Contents

Chapter 1

Introduction to the Kontinua Sequence

The purpose of this book is to help you along the long and difficult trek to becoming a modern problem solver. As you explore this path, you will learn how to use the tools of math, computers, and science.

If this path is so arduous, it is only fair to ask why you should bother in the first place. There are big problems out there that will require expert problem solvers. Those people will make the world a better place, while also enjoying interesting and lucrative careers. We are talking about engineers, scientists, doctors, computer programmers, architects, actuaries, and mathematicians. Right now, those occupations represent about 6% of all the jobs in the United States. Soon, that number is expected to rise above 10%. On average, people in that 10% of the population are expected to have salaries twice that of their non-technical counterparts.

Solving problems is difficult. At some point on this journey, you will see people who are better at solving problems than you are. You, like every other person who has gone on this journey, may think "I have worked so hard on this, but that person is better at it than I am. I should quit." *Don't*.

Instead, remember these two important facts. First, solving problems is like a muscle. The more you do, the better you get at it. It is OK to say "I am not good at this yet." That just means you need more practice.

Second, you don't need to be the best in the world. 10 million people your age can be better at solving problems than you, *and you can still be in the top 10% of the world*. If you complete this journey, there will be problems for you to solve and a job where your problem-solving skills will be appreciated.

Where do we start?

The famous physicist Richard Feynman once asked, "If, in some cataclysm, all of scientific knowledge were to be destroyed, and only one sentence was passed on to the next generation of creatures, what statement would contain the most information in the fewest words?"

His answer was "All things are made of atoms—little particles that move around in perpetual motion, attracting each other when they are a little distance apart, but repelling upon being squeezed into one another."

That seems like a good place to start.

Chapter 2

Matter and Energy

The universe is made of matter and energy. Current models posit that the universe is approximately 68% dark energy, 27% dark matter, and 5% ordinary matter. Everything you can see and touch is part of the small part of the universe made of ordinary matter. Most science deals with ordinary matter and its interactions; highly trained theoretical physicists are currently debating the nature and effects of dark matter and dark energy.

What is this ordinary matter made of? All things (including the air around you) are made of *atoms*. Atoms are incredibly tiny — there are more atoms in a drop of water than there are drops of water in all the oceans. It is not easy to visualize the scale, but computers allow us to make simulations that can give us an idea. See this cool website by Cary and Michael Huang shows different size by scales! It may help to start at a length of $10^{-15.5}$ to see protons and neutrons for scale.

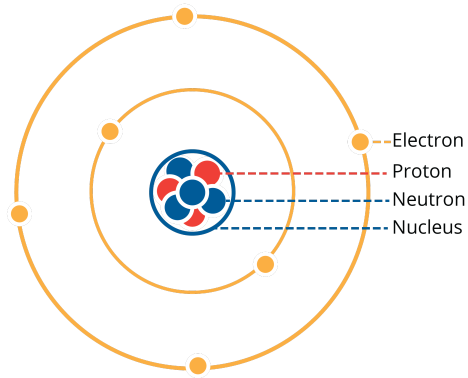


Figure 2.1

Every atom has a *nucleus* that contains *protons* and *neutrons*. Orbiting around the nucleus is a cloud of *electrons*. However, the mass of the atom comes mainly from the protons and neutrons, since they are about 2000 times as massive as an electron! These three particles, protons, neutrons, and electrons, are called *subatomic particles*. (See figure 2.1.)

Studies such as particle physics, nuclear physics, and astrophysics analyze how these particles interact.

Notice how these atoms are mostly empty space in between the subatomic particles. The force, or attractive pull, keeping them together is called *electromagnetic force*. What keeps these charged atoms in their orbits is the **electromagnetic field**.

The charges of every particle create an *electric field* around it. The orbitting and movement of an subatomic particles causes its *magnetic field. This is why electric current* (*moving electrons in a wire*) *produces a magnetic force. The faster the particles move, the stronger the field.*

2.1 Atoms and Their Models

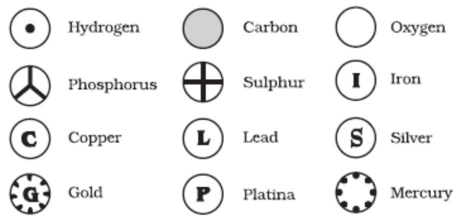


Figure 2.2

Over the history of science, there have been many ideas about the structure of atoms. The first model of the atom was rudimentary; it was modeled as one large, solid, uniform, and neutral object. Then British physicist J.J. Thomson discovered that atoms could be split into a light, negatively charged particle and a heavier, positively charged particle (we now know this is the nucleus, the dense grouping of protons and neutrons in the center of an atom).

