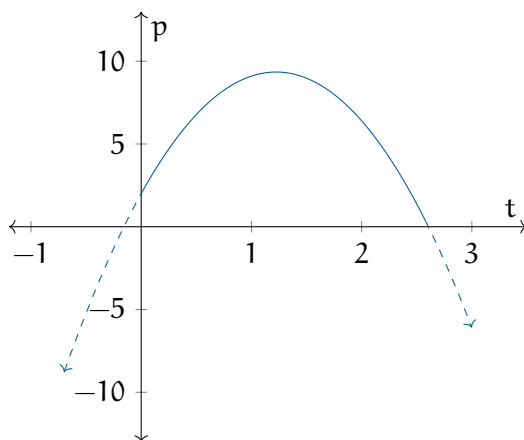
Since $g < 0$ and $F_{\text{air resistance}}$ is positive, the greater the air resistance the slower the paper falls.

3.3.2 Calculating Position

If you let go of the hammer when it is 2 meters above the ground, the height of the hammer is given by:

$$p = -\frac{9.8}{2}t^2 + 12t + 2$$

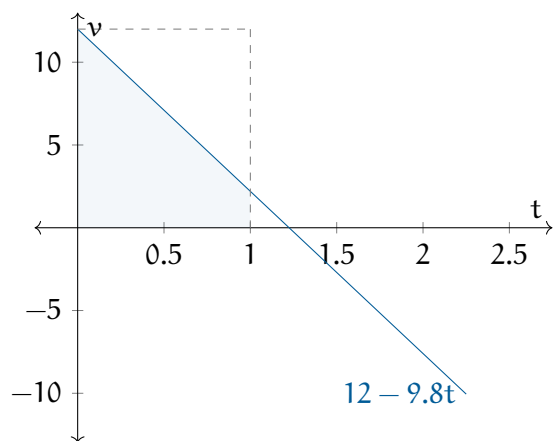
Here is a graph of this function:



How do we know? **The change in position between time 0 and any time t is equal to the area under the velocity graph between $x = 0$ and $x = t$.**

Let us use the velocity graph to figure out how much the position has changed in the first second of the hammer's flight. Here is the velocity graph with the area under the graph

for the first second filled in:



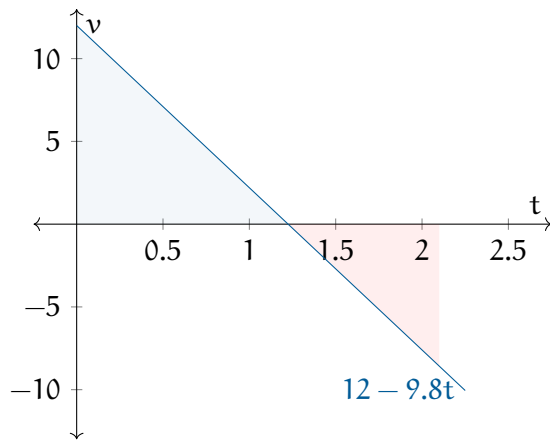
The blue filled region is the area of the dashed rectangle minus that empty triangle in its upper left. The height of the rectangle is twelve and its width is the amount of time the hammer has been in flight (t). The triangle is t wide and $9.8t$ tall. Thus, the area of the blue region is given by $12t - \frac{1}{2}9.8t^2$.

That is the change in position. Where was it originally? 2 meters off the ground. This means the height is given by $p = 2 + 12t - \frac{1}{2}9.8t^2$. We usually write terms so that the exponent decreases, so:

$$p = -\frac{1}{2}9.8t^2 + 12t + 2$$

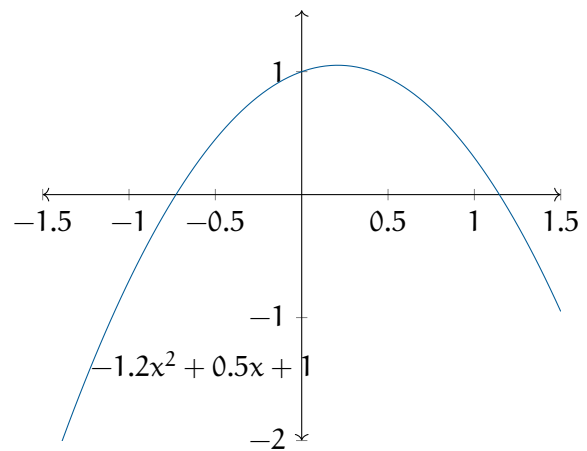
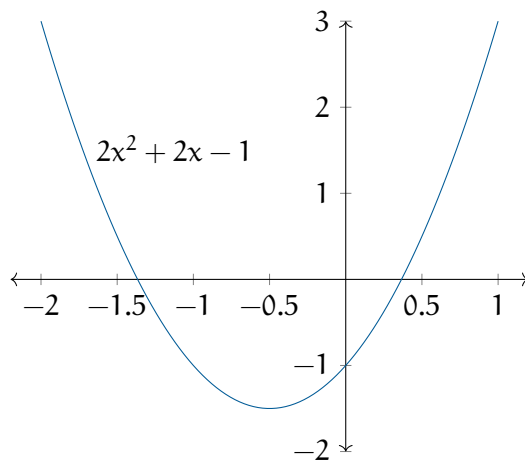
Finding the area under the curve like this is called *integration*. We say “To find a function that gives the change in position, we just integrate the velocity function.” Much of the study of calculus is learning to integrate different sorts of functions.

One important note about integration: Any time the curve drops under the x -axis, the area is considered negative. (Which makes sense, right? If the velocity is negative, the hammer’s position is decreasing.)



3.3.3 Quadratic functions

Functions of the form $f(x) = ax^2 + bx + c$ are called *quadratic functions*. If $a > 0$, the ends go up. If $a < 0$, the ends go down.



The graph of a quadratic function is a *parabola*.

3.3.4 Simulating a Falling Body

There is another way to find the hammer's height and speed: *simulate* it. Start the hammer at 2 meters with a speed of 12 m/s upward. Every hundredth of a second, change the height by the speed times 0.01 s, and change the speed by the acceleration times 0.01 s. Repeat until the height reaches 0. Each step is simple arithmetic, so a computer can do thousands of them in an instant. After you learn Python, the Making Plots with matplotlib chapter walks you through this simulation and graphs the results. You will find that the

hammer hits the ground just after 2.61 seconds.

A natural question at this point is “When exactly will the hammer hit the ground?” In other words, when does height = 0? The values of t where a function is zero are known as its *roots*. Height is given by a quadratic function. In a later chapter, you will learn a trick for finding the roots of any quadratic function.

Forces and Motion

4.1 Force, Mass, and Acceleration

A force is a push or a pull. Kick a ball, and your foot pushes it. Hold a dog's leash, and the leash pulls your hand. Forces are *vectors*: each has a size and a direction, and we can add them up, even when they point opposite ways. Newton's three laws of motion tell us how forces change the way things move.

4.1.1 Newton's First Law

Newton's First Law of Motion states that an object at rest will stay at rest, and an object in motion will stay in motion with a constant velocity (same speed and direction) unless acted upon by a *net external force*. This means that if there is no net force on an object, it will not *accelerate*.

For example, a car that stops pressing the gas on a flat road will eventually stop due to friction, which is a force that acts opposite to the direction of motion. If there were no friction, the car would continue moving at a constant speed forever.

This law is sometimes also referred to as the law of *inertia*, which is the tendency of objects to resist changes in their motion.

4.1.2 Mass, Acceleration, and Newton's Second Law

Each atom has a mass, which means everything made up of those atoms has mass as well (and that is pretty much everything!). We measure mass in grams. A paper clip is about 1 gram of steel. An adult human can have a mass of 70,000 grams, so for larger things, we often talk about kilograms, which is 1000 grams.

The first interesting thing about mass is that objects with more mass require more force to accelerate. For example, pushing a bicycle so that it accelerates from a standstill to jogging speed in 2 seconds requires much less force than pushing a train so that it accelerates at the same rate.

Newton's Second Law of Motion

The force necessary to accelerate an object of mass m at an acceleration of a is given by:

$$F = ma$$

This is said as “the force is equal to the mass times the acceleration.”

This is Newton’s Second Law of Motion. It may be the most useful equation in all of physics.

What are the units here? We already know that mass is measured in kilograms. We can measure velocity in meters per second, but that is different from acceleration. So if we want to go from 0 to 5 meters per second (that is jogging speed) in two seconds, that is a change in velocity of 2.5 meters per second every second. We would say this acceleration is 2.5m/s^2 .