Suddenly, the atom went from neutral and indivisible to made of different types of charged particles. Further experiments by Ernest Rutherford showed the atom to be mainly empty space, further updating scientists' model of the atom. Subsequently, Niels Bohr explained the phenomena of spectral lines (we will discuss this further in Sequence 2) by proposing that electrons orbit at specific distances from the nucleus.

Figure 2.3

This is likely the model you are most familiar with seeing, and it is the one we will use most often in this text. The first figure shown in this chapter is a Bohr model: it shows the protons and neutrons in the nucleus, and models the electrons as moving in discrete orbits around the nucleus.

However, the Bohr model is slightly inaccurate. While it is a convenient model for thinking about atoms, in reality, electrons don't neatly orbit the nucleus. Scientists don't know exactly where an electron will be in relation to the nucleus, but they do know where it is most likely to be. They use a cloud that is thicker in the center but fades out at the edges to represent an electron's position (see figure 2.3). This cloud represents the probability distribution of finding an electron in a particular region around the nucleus. The figure is only in two dimensions, but the actual electron cloud is three-dimensional. The shape of the cloud depends on the energy level of the electron, and different energy levels have different shapes and sizes of clouds.

While the cloud model is more accurate, we will use the Bohr model as it allows you to easily and quickly assess the number and arrangement of electrons.

2.1.1 Classifying Atoms

We classify atoms by the numbers of protons they have. An atom with one proton is a hydrogen atom, an atom with two protons is a helium atom,

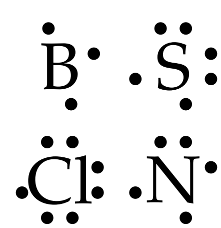


Figure 2.4

and so forth (refer to Table **??** on page **??**)). We say that hydrogen and helium are *elements* because the classification of elements is based on the proton number. And we give each element an atomic symbol. Hydrogen gets H, helium gets He, oxygen gets O, carbon gets C, and so on. You can see an element's symbol on the periodic table.

An atom will typically have the same number of electrons as proton (with an overall electric charge of 0), but when it gains or loses electrons, it becomes an *ion* with a net electric charge. **Ionization** is the addition or removal of certain particles to reach a charge imbalance. Atoms can gain or lose electrons through various processes, such as chemical reactions or ionizing radiation. An atom likes to gain or lose electrons to achieve a full outer electron shell, typically with 8 electrons (the octet rule). When an atom gains electrons, it becomes a negatively charged ion, or *anion*. When it loses electrons, it becomes a positively charged ion, or *cation*. .

2.1.2 When Atoms Combine

When atoms of different elements combine, they make *compounds*. Compounds are substances made up of more than one element. Compounds can be *molecules* or *crystal lattices*. In the next section you'll learn *why* these different structures form.

There are many kinds of compounds. You know a few:

- Table salt is crystals made of Na^+ and Cl^- ions: a sodium atom that lost an electron and a chlorine atom that gained an electron
- Baking soda, or sodium bicarbonate, is $NaHCO_3$.
- O_2 is the oxygen molecules that you breathe out of the air (air, a blend of gases, is mostly N_2.).
- Common quartz is SiO_2: silicon dioxide

The subscripts indicate what quantities of the elements are present in the compound. Each number indicates the quantity of the preceding element. A drop of water, H_2O, has two hydrogen atoms and one oxygen atom. Often times we'll see these numbers used to determine the ratio of elements in a compound. To do this, we compare the quantities of each element to find their simplest whole-number ratio. This is called the *empirical formula* of the compound.

Example: What is the ratio of elements present in Epsom salt?

Solution: Epsom salt, chemical name magnesium sulfate, has the chemical formula $MgSO_4$.

Therefore, the ratio of Mg:S:O is 1:1:4.

Exercise 1 Ratios of Atoms in Molecules

Give the elemental ratio for each compound.