Velocity versus Acceleration

Acceleration is the change in velocity. If an object is speeding up or slowing down, it is accelerating. In everyday language, we often use *decelerate* to indicate slowing down, but in physics you can use the word accelerate (slowing down is just negative acceleration). Since velocity is a vector (it has a magnitude and direction), changing direction is *also acceleration*. On the other hand, an object moving at a constant velocity (same speed, same direction) is *not accelerating*!

Exercise 13 Is it accelerating?

State whether the described object is accelerating or not.

Working Space

1. A satellite orbiting the Earth at a constant speed.
2. A car moving due west at a constant 30 mile per hour.
3. A child coming to a stop on their bicycle.
4. A roller coaster going around a loop at a constant speed.
5. A roller coaster speeding up as it goes down the initial hill.

6. A book sitting on a table.

Answer on Page 66

It is a common misconception that all objects in motion are accelerating. If an object is moving with a constant velocity (same speed, same direction), then it is not accelerating.

Calculating Acceleration

When an object is speeding up or slowing down, we can calculate the acceleration by dividing the change in velocity by the time it takes to make that change.

Calculating Acceleration

The acceleration of an object from an initial velocity, v_i , to a final velocity, v_f , over a period of time, t , is given by:

$$a = \frac{v_f - v_i}{t}$$

Notice that if the velocity does not change, then $v_f - v_i = 0$ and the acceleration is also zero.

Example: Your car can go from zero to 60 mph in 3 seconds. What is the acceleration in m/s^2 ?

Solution: First, let us convert from the imperial units of miles per hour to the SI units of meters per second. You can do this using a search engine, but we will show how to do it by hand below. (You will learn more about this method in the Units chapter).

$$\frac{60 \text{ miles}}{1 \text{ hour}} \cdot \frac{1.61 \text{ km}}{1 \text{ mile}} \cdot \frac{1000 \text{ m}}{1 \text{ km}} \cdot \frac{1 \text{ hour}}{3600 \text{ seconds}} \approx \frac{26.82 \text{ m}}{\text{s}}$$

Now we have the starting velocity (0 m/s), the ending velocity (26.82 m/s), and the time (3 s), and we can find the acceleration:

$$a = \frac{v_f - v_i}{t} = \frac{26.82 \frac{\text{m}}{\text{s}} - 0 \frac{\text{m}}{\text{s}}}{3 \text{ s}} \approx 8.94 \frac{\text{m}}{\text{s}^2}$$

Determining Force

What about measuring force? The unit is named after Newton: the force needed to accelerate one kilogram at 1m/s^2 is *one newton*, written N. An apple sitting in your hand pushes down with about one newton.

$$1\text{N} = 1 \frac{\text{kg} \cdot \text{m}}{\text{s}^2}$$

Example: If the car in the above example has a mass of 1500 kg, how much force does the engine use to accelerate the car?

Solution: We have already found the car's acceleration: 8.94 m/s^2 . With the mass and acceleration, we can use Newton's Second Law to find the force needed to accelerate the car:

$$F = m \cdot a = 1500 \text{ kg} \cdot 8.94 \frac{\text{m}}{\text{s}^2} = 13410 \text{ N}$$

Exercise 14 Acceleration

Working Space

While driving a bulldozer, you come across a train car (with no brakes and no locomotive) sitting on a track in the middle of a city. The train car has a label telling you that it has a mass of 2,400 kg. There is a time-bomb welded to the interior of the train car, and the timer tells you that you can safely push the train car for 120 seconds. To get the train car to where it can explode safely, you need to accelerate it to 20 meters per second. Fortunately, the track is level and the train car's wheels have almost no rolling resistance.

With what force, in newtons, do you need to push the train for those 120 seconds?

Answer on Page 66

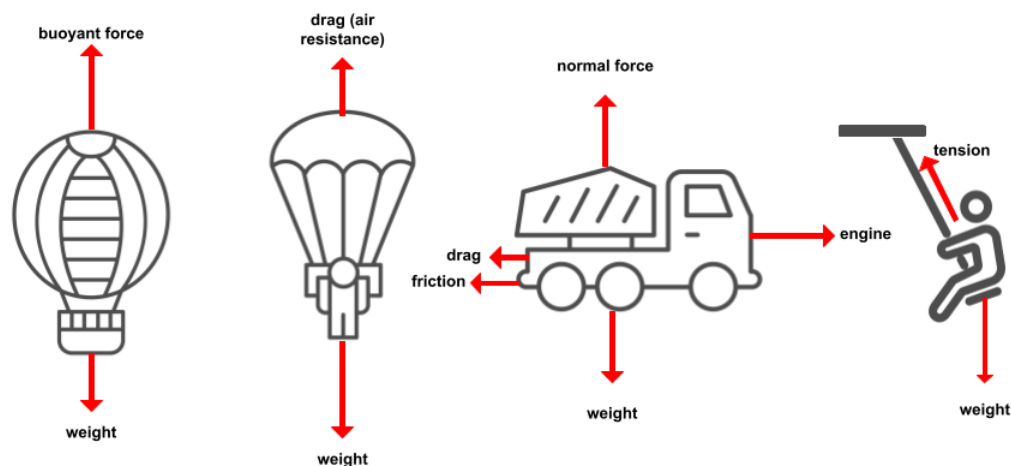


Figure 4.1: The hot air balloon has an upwards net force. The parachuter has a downwards net force. The truck has a rightward net force. The swinger has a leftward net force.

4.1.3 Net Force

So far, we have looked at examples where only one force is acting on an object. In reality, there are usually multiple forces acting on an object. For example, the engine pushes your car forward while friction pulls it backwards. Or your chair is pushing up on you while gravity pulls you down. How then can we describe the motion of an object if more than one force is acting on it?

We can rearrange Newton's Law:

$$a = \frac{F_{\text{net}}}{m}$$

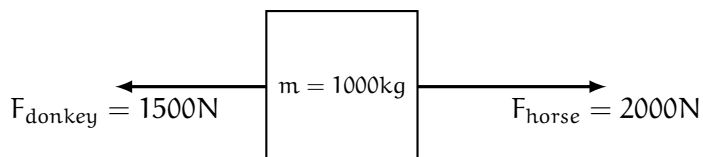
This means that an object's acceleration is directly proportional to the *net force* acting on the object and inversely proportional to the object's mass. The *net force* is the vector sum of all the forces acting on an object. The vector sum just means we have to take the direction of the force into account. Usually, up and right are positive while down and left are negative. You can see some examples in figure 4.1.

For now, we will only look at parallel forces (up/down or left/right). You will learn to use vector addition to combine orthogonal (at right angles) and skew (at angles other than parallel or right) forces in a later chapter.

Example: A donkey and a horse are each pulling a cart. The donkey pulls to the left with

a force of 1500 N, while the horse pulls to the right with a force of 2000 N. What is the net force on the cart? If the cart has a mass of 1000 kg, in what direction and with what magnitude will the cart accelerate?

Solution: We begin by drawing a diagram (this is good practice - while a diagram may not be necessary for such a simple question, it will be very useful as we examine more complex scenarios in future chapters).



It is customary to take right as positive, so the horse applies a positive force while the donkey applies a negative force to the cart. Therefore, the net force is:

$$F_{\text{net}} = F_{\text{horse}} - F_{\text{donkey}} = 2000\text{N} - 1500\text{N} = 500\text{N}$$

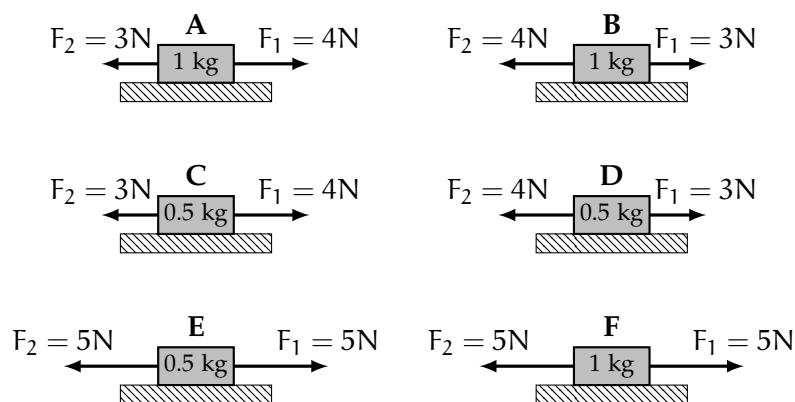
Since the net force is positive, it points to the right and the cart will accelerate to the right at a rate of:

$$a = \frac{F_{\text{net}}}{m} = \frac{500\text{N}}{1000\text{kg}} = 0.5 \frac{\text{m}}{\text{s}^2}$$

Exercise 15 Net Force and Acceleration

Rank the acceleration of the boxes shown below from greatest to least. All surfaces are frictionless and each box starts at rest. Take left as negative, and negative accelerations are less than a zero acceleration.

Working Space

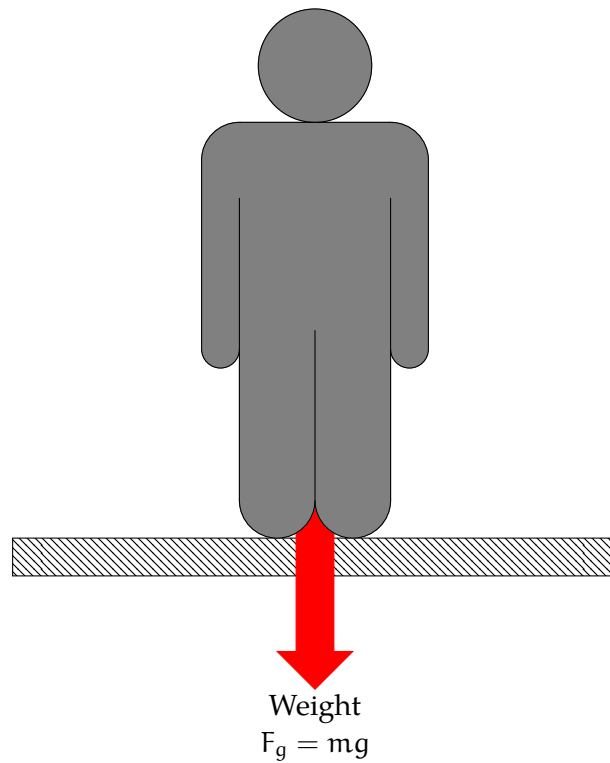


Answer on Page 66

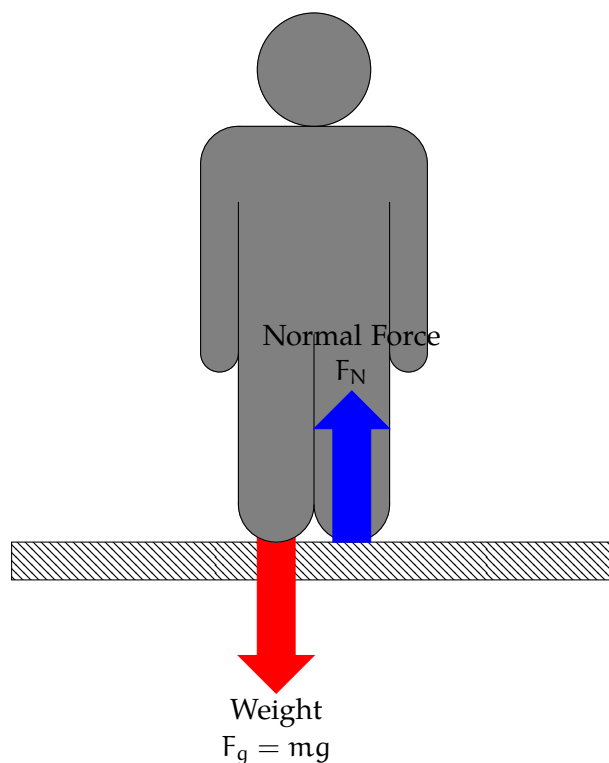
4.1.4 Normal Force and Apparent Weight

If you were to stand on a scale while riding an elevator, you would see the scale fluctuate as the elevator accelerates up and down (if you have a bathroom scale and live in a building with an elevator, you can try this yourself!). The reading on the scale is your *apparent weight* and is equal to the normal force between you and the floor.

What is a *normal force*? First, the word “normal” does not have the colloquial meaning of average or usual. In mathematics, “normal” means perpendicular. A normal force is perpendicular to the *contact* surface between two objects. Let us look at a person standing at rest. We know that gravity is pulling down on the person:



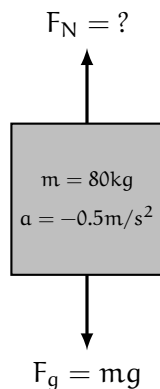
Since the person is not accelerating, there must be some other force acting on them in the upwards direction that balances out the person's weight (recall that if $a = 0\text{m/s}^2$, then it must be true that $F_{\text{net}} = 0\text{N}$). In this case, the balancing force is the *normal force* of the floor pushing up on the person's feet:



When you step on a bathroom scale, it is actually measuring the normal force! When you are not accelerating, your weight and the normal force between you and the scale are equal, so the scale gives you an accurate measure of your weight. Let us look at what happens if you are accelerating, as in an elevator.

Example: Maria has a mass of 80. kg. If the elevator in her apartment building initially accelerates at 0.50 m/s^2 , what is her apparent weight as the elevator begins to move down?

Solution: We begin with a diagram:



Her apparent weight is the normal force. Seeing that the net force acting on Maria is given

by $F_N - F_g$, we can use Newton's Second Law to find F_N :

$$\begin{aligned} F_{\text{net}} &= F_N - F_g = ma \\ F_N - (80\text{kg}) \left(9.8 \frac{\text{m}}{\text{s}^2} \right) &= (80\text{kg}) \left(-0.5 \frac{\text{m}}{\text{s}^2} \right) \\ F_N &= (80\text{kg}) \left(9.8 \frac{\text{m}}{\text{s}^2} - 0.5 \frac{\text{m}}{\text{s}^2} \right) = 744\text{N} \end{aligned}$$

Maria's apparent weight is 744 newtons when the elevator is accelerating downwards.

Exercise 16 Moving Elevators

A 700-N person is standing on a scale in an elevator. Rank the reading on the scale from greatest to least for the following scenarios:

1. The elevator has an initial upward velocity of 2 m/s and an upward acceleration of 3 m/s²
2. The elevator is initially at rest and has an upward acceleration of 3 m/s²
3. The elevator has an initial downward velocity of 2 m/s and an upward acceleration of 5 m/s²
4. The elevator is initially at rest and has a downward acceleration of 9.8 m/s²
5. The elevator has an initial upward velocity of 2 m/s and a downward acceleration of 5 m/s²
6. The elevator has an initial downward velocity of 2 m/s and is not accelerating

Working Space

Answer on Page 67

On flat ground, for an object that is not accelerating up or down, the normal force is equal

in size to the object's weight. The normal force also controls another contact force, friction, which you will meet later in this chapter. On a ramp, the normal force points *perpendicular* to the ramp, not straight up.

4.1.5 Newtons and Joules

Energy is measured in joules. Compare the units of a newton and a joule:

$$\text{N} = \frac{\text{kg} \cdot \text{m}}{\text{s}^2} \quad \text{J} = \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2} = \text{N} \cdot \text{m}$$

A joule is a newton times a meter. That is a hint: push with a force (newtons) through a distance (meters), and you change an object's energy (joules). This idea, the work-energy theorem, comes in a later chapter.

4.1.6 Newton's Third Law

Newton's Third Law says that every action has an equal and opposite reaction. In other words, if body A exerts a force on body B, then body B simultaneously exerts a force of equal magnitude and opposite direction on body A. Mathematically, we can express this as:

$$\mathbf{F}_{AB} = -\mathbf{F}_{BA}$$

Forces always come in pairs. If box 1 pushes on box 2, then box 2 pushes back on box 1 just as hard, no matter which box is heavier: $\mathbf{F}_{\text{Box 1 on 2}} = -\mathbf{F}_{\text{Box 2 on 1}}$. You will see these pairs everywhere: air pushing back on a moving car, coupled train cars tugging on each other, and planets pulling on each other.

Note the following:

- Action-reaction pairs do *not* cancel when applying Newton's Second Law to a single body.
- The forces are always of the same type (both contact, both gravitational, etc).
- They occur simultaneously; there is no time delay.

Watch out for a classic mistake: a book's weight and the normal force on it are *not* an action-reaction pair, even though they are often equal and opposite. Both act on the book, and they are different types of force. The partner of the Earth pulling down on the book is the book pulling up on the Earth. The partner of the table pushing up on the book is the book pushing down on the table. **Example:** Consider a swimmer pushing the wall of a pool. What forces are in play?