1. methane, CH_4
2. copper (II) sulfate, $CuSO_4$
3. glucose, $C_6H_{12}O_6$

Working Space

Answer on Page 65

2.2 Types of Matter

One way to classify matter is by the types of chemical bonds that hold a material's atoms together. The nature of these bonds, in turn, affects the properties of the material. For now, all you need to know is there are three types of chemical bonds: *metallic*, *covalent*, and *ionic*. Materials held together with the same type of bonds tend to have similar properties. For example, copper, bronze, iron, and steel (all containing metallic bonds) are all shiny, ductile, malleable, and good conductors of heat and electricity. On the other hand, Epsom salt and table salt form large crystals, have very high melting points, and are poor conductors of electricity in their pure form. These two substances (Epsom and table salt) both contain ionic bonds.

2.2.1 Covalent Compounds

Water is an example of a covalent compound: it is made of two hydrogen atoms covalently bonded to one oxygen atom (see figure 2.5). The result is a water molecule. The different atoms cluster together because they share electrons in their clouds. This is the nature of a *covalent bond*: it is formed when atoms share electrons. Sometimes, electrons are shared evenly, but in water, they are shared unevenly. Oxygen is better at attracting electrons to itself than hydrogen, and so the shared electrons spend more time on the oxygen atom than the hydrogen atoms. As a result, the oxygen side of a water molecule has a slight negative charge, while the hydrogen atoms have a slight positive charge. These slight charges are represented with a lower case Greek letter delta, δ. When electrons are shared unevenly, we call this a *polar* covalent bond, because there are positive and negative poles at either end of the bond.

Whether electrons are shared evenly or unevenly is based on the elements' relative *electronegativities*. Electronegativity is simply a measure of how strongly an atom can attract electrons to itself. In general, elements on the right side of the periodic table are more electronegative than elements on the left side. When covalent bonds form between two elements of similar electronegativities, the electrons are shared evenly. We call this a *non-polar* covalent bond. Oil is an example of a non-polar covalent substance. Different oils have different ratios and structures, but all oils are made mainly of carbon and hydrogen, which have similar electronegativities. What happens when you try to mix oil and water? They don't mix well! This is due to the difference between their bond types. Polar substances, like water, mix best with other polar substances, while non-polar substances, like oils, mix best with non-polar substances. You'll learn more about why this is in Sequence 2.

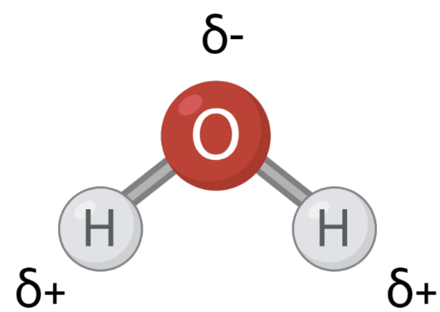
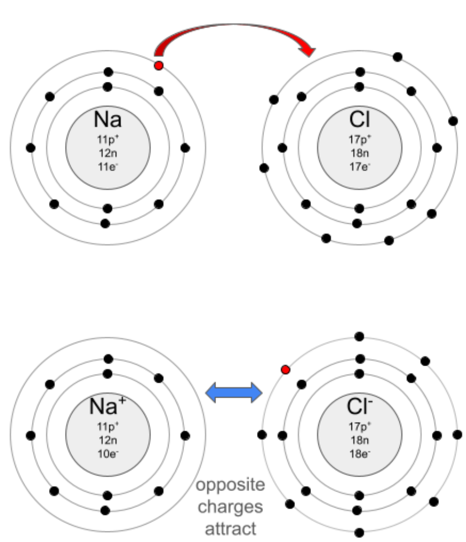


Figure 2.5

For both polar and non-polar covalent bonds, the electrons are held tightly to the nuclei, even if they are shared among atoms. Those electrons don't move to another molecule: they move around within the molecule they are already a part of. Since electrons don't flow freely in covalent substances, they are also poor conductors of electricity. Covalent compounds also tend to have lower melting and boiling points compared to ionic compounds.