Solution: While the swimmer exerts a force F_{wall} on the wall, the wall exerts the reaction $-F_{\text{wall}}$ on the swimmer, propelling the swimmer forward. Although the wall does not move due to mass and acceleration differences, the forces are equal and opposite.

4.2 Free Body Diagrams

Now that you have mastered modeling *how things move*, you can learn to model *why things move*. Recall Newton's Second Law:

$$F = ma$$

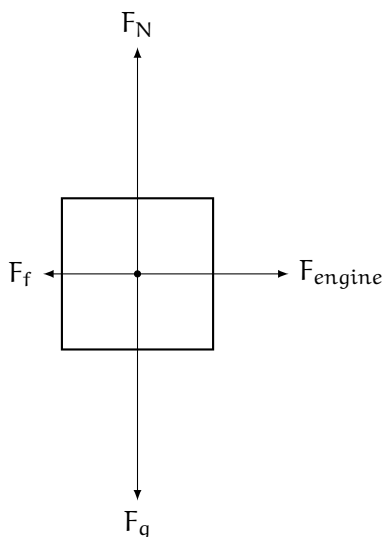
This is a simplification: what Newton's Second Law says in total is "the acceleration of a body is directly proportional to the *net force* acting on the body and inversely proportional to the mass of the body." Mathematically, that would be:

$$a = \frac{\Sigma F}{m}$$

Where ΣF is the vector sum of all the individual forces acting on the body. This sum is also called the *net force*. To visualize the magnitude and direction of the net force, it can be very helpful to draw a free body diagram.

4.2.1 Interpreting Free Body Diagrams

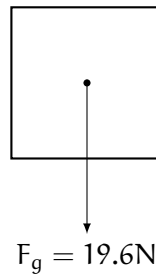
Before you learn to draw your own free body diagrams (FBDs), let us examine and analyze a few. Here is a FBD for an accelerating car:



We can consider the x and y axes separately. On the y -axis, we see that the weight (F_g) is the same magnitude as the normal force (F_N) because the arrows are the same length. This means in the y direction, the forces are balanced, and thus we do not expect to see acceleration in the y direction. This makes intuitive sense: if you are accelerating your car on a flat road, your car does not go up into the air. How about the x direction? Here we see friction (F_f) pulling the car backwards while the engine is pushing the car forwards. In this case, the forces are *not* balanced. See how the F_{engine} arrow is longer than the F_f arrow? This means the forward force from the engine is *greater in magnitude* than the backward force of F_f . Therefore, the *net force* in the x direction is to the right, and the car will accelerate to the right.

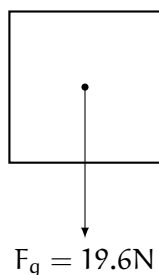
4.2.2 Drawing Free Body Diagrams

A free body diagram consists of the object, usually represented as a simple square, with various arrows representing the forces acting on the object. The direction and relative lengths of the arrows show the direction and relative strength of the forces. Here is a free body diagram for a 2-kg hammer in free fall through the air on Earth:



Objects At Rest

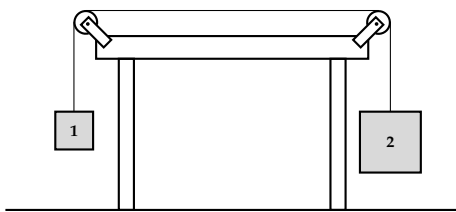
If an object is at rest, then its acceleration must be $0 \frac{\text{m}}{\text{s}^2}$. By Newton's Second Law, the *net force* acting on that object must also be zero. Does this mean there are no forces acting on the object? NO! It means all the forces are *balanced*: for every upwards force, there is an equal downwards force, and so on. Consider the same 2-kg hammer, but this time it is sitting on a table. We know gravity must be acting on the hammer, so let us begin with that:



Exercise 17 Blocks on a Table

[This exercise was originally presented as a free-response question on the 2015 AP Physics 1 exam.] Two blocks are connected with a string and two pulleys over a table, as shown in the figure. Block 1 has a mass of 3 kg and Block 2 has a mass of 9 kg. The blocks are released from rest. Treat the string as massless and the pulleys as massless and frictionless when answering the questions below.

Working Space

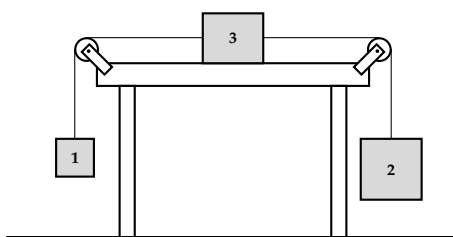


1. Draw free body diagrams for both blocks. Correctly show the relative magnitude of the forces using the relative lengths of the vectors.
2. Before doing any calculations, describe each block's acceleration as (a) positive or negative, (b) having a magnitude greater or less than g . Take the upward direction as positive.
3. Calculate the acceleration of each block. Take the upward direction as positive and express your answer in

terms of g .

Answer on Page 67

Now here is an interesting question: what happens if we add another mass to the system? Imagine that a third block with a mass of 15 kg is laid on the table from the previous exercise and the string is attached to each side, as shown below.



We could redraw free body diagrams for all three blocks, noting that the tensions on either side of Block 3 will be different. Then, we would have another system of equations to solve and we could find the acceleration. Or, we could think about the entire system. Originally, the net force on the two-block system was 58.8 Newtons (the weight of the larger block minus the weight of the smaller block). And the mass of the entire two-block system was 12 kilograms. We can apply Newton's Second Law:

$$a = \frac{58.8\text{N}}{12\text{kg}} = 4.9 \frac{\text{m}}{\text{s}^2} = \frac{1}{2}g$$

(Notice, this is the same answer we found in the previous exercise). What changes when we add a third, 15-kg block? The net force on the system does not change: the table pushes back up on the third block with a force equal to the block's weight. However, the total mass of the system has changed: it is increased. Now the total mass is 27 kg, and we can again apply Newton's Second Law:

$$a = \frac{58.8\text{N}}{27\text{kg}} \approx 2.18 \frac{\text{m}}{\text{s}^2}$$

This answer makes sense: the mass of the system has increased while the net force acting on the system has stayed the same, resulting in a slower acceleration.

Now, what if instead of blocks resting on level tables, the blocks were on an incline? Ramps need one more tool, trigonometry, so they get their own chapter, Inclined Planes, in Volume 7.

Friction

Imagine there is a large and heavy steel box resting in the middle of a floor, and you push it hard enough to get it moving. If you stop pushing, will it continue to glide gracefully across the floor?

Probably not. Unless the floor is very slippery for some reason, the box will come to a halt immediately after you stop pushing. We would say that it is stopped by the force of *friction*.

What is really happening? The box's energy of motion is turning into heat where the box rubs the floor. Rub your hands together fast, and you can feel the same thing. As the surfaces warm up, the box slows down.

5.1 The Coefficient of Friction

The amount of friction is proportional to the force with which the box is pressing against the floor — so you should expect a box that is twice as heavy to experience twice as much frictional force.

In other words, the frictional force is proportional to the normal force. On a level floor, the normal force is equal in size to the box's weight and points the opposite way, straight up. On a slope, the normal force points perpendicular to the slope; the Inclined Planes chapter covers that case, so stick to level floors for now.

The amount of friction is also determined by the materials that are sliding against each other. For example, if the floor is ice, the frictional force will be less than if the floor is made of wood. Written mathematically, we can express the force of friction as

$$F_f = \mu N$$

where F_f is the force of friction, N is the normal force (you may see written as R , F_n , or just N), and μ is a coefficient that depends on the materials in contact with each other.

If you are pushing the box with a force of F and it is moving but neither accelerating nor decelerating, then the force you are applying is exactly balanced by the frictional force. If the box presses on the floor with a force N , the *coefficient of friction* is the ratio of the friction force to that normal force:

$$\mu = \frac{F_f}{N}$$

Exercise 18 **Bicycle Stopping**

Working Space

You are riding your bicycle on a flat road at 11 meters per second when you suddenly slam on the brakes and lock up the wheels.

You and the bike have a mass of 55 kg.

When any piece of rubber is skidding across a dry road, the coefficient of friction will be about 0.7.

Answer the following questions:

- How much kinetic energy do you have when you engage the brakes?
- As you skid, how much frictional force is decelerating you?
- For how many meters will you slide?

Answer on Page 68

Notice that the force of friction is not determined by how much of the tire is touching the ground. The coefficient of friction of the two materials and the normal force is all you need to compute the friction.

5.1.1 Static vs Kinetic Friction Coefficients

Let us return to the box on the floor we discussed earlier. As you start to push it, it will sit still until your force is greater than the force of friction. However, once it starts moving, the force of friction seems to be less.

Between the two materials, there are actually two different friction coefficients:

- Static friction coefficient: The coefficient you use to figure out how much force you need to get the box to start to move.
- Kinetic friction coefficient: The coefficient you use once the box is sliding against the floor.

The kinetic friction coefficient is always less than the static friction coefficient:

- *Static*, μ_s : When the car is parked with its brakes on, it has a friction coefficient of about 1.0.
- *Kinetic*, μ_k : For a car skidding on a dry road, the friction coefficient is about 0.7.

It is important to note once the force of static friction is overcome, there will be a force of kinetic friction less than the static friction force.

Exercise 19 Rocket Sled

Working Space

You built a rocket sled with steel runners that is resting on a flat, level wooden floor. The sled has a mass of 50 kg and you have a mass of 55 kg.

Before you get on the sled, you try pushing it around the floor. You find that you can get it to move from a standstill if you push it with a force of 270 N. Once it is moving, you can keep it moving at the same speed using a force of 220 N.

What are μ_s and μ_k of your sled's runners on your wooden floor?

Next, you get on the sled and gradually increase the thrust of the rocket mounted on the sled until it starts to move. You then keep the thrust constant.

How much force was the rocket exerting on you and the sled when it started

to move?

How fast do you accelerate now that the sled is moving?

Answer on Page 68

5.1.2 Skidding and Anti-Lock Braking Systems

When a car goes through a curve, the friction of the tire on the road is what changes the direction of the car's travel. Even though the wheel is turning, this is the *static friction coefficient* because the surface of the tire is not sliding across the road. The part of the tire touching the road at any instant is at rest relative to the road. The force of friction is the so-called *centripetal force* keeping the object on the road.

If you go into the curve too fast, the tire may not have enough friction to turn the car. In this case the car will start to slide sideways. Now, the friction between the tire and road uses the kinetic coefficient. In other words, you have significantly less friction than you had before you started to skid.

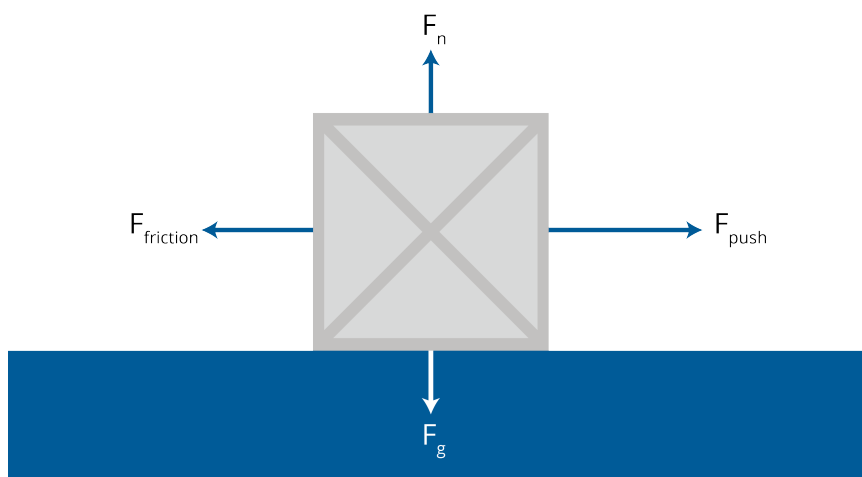
When you are driving a car, the force of friction that your tires create is your friend. It lets you steer, accelerate, and stop.

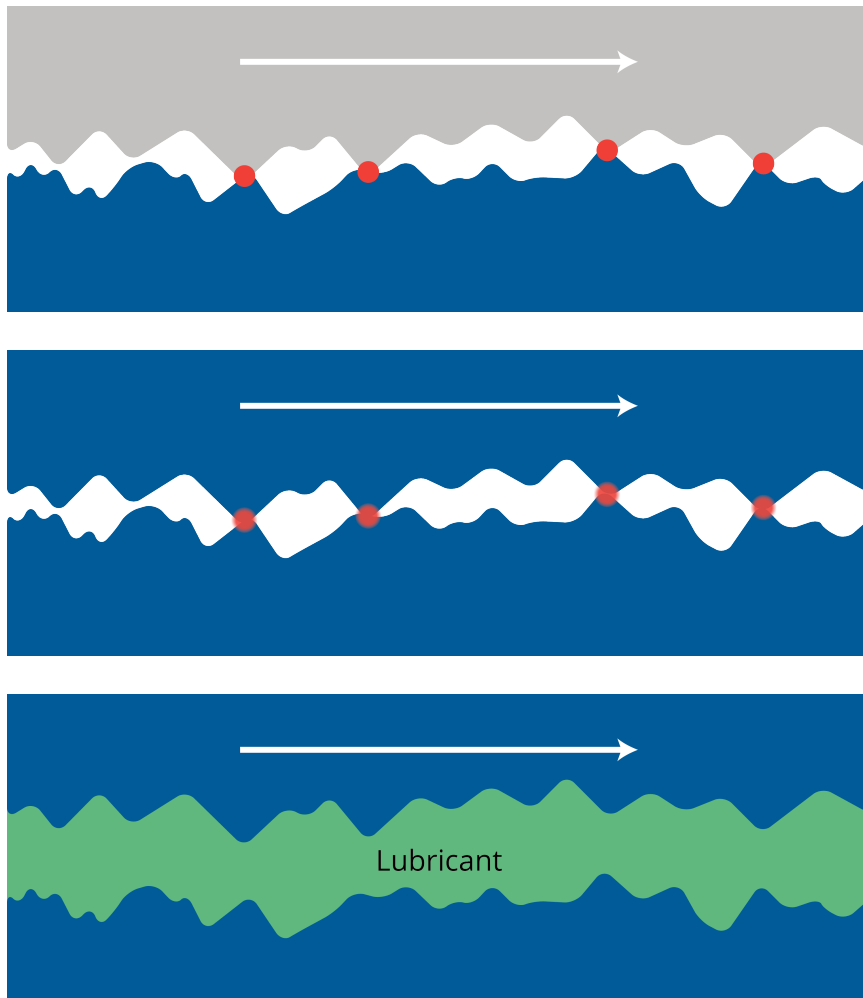
In older cars, if you panicked and slammed on the brakes, you would probably lock up the wheels: they would stop turning suddenly. And the surface of the tire would begin to slide across the pavement. At that moment, two problems occurred:

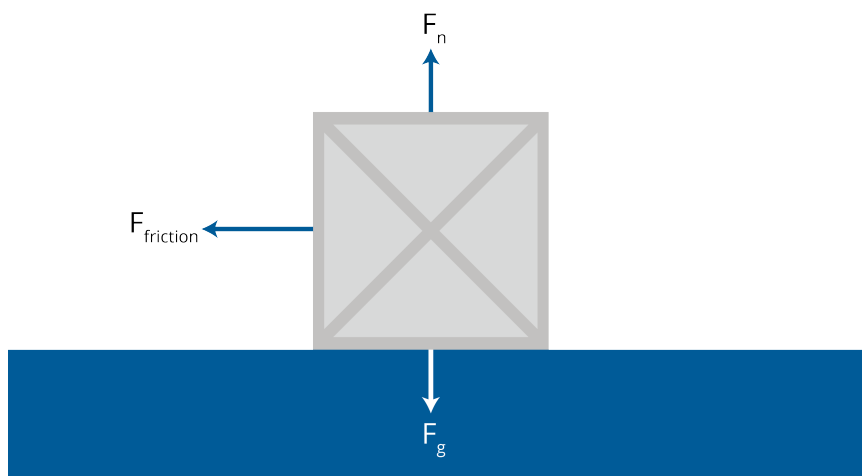
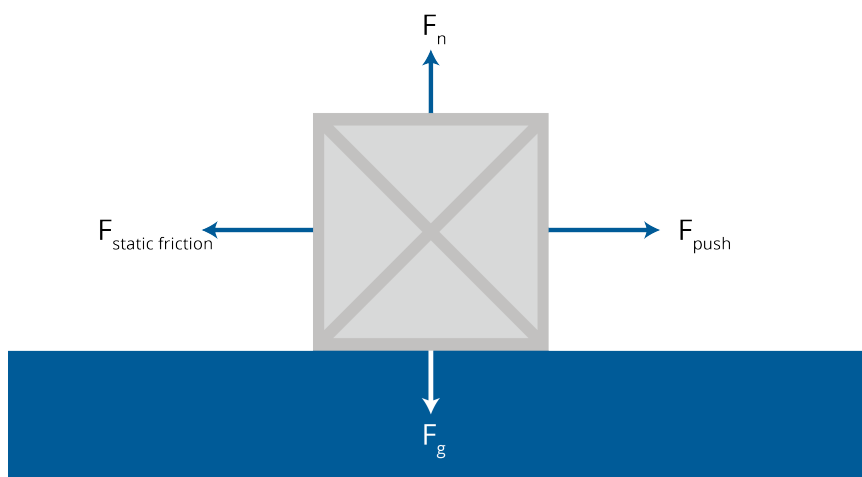
- You do not stop as quickly, because now the friction between your tires and the road is based on the kinetic friction coefficient instead of the static friction coefficient.
- You cannot steer the car. Steering only happens because the wheels are turning in a particular direction.