2.2.2 Ionic Compounds

Ionic bonds are the electrical attraction between opposite-charged ions. When a neutral atom gains or loses an electron it becomes an *ion* (a charged atom), and oppositely-charged ions are attracted to each other. Which atom gets the electron and which loses it is based on their electronegativities: the more electronegative atom steals one or more electrons from the less electronegative atom.



Let's examine how a simple ionic compound is formed: sodium chloride, also known as table salt, is made up of sodium and chlorine atoms (see figure 2.6). When sodium and chlorine come in contact with each other, electrons move from the sodium to the chlorine, making a sodium *cation* (positively-charged ion) and a chloride *anion* (negatively-charged ion). Yes — chlor*ide* is correct! When naming an anion, the ending of the element name changes to *-ide*. Once the sodium cation and chloride anion are formed, their opposite charges attract them to each other, like north and south magnet poles.

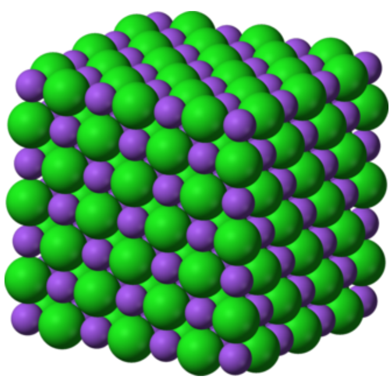


Figure 2.7

When there are many, many sodium and chloride ions around, they spontaneously arrange themselves in a pattern, giving ionic compounds their characteristic crystal structure (see figure 2.7). Because the electrons are tightly held by each ion, ionic substances don't conduct electricity well as solids. The atomic crystal lattice also determines the shape of the macroscopic crystals. Salt crystals are generally cubic, while other crystals (like quartz) form hexagonal prisms. You'll learn how to predict the atomic and macroscopic crystal structure of different compounds in Sequence 2.

Modeling Ionic Compounds

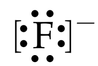
You can represent ions with Lewis dot diagrams by adding or subtracting electrons from the model. Since electrons are negative, anions have gained electrons while cations have lost them. Additionally, the ion is in brackets and the overall charge is indicated outside the top right corner of the brackets.

Example: Create Lewis dot diagrams for Na^+, F^-, O^{2+}, and Mg^-.

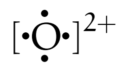
Solution: Sodium is in column 1, so a neutral sodium atom has 1 valence electron. A +1 charge means it has lost 1 electron, leaving zero.

$$[\mathrm{Na}]^+$$

Fluorine is in column 17 and has 7 valence electrons when neutral. The anion F^- has gained one electron, for a total of 8.



Neutral oxygen has 6 valence electrons, therefore O^{2+} has 4.



Neutral magnesium has 2 valence electrons, so Mg^{-} has 3 valence electrons.

To make Lewis dot diagrams of ionic compounds, you show both ions in the ratio given in the formula. For example, a Lewis dot diagram of $MgCl_2$ would show one magnesium cation and two chloride anions.

2.2.3 Metallic Compounds

You may already know that metals are excellent conductors of electricity and heat. This is a consequence of their metallic bonds. In pure metals and alloys, the outermost layer of electrons can move freely from one atom to the next. As a result, at the atomic level, metals are best characterized as a lattice of cations surrounded by a ”sea of electrons” (see figure fixme metallic bonding figure)

The free-flowing sea of electrons in pure and alloyed metal means the cation lattice can be rearranged without breaking the metallic bond. As a result, metals are ductile (able to be drawn out into a wire) and malleable (able to be hammered into a new shape without cracking or breaking). (fixme figure showing deformation of cation lattice in bending metal)

Metals can be pure, like copper or iron, or *alloys*, like bronze or steel. *Alloys* are mixtures between two or more elements where at least one element is a metal. Steel is an alloy of iron and carbon; bronze is an alloy of copper and tin. Alloying a metal changes its properties because of the change in the cation lattice (see figure FIXME pure vs alloyed lattice and structural changes).

2.3 Energy and Work

Look around you: things are moving, warming up, lighting up, and changing. No physical property changes without *work* being done. What is work? No, we are not talking about having a job. Work is defined as a change in Energy. Every object has some type of energy. When you climb a flight of stairs, you feel tired. That tiredness is evidence that energy has left your body and gone somewhere else—into lifting your body upward. When the Sun warms your skin, energy is traveling through space and into you. When a moving car comes to a stop, its motion doesn't simply disappear; energy is transferred to the brakes and the surrounding air, mostly as heat.