To prevent this problem, car companies developed the anti-lock brake system, or ABS.

An ABS computer watches each wheel. The instant a wheel starts to lock, it eases off that brake, then squeezes again, many times per second. The tires stay just short of skidding, so you keep the stronger static friction *and* you can still steer. If you have ever felt a brake pedal buzz under your foot in a hard stop, that was ABS at work. The figures below show what happens.







Units and Conversions

In 1999, NASA lost the \$125 million Mars Climate Orbiter because one team used pounds and another used newtons. Units matter.

You are already juggling a lot of them: grams for mass, newtons for force, meters for distance, seconds for time, joules for energy. Worse, each kind of measurement has several units. Distance alone comes in inches, feet, miles, kilometers, and light-years. This chapter shows you how to move between them without getting lost.

Some Equivalencies

Distance	
1 mile	1.6093 kilometers
1 foot	0.3048 meters
1 inch	2.54 centimeters
1 light-year	9.461×10^{12} kilometers
Volume	
1 milliliter	1 cubic centimeter
1 quart	0.9464 liters
1 gallon	3.7854 liters
1 fluid ounce	29.6 milliliters
Mass	
1 pound	0.4535924 kilograms
1 ounce	28.35 grams
1 metric ton	1000 kilograms
Force	
1 newton	1 kilogram meter per sec ²
Pressure	
1 pascal	1 newton per square meter
1 bar	0.98692 atmosphere
1 pound per square inch	6895 pascals
Energy	
1 joule	1 newton meter
1 calorie	4.184 joules
1 kilowatt-hour	3.6×10^6 joules

(You do not need to memorize these! Just remember that this page is here.)

In the metric system, prefixes are often used to express a multiple. Here are the common

prefixes:

Common Prefixes for Metric Units

giga	$\times 10^9$
mega	$\times 10^6$
kilo	$\times 10^3$
milli	$\div 10^3$
micro	$\div 10^6$
nano	$\div 10^9$

(These are worth memorizing. Here is a mnemonic for the most common ones: “King Henry Died By Drinking Chocolate Milk.” The letters stand for kilo ($\times 10^3$), hecto ($\times 10^2$), deca ($\times 10^1$), base unit such as the gram ($\times 10^0$), deci ($\times 10^{-1}$), centi ($\times 10^{-2}$), and milli ($\times 10^{-3}$.)

6.1 Conversion Factors

Here is the one trick you need.

Suppose someone asks, “How many miles are in 0.23 light-years?” From the table, 1 mile is 1.6093 kilometers and 1 light-year is 9.461×10^{12} kilometers. How do you get from one to the other?

Turn each equality into a fraction that equals 1:

$$\frac{1 \text{ mile}}{1.6093 \text{ km}} = \frac{1.6093 \text{ km}}{1 \text{ mile}} = 1$$

and

$$\frac{1 \text{ light-years}}{9.461 \times 10^{12} \text{ km}} = \frac{9.461 \times 10^{12} \text{ km}}{1 \text{ light-years}} = 1$$

We call these fractions *conversion factors*.

Now, your problem is

$$0.23 \text{ light-years} \times \text{Some conversion factors} = ? \text{ miles}$$

Note that when you multiply fractions together, things in the numerators can cancel with

things in the denominator:

$$\left(\frac{31\pi}{47}\right) \left(\frac{11}{37\pi}\right) = \left(\frac{31\cancel{\pi}}{47}\right) \left(\frac{11}{37\cancel{\pi}}\right) = \left(\frac{31}{47}\right) \left(\frac{11}{37}\right)$$

Units cancel the same way. Multiplying by 1 never changes a value, so you can multiply by as many conversion factors as you like. Choose them so the units you do not want cancel out:

Worked example: converting 0.23 light-years to miles.

$$\begin{aligned} &0.23 \text{ light-years} \left(\frac{9.461 \times 10^{12} \text{ km}}{1 \text{ light-years}}\right) \left(\frac{1 \text{ mile}}{1.6093 \text{ km}}\right) = \\ &0.23 \cancel{\text{light-years}} \left(\frac{9.461 \times 10^{12} \cancel{\text{km}}}{1 \cancel{\text{light-years}}}\right) \left(\frac{1 \text{ mile}}{1.6093 \cancel{\text{km}}}\right) = \frac{(0.23)(9.461 \times 10^{12})}{1.6093} \text{ miles} \approx 1.35 \times 10^{12} \text{ miles} \end{aligned}$$

Exercise 20 Simple Conversion Factors

Working Space

How many calories are in 4.5 kilowatt-hours?

Answer on Page 69

6.2 Conversion Factors and Ratios

Conversion factors also work on ratios. For example, if you are told that a bug is moving 0.5 feet every 120 milliseconds, what is that in meters per second?

The problem then is

$$\frac{0.5 \text{ feet}}{120 \text{ milliseconds}} = \frac{? \text{ m}}{\text{second}}$$

So you will need conversion factors to replace the “feet” with “meters” and to replace “milliseconds” with “seconds”:

$$\left(\frac{0.5 \text{ feet}}{120 \text{ milliseconds}}\right) \left(\frac{0.3048 \text{ meters}}{1 \text{ foot}}\right) \left(\frac{1000 \text{ milliseconds}}{1 \text{ second}}\right) = \frac{(0.5)(0.3048)(1000)}{120} \text{ m/second} \approx 1.27 \text{ m/second}$$

Exercise 21 Conversion Factors

Working Space

The hole in the bottom of the boat lets in 0.1 gallons every 2 minutes. How many milliliters per second is that?

Answer on Page 69

6.3 When Conversion Factors Fail

Conversion factors only work when the units being converted are proportional to each other. Gallons and liters, for example, are proportional to each other: If you have n gallons, you have $n \times 3.7854$ liters.

Degrees Celsius and degrees Fahrenheit are *not* proportional. Zero degrees Celsius is 32 degrees Fahrenheit, not zero. If your soup is n degrees Celsius, it is $n \times \frac{9}{5} + 32$ degrees Fahrenheit. That “+32” is why no conversion factor can do this job.

6.4 SI units

Scientists everywhere use one standard set of units, so that a measurement made in Tokyo means the same thing in Toronto. It is called the **International System of Units**, or **SI**. It has seven base units:

Time	second (s)
Length	meter (m)
Mass	kilogram (kg)
Electric Current	ampere (A)
Temperature	Kelvin (K)
Amount of substance	mole (mol)
Luminous intensity	candela (cd)

Every other unit is built from these seven.

You will often see units written with exponents. Velocity is meters *per* second, which you can write as a fraction or with seconds to a negative power:

$$v = \frac{\text{m}}{\text{s}} = \text{m} \cdot \text{s}^{-1}$$

Acceleration, meters per second per second, works the same way:

$$a = \frac{\text{m}}{\text{s}^2} = \text{m} \cdot \text{s}^{-2}$$

6.4.1 Converting from SI units

Many everyday units are SI units in disguise, built from the base units:

Physical Quantity	Name	Symbol	Expression in Base Units
Force	Newton	N	$\text{kg} \cdot \text{m} \cdot \text{s}^{-2}$
Energy/Work	Joule	J	$\text{kg} \cdot \text{m}^2 \cdot \text{s}^{-2}$
Power	Watt	W	$\text{kg} \cdot \text{m}^2 \cdot \text{s}^{-3}$
Frequency	Hertz	Hz	s^{-1}
Electric Charge	Coulomb	C	$\text{A} \cdot \text{s}$

You can derive all of these yourself. Try the newton: force is mass times acceleration, so what are its base units?