Energy Transformations are all around us:

1. Feeling less tired after consuming calories from food
2. The light and the feelin of heat from the Sun
3. A ball speeding up while rolling down a hill
4. Being out of breath after climbing 5 flights of stairs
5. A car stopping when brakes are applied

Some types of energy are easy to visualize, while others are not. Energy is what moves from one object to another when work is being done. There are two main types of energy: energy that *stored* and energy that is motion. Most times, energy that is stored is transformed into energy in motion. This stored energy is called *Potential Energy* and the energy in motion is referred to as *Kinetic Energy*.

When you lift something, the energy stored in your body (in the form of sugar and fat) is transferred to the object, accelerating it upwards. Your body continues to transfer energy as you lift the object against gravity. When you've lifted it as high as you can, most of the energy your body lost (we call this "burning Calories" colloquially) is stored as *Potential Energy*, due to the object's height. Potential energy is the stored energy an object has due to its position or configuration, rather than its motion. *Kinetic Energy* is the term we use to describe the energy an object possesses due to its motion; you'll learn more about different forms of energy in 3, and in Sequence 2.

Another example of energy is a simple circuit connecting a battery and a light bulb. The battery has stored potential energy. When the circuit is complete, the potential energy in the battery is transferred to electrons in the light bulb, causing them to move and gain kinetic energy. In the light bulb's filament (we are referencing old, non-LED light bulbs here!), the electrons encounter resistance, which slows them down. The energy the electrons lose in this process is being transformed into light and heat, lighting your room.

The Work-Energy theorem explains the relationship between work and energy, and we will introduce the theorem and use it to explain energy transfer in a subsequent chapter.

2.4 Mass-Energy Equivalence

You've probably seen the equation

$$E = mc^2$$

E is energy, m is mass, and c is the speed of light in a vacuum (about $3 \times 10^8 \frac{m}{s}$). So far, we've been discussing matter and energy separately. This equation shows that matter can be converted to energy, and vice-versa. This is the source of the light energy emitted by the sun.

The Sun is mostly hydrogen. At the very center, it is so hot and dense that the nuclei of hydrogen atoms are fused together to form helium atoms, a process called the *proton-proton chain reaction*. The actual reaction involves several steps and is more complicated, but the overall process can be summarized as:

$$4\text{H} \rightarrow \text{He}$$

If every hydrogen atom has a mass of approximately $1.6735575 \times 10^{-27}$ kg and every helium atom has a mass of approximately $6.6464731 \times 10^{-27}$ kg, how much energy is released when one atom of helium is created? First, notice that one helium has less mass than 4 hydrogen atoms:

$$4 \times \left(1.6735575 \times 1{-}^{27}\right) - 6.6464731 \times 10^{-27} = 4.77569 \times 10^{-29}$$

Now, we can use $E = mc^2$ to find out how much energy is equivalent to 4.77569×10^{-29} kg:

$$E = \left(4.77569 \times 10^{-29}\text{kg}\right)\left(2.99792458 \times 10^8 \frac{\text{m}}{\text{s}}\right)^2$$
$$E \approx 4.292 \times 10^{-12}\text{joules}$$

All of these numbers are very small and hard to visualize. We could ask this: if 1 kilogram of hydrogen (about enough to fill a standard beach ball) were fused to make helium, how much energy would be released? For every kilogram of hydrogen that enters the proton-proton chain reaction, about 0.02854 kilograms of mass are converted to energy (the mass of about 5 quarters).

$$E = (0.02854 \text{ kg})\left(2.99792458 \times 10^8 \frac{\text{m}}{\text{s}}\right)^2 \approx 2.5647 \times 10^{15} \text{ joules}$$

This is more than 700,000,000 kWh (kilowatt hours); the average US household uses only 30 kWh per day. So, fusing one kilogram of hydrogen releases enough energy to power an average US home every day for *over 65,000 years*! This relatively huge release of energy is why scientists continue to research nuclear fusion energy sources. The nuclear power plants currently running around the world today rely on *nuclear fission*: the splitting apart of atoms, the opposite process of *nuclear fusion*. Nuclear fission releases much, much less energy per kilogram of input material than nuclear fusion, and thus stable, affordable nuclear fusion power plants remain a ”holy grail” of scientific research.