Exercise 22 Miles to Feet

How many feet are in 3 miles? Use 1 mile = 1.6093 kilometers and 1 foot = 0.3048 meters.

Working Space

Answer on Page 70

The Greek Alphabet

Mathematicians and scientists ran out of Latin letters a long time ago, so they borrowed the Greek alphabet. You will see these letters constantly from now on. Here they all are:

Capital	Lower	Pronounced	Capital	Lower	Pronounced
A	α	Alpha	N	ν	Nu
B	β	Beta	Ξ	ξ	Xi ("ksee" or "zy")
Γ	γ	Gamma	O	$\omicron$	Omicron
Δ	δ	Delta	Π	π	Pi
E	ϵ	Epsilon	P	ρ	Rho
Z	ζ	Zeta	Σ	σ	Sigma
H	η	Eta	T	τ	Tau
Θ	θ	Theta	Υ	υ	Upsilon
I	ι	Iota	Φ	ϕ	Phi
K	κ	Kappa	X	χ	Chi ("Kai")
Λ	λ	Lambda	Ψ	ψ	Psi ("Sigh")
M	μ	Mu	Ω	ω	Omega

7.1 Greek Letters in Math and Science

Some letters show up so often that they have become nicknames for particular ideas:

Symbol	Common meaning
π	circle constant, approximately 3.14159
θ	an angle
Δ	change in a quantity
Σ	sum of many terms
λ	wavelength
μ	coefficient of friction, mean, or micro-
ρ	density
ω	angular velocity
Ω	ohms, a unit of electrical resistance

A Greek letter can stand for a fixed *constant* or for a *variable* that changes. For example, the area of a circle is

$$A = \pi r^2$$

where r is the radius. Here π is a constant (it is always about 3.14159) and r is a variable. Or take this equation from physics:

$$\Delta x = v\Delta t + \frac{1}{2}a\Delta t^2$$

It gives the change in position, Δx , for something that starts at velocity v and speeds up with constant acceleration a for a time Δt . The Δ (“delta”) means “change in,” so Δx is read “delta x.” All four quantities here are variables.

One more, which you will meet again when we study light: the wavelength of a light wave depends on its frequency,

$$\lambda = \frac{c}{f}$$

where λ is the wavelength, c is the speed of light, and f is the frequency. Here λ (“lambda”) and f are variables, while c is a constant.

Worked example: wavelength of a radio wave. A radio station broadcasts at a frequency of 100 MHz = 1.00×10^8 Hz. The speed of light is $c = 3.00 \times 10^8$ m/s. Using $\lambda = \frac{c}{f}$,

$$\lambda = \frac{3.00 \times 10^8 \text{ m/s}}{1.00 \times 10^8 \text{ Hz}} = 3.00 \text{ m}$$

Worked example: distance under constant acceleration. A cart starts at $v = 2$ m/s and speeds up at $a = 3$ m/s² for $\Delta t = 4$ s. Using $\Delta x = v\Delta t + \frac{1}{2}a\Delta t^2$,

$$\Delta x = (2)(4) + \frac{1}{2}(3)(4)^2 = 8 + 24 = 32 \text{ m}$$

Exercise 23 Using Greek Symbols

A wave has a wavelength of $\lambda = 0.5$ m and travels at $c = 3.00 \times 10^8$ m/s. Using $\lambda = \frac{c}{f}$, find its frequency f in hertz.

Working Space

Answer on Page 70

We will always say what each letter means when we use it. Still, it pays to know the alphabet: an equation is much easier to read when you can say its symbols out loud.

Answers to Exercises

Answer to Exercise 1 (on page 4)

$\frac{250}{18} \approx 13.9$ mol, so the number of molecules is about $13.9 \times 6.02 \times 10^{23} \approx 8.4 \times 10^{24}$.

Answer to Exercise 2 (on page 4)

Atoms are tiny, so any amount you can see or weigh contains a huge number of them. Moles turn a mass you can measure in a lab into a count of particles, which lets you move from a measurement to a statement about atoms.

Answer to Exercise 3 (on page 7)

The gravitational acceleration is 9.8 m/s^2 , no matter the mass of the object.

Answer to Exercise 4 (on page 8)

The mass of the coin is $\frac{0.0198 \text{ N}}{9.8 \text{ m/s}^2} \approx 0.00202 \text{ kg}$, or about 2.0 grams.

Answer to Exercise 5 (on page 9)

(a)

$$\begin{aligned} F &= G \frac{m_1 m_2}{r^2} \\ &= (6.674 \times 10^{-11}) \frac{(6.0 \times 10^{24})(8.0 \times 10^{22})}{(4.0 \times 10^6)^2} \\ &= 2.0022 \times 10^{24} \end{aligned}$$

(b) Now we have a new distance, $r' = 8.0 \times 10^6 \text{ m}$, double the original. Notice that

gravitational force is inversely proportional to the square of the distance:

$$F \propto \frac{1}{r^2}$$

So if distance doubles,

$$r' = 2r \Rightarrow F' = \frac{F}{4}$$

Which results in 5.0055×10^{23} N

Answer to Exercise 6 (on page 9)

First convert the distance to meters: $100,000 \text{ km} = 1.0 \times 10^8 \text{ m}$. Then

$$F = G \frac{m_1 m_2}{r^2} = (6.674 \times 10^{-11}) \frac{(6.8 \times 10^3)(6 \times 10^{24})}{(10^8)^2} \approx 270 \text{ N}$$

About 270 newtons, roughly the weight of a 28-kg backpack on Earth.

Answer to Exercise 7 (on page 12)

The seesaw balances when its center of mass is right over the pivot. Put the origin at the pivot, take right as positive, and call the unknown position d :

$$\begin{aligned} x_{\text{CM}} &= \frac{\sum m_i x_i}{\sum m_i} \\ 0 &= \frac{(30)(40) + (50)(-20) + (15)d}{30 + 50 + 15} \\ &= \frac{1200 - 1000 + (15)d}{85} \\ &= \frac{200 + (15)d}{85} \\ d &= -13\frac{1}{3} \text{ cm} \end{aligned}$$

The answer is negative, so M_3 goes $13\frac{1}{3}$ cm to the *left* of the pivot.

Answer to Exercise 8 (on page 13)

On Earth, $W = mg = 70(9.8) = 686 \text{ N}$. On the Moon, $W = 70(1.62) = 113.4 \text{ N}$. The mass is the same in both places; only the weight changes.

Answer to Exercise 9 (on page 13)

$$\bar{x} = \frac{2(0)+6(4)}{2+6} = 3 \text{ m.}$$

Answer to Exercise 10 (on page 14)

$$g = \frac{GM}{R^2} = \frac{(6.674 \times 10^{-11})(5.972 \times 10^{24})}{(6.371 \times 10^6)^2} \approx 9.82 \text{ m/s}^2.$$

Answer to Exercise 11 (on page 21)

1. D, B/C/F, A, E; displacement is the difference in position between the starting and ending points. Object D moves backwards and therefore has a negative displacement. Objects B, C, and F all end in the same position they started and therefore have zero displacement. Objects A and E both move forward and have positive displacement. Object E moves forward 4 units while Object A only moves forward by 1 unit. Therefore, object E has a greater positive displacement than object A.
2. C, D, B/F, A, E; instantaneous velocity is given by the slope of a position-time graph at the indicated time (in this case, $t = 7.5\text{s}$). Since the graphs show 10 seconds of motion and there are 4 tick marks on each x-axis, each unit on the x-axis represents 2.5 seconds of time. Therefore, $t = 7.5\text{s}$ is the third tick mark on the x-axis. At $t = 7.5\text{s}$, the graphs of objects C and D have negative slopes and therefore negative velocities. Since the slope for object C is steeper than the slope for object D, object C's *speed* is greater than object D's, and object C's *velocity* is more negative than object D's. The graphs of object's B and F are horizontal at $t = 7.5\text{s}$ and therefore their velocities are zero. Graphs A and E have positive slopes, and since E is steeper, object E has a greater speed and more positive velocity than object A.
3. B/F, A, D, E, C; distance is a scalar and therefore always positive. B and F do not change position, and therefore travel a distance of 0. A moves forward 1 unit. D moves backwards 3 units (for a *distance* of 3). E moves forward 4 units. C moves forwards 3 units, then backwards 3 additional units, for a total of 6 units of distance.