2.5 Conclusion

We have seen that the universe is made of dark energy, dark matter, and ordinary matter. Ordinary matter is made of atoms, which can be classified based on their number of protons. Atoms combine in different ways to make compounds, and the manner of combination (ionic, covalent, or metallic bonding) affects the macroscopic properties of the substance. Energy allows matter to do work, and work is the transfer of energy.

Matter and energy do share a fundamental property: they are both conserved. Neither matter nor energy can be created or destroyed. This means the total amount of ordinary matter and energy in the universe is constant. This great scientific truth *something about its application* In the next chapter,

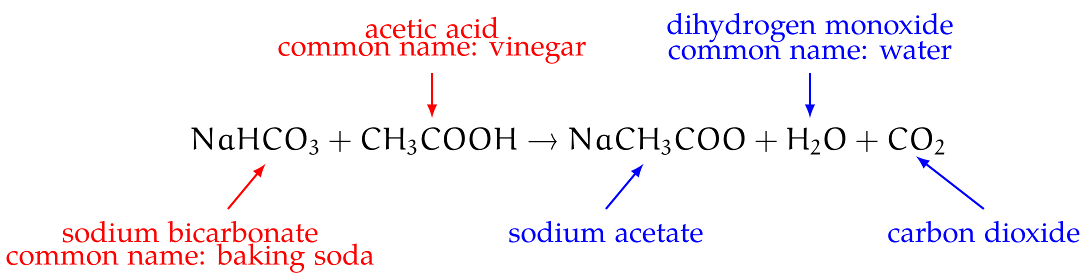
Chapter 3

Conservation of Mass and Energy

One of the most fundamental laws in science is the conservation of mass and energy. This law states that mass (matter) and energy cannot be created or destroyed. This means the total matter and energy in the universe always stays the same.

3.1 Conservation of Mass

Since matter cannot be created or destroyed, a chemical reaction does not change the mass of the *reactants* as they form *products*. Consider the reaction between vinegar and baking soda, which produces carbon dioxide, water, and sodium acetate:



The reactants are labeled with red, and the products with blue. This equation is *balanced*: that is, it shows the same number of each element on each side of the arrow. This shows that the atoms are not created or destroyed during a chemical reaction, they are only rearranged. Take a minute to count up each element on each side. You should find there are 5 hydrogens, 1 sodium, 3 carbons, and 5 oxygens on each side.

Now, let's look at an *unbalanced* chemical reaction. As you know, hydrogen and oxygen combine to form water. Additionally, hydrogen and oxygen both exist as *diatomic gases*. When we say "oxygen gas" or "hydrogen gas", we mean the diatomic molecules, O_2 and H_2, respectively. Here is an unbalanced chemical reaction between hydrogen gas and oxygen gas to form water:

$$H_2 + O_2 \rightarrow H_2O$$

How do we know this equation is unbalanced? Count up the elements: there are two oxygen atoms on the reactant side, but only one on the product side. This violates the conservation of matter: that oxygen atom cannot just disappear!

3.1.1 Balancing Chemical Reactions

We solve this by *balancing* the chemical reaction: adjusting the number of products and reactants to comply with the Law of Conservation of Matter. You'll learn strategies for balancing chemical reactions in Sequence 2, but for now we'll briefly balance this chemical reaction so that it complies with the Law of Conservation of Matter. You may be tempted to simply add a lone oxygen atom to the products side:

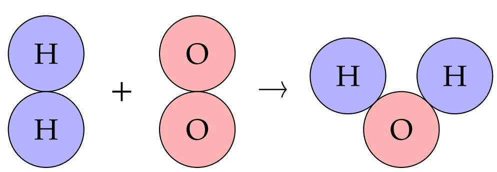
$$H_2 + O_2 \rightarrow H_2O + O$$

The major reason this is incorrect is that oxygen does not exist as a lone atom - as discussed above - so it doesn't make sense to have a lone oxygen as a product. So maybe we should add a molecule of oxygen gas to both sides?