Answer to Exercise 12 (on page 25)

At its apex, the hammer's velocity is 0m/s , so we will find the time where $v = 0\text{m/s}$:

$$0 \frac{\text{m}}{\text{s}} = 12 \frac{\text{m}}{\text{s}} - 9.8 \frac{\text{m}}{\text{s}^2} \cdot t$$

$$t = \frac{12 \frac{\text{m}}{\text{s}}}{9.8 \frac{\text{m}}{\text{s}^2}} \approx 1.22 \text{s}$$

Answer to Exercise 13 (on page 32)

1. Acceleration: the satellite is moving in a circle, therefore changing direction and accelerating.
2. Not acceleration: the car is not changing speed or direction.
3. Acceleration: the child is changing speed.
4. Acceleration: the roller coaster is changing direction.
5. Acceleration: the roller coaster is changing speed.
6. Not acceleration: the book is not changing speed or direction.

Answer to Exercise 14 (on page 34)

If you accelerate to 20 m/s in 120 s, the acceleration is:

$$a = \frac{v_f - v_i}{t} = \frac{20 \text{ m/s} - 0 \text{ m/s}}{120 \text{ s}} = \frac{1}{6} \frac{\text{m}}{\text{s}^2}$$

To achieve this acceleration, you will need to apply a force of:

$$F = m \cdot a = 2400 \text{ kg} \cdot \frac{1}{6} \frac{\text{m}}{\text{s}^2} = 400 \text{ N}$$

Answer to Exercise 15 (on page 36)

C, A, E/F, B, D

$$\begin{aligned} a_A &= \frac{F_{\text{net},A}}{m_A} = \frac{4\text{N} - 3\text{N}}{1\text{kg}} = \frac{1\text{N}}{1\text{kg}} = 1 \frac{\text{m}}{\text{s}^2} \\ a_B &= \frac{F_{\text{net},B}}{m_B} = \frac{3\text{N} - 4\text{N}}{1\text{kg}} = \frac{-1\text{N}}{1\text{kg}} = -1 \frac{\text{m}}{\text{s}^2} \\ a_C &= \frac{F_{\text{net},C}}{m_C} = \frac{4\text{N} - 3\text{N}}{0.5\text{kg}} = \frac{1\text{N}}{0.5\text{kg}} = 2 \frac{\text{m}}{\text{s}^2} \end{aligned}$$

$$a_D = \frac{F_{\text{net},D}}{m_D} = \frac{3\text{N} - 4\text{N}}{0.5\text{kg}} = \frac{-1\text{N}}{0.5\text{kg}} = -2\frac{\text{m}}{\text{s}^2}$$

$$a_E = \frac{F_{\text{net},E}}{m_E} = \frac{5\text{N} - 5\text{N}}{0.5\text{kg}} = \frac{0\text{N}}{0.5\text{kg}} = 0\frac{\text{m}}{\text{s}^2}$$

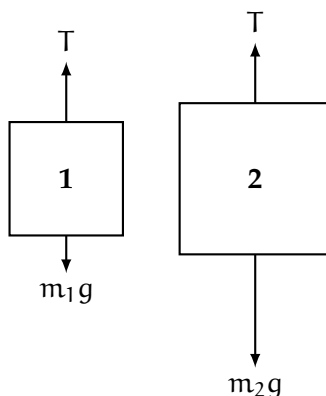
$$a_F = \frac{F_{\text{net},F}}{m_F} = \frac{5\text{N} - 5\text{N}}{1\text{kg}} = \frac{0\text{N}}{1\text{kg}} = 0\frac{\text{m}}{\text{s}^2}$$

Answer to Exercise 16 (on page 40)

3, 1/2, 6, 5, 4 The velocity does not affect the apparent weight, only the acceleration. If the elevator is accelerating upwards, the apparent weight increases. If the elevator is accelerating downwards, the apparent weight decreases. In fact, for scenario 4, the elevator is in free-fall and the person has no apparent weight. When the elevator is not accelerating, the person's apparent weight is their true weight.

Answer to Exercise 17 (on page 44)

1. Each block is acted on by gravity and the tension of the string. The force of gravity on block 2 is 3 times that of block 1, and the tension vector should be between the lengths of the gravity vectors:



2. Block 1 will have a positive acceleration and Block 2 a negative one. Both magnitudes are less than g : Block 2 falls, but the string holds it back, so it falls more slowly than it would on its own.
3. Since the blocks are connected, they will move as a system and therefore have the same magnitude acceleration. From this, the free body diagrams, and Newton's Second Law, we know that:

$$m_1 a = T - m_1 g$$

$$m_2(-a) = T - m_2g$$

We know m_1 and m_2 , so the two unknowns are T and a . One way to solve this system of equations would be to subtract equation 2 from equation 1:

$$m_1a + m_2a = m_2g - m_1g$$

$$a = \frac{m_2 - m_1}{m_1 + m_2}g$$

Substituting for the masses, we see that:

$$a = \frac{9 - 3}{9 + 3}g = \frac{6}{12}g = \frac{1}{2}g$$

Therefore, Block 1 is accelerating at $+\frac{1}{2}g$ and Block 2 is accelerating at $-\frac{1}{2}g$.

Answer to Exercise 18 (on page 48)

Kinetic energy? $E = \frac{1}{2}mv^2 = (.5)(55)(11^2) = 3,327.5 \frac{\text{kgm}^2}{\text{s}^2} = 3,327.5 \text{ joules}$.

Frictional force? $F = \mu N = (0.7)(55)(9.8) = 377.3 \text{ newtons}$.

Distance? $D = \frac{3,327.5}{377.3} \approx 8.8 \text{ meters}$. (Friction does work $W = F_f D$, and that work removes all of the kinetic energy.)

Answer to Exercise 19 (on page 49)

The empty sled is pushing directly down on the floor with a force of $(50)(9.8) = 490 \text{ N}$.

The force to overcome the static friction is:

$$270 = 490\mu_s$$

Thus, $\mu_s = 0.551$

The force to match kinetic friction is:

$$220 = 490\mu_k$$

Thus, $\mu_k = 0.449$

Once you are on the sled, it is pressing directly down on the floor with a force of $(50 + 55)(9.8) = 1,029$ N.

The force to overcome the static friction is:

$$F = (1,029)(0.551) = 567 \text{ N}$$

Once the sled is moving, friction is counteracting some of your force. How much?

$$F_f = (1,029)(0.449) = 462 \text{ N}$$

All of your acceleration comes from the leftover $567 - 462 = 105$ N.

We know that $F = ma$. In this case $F = 105$ N and $m = 105$ kg. So

$$a = \frac{105}{105} = 1.0 \text{ meters per second per second}$$

Answer to Exercise 20 (on page 57)

$$4.5 \text{ kWh} \left(\frac{3.6 \times 10^6 \text{ joules}}{1 \text{ kWh}} \right) \left(\frac{1 \text{ calories}}{4.184 \text{ joules}} \right) = \frac{(4.5)(3.6 \times 10^6)}{4.184} = 3.87 \times 10^6 \text{ calories}$$

Answer to Exercise 21 (on page 58)

$$\frac{0.1 \text{ gallons}}{2 \text{ minutes}} \left(\frac{3.7854 \text{ liters}}{1 \text{ gallons}} \right) \left(\frac{1000 \text{ milliliters}}{1 \text{ liters}} \right) \left(\frac{1 \text{ minutes}}{60 \text{ seconds}} \right) = \frac{(0.1)(3.7854)(1000)}{(2)(60)} \text{ ml/second} = 3.1545 \text{ ml/second}$$

Answer to Exercise 22 (on page 59)

Convert miles to meters: $3 \text{ mile} \times \frac{1.6093 \times 10^3 \text{ m}}{1 \text{ mile}} = 4827.9 \text{ m}$. Then convert meters to feet:
 $4827.9 \text{ m} \times \frac{1 \text{ ft}}{0.3048 \text{ m}} \approx 15,840 \text{ ft}$.

Answer to Exercise 23 (on page 62)

$$f = \frac{c}{\lambda} = \frac{3.00 \times 10^8}{0.5} = 6.00 \times 10^8 \text{ Hz}.$$



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