$$H_2 + O_2 \rightarrow H_2O + O_2$$

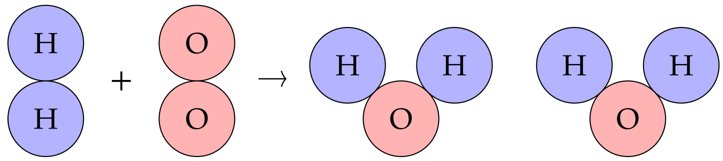
Well, now we have the same problem we started with: the oxygens are unbalanced. When balancing chemical reactions, we can only add *whole molecules* that are already in the reaction. Let's take another look at our unbalanced reaction with some molecular models for visualization:

$$H_2 + O_2 \rightarrow H_2O$$



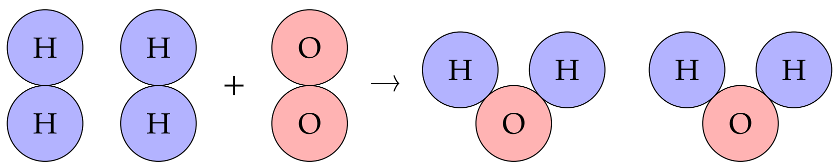
You can clearly see we need more oxygens on the product side. Since we can only add *whole molecules*, our only option is to add another water. We do this by adding a coefficient of 2 in front of H_2O in our equation, which indicates 2 water molecules (just like $2x$ means two x's):

$$H_2 + O_2 \rightarrow 2H_2O$$



We've fixed our oxygen problem: now there are two oxygen atoms on both sides. But now we have a hydrogen problem: there are 2 on the reactant side and 4 on the product side. We can address this by adding another hydrogen gas molecule to the reactant side:

$$2H_2 + O_2 \rightarrow 2H_2O$$



And now we have the same number of hydrogens and oxygens on each side! Notice we have all the same reactants and products that we started with, but now in ratios that reflect the conservation of matter.

A final note: if atoms are in parentheses followed by a subscript, the subscript applies to every atom in the parentheses. For example, zinc nitrate, $Zn\,(NO_3)_2$ is made of 1 zinc, 2 nitrogens, and 6 oxygens.

Exercise 2 Balanced and Unbalanced Reactions

Classify the following chemical reactions as balanced or unbalanced. If it is unbalanced, state what element(s) are not conserved.

1. $NiCl_2 + 2NaOH \rightarrow Ni\,(OH)_2 + 2NaCl$
2. $HgO \rightarrow Hg + O_2$
3. $BaSO_4 + 2C \rightarrow 2BaS + CO$
4. $Cd\,(NO_3)_2 + H_2S \rightarrow CdS + 2HNO_3$

Working Space

Answer on Page 65

3.2 Conservation of Energy

Just like matter, energy is also conserved: it cannot be created or destroyed, only change forms. You'll learn more about the types of energy in a subsequent chapter, Work and Energy. The transformation of energy from one type to another drives our modern world: your phone transforms electrical potential energy into light and sound energy, a nuclear power plant transforms nuclear energy to electrical energy, and your car transforms chemical potential energy (in the gasoline) into kinetic energy (motion).

3.2.1 Friction, Heat, and Energy "Loss"

Imagine rolling a ball across a flat surface: you have given the ball some kinetic energy in its motion. If the kinetic energy were conserved, the ball would keep rolling at the same speed forever, as long as it was on a flat surface. Experience tells us this isn't what happens: the ball will eventually come to a stop. Why doesn't this violate the Conservation of Energy?

Friction is the force that opposes motion: whenever you slide two objects past each other, friction transforms kinetic energy into heat. Rub the palms of your hands together. You should feel warmth, a product of the friction between your hands. As the ball in the example above rolls, it also experiences friction between itself and the ground. The friction slowly transforms the kinetic energy of the ball into heat, causing the ball to lose kinetic energy. When all of the ball's kinetic energy is transformed to heat, the ball comes to a rest. So, the kinetic energy of the ball wasn't destroyed and didn't disappear: it became heat.

In fact, nearly all energy in the universe will eventually be transformed to heat, resulting in the inevitable "heat death" of the universe. Here is a short video about heat, entropy, and the heat death of the universe: https://www.youtube.com/watch?v=gOWt_Hq3yrE/.

3.2.2 Energy Conservation in Falling Objects

When an object is positioned above the ground, it has *gravitational potential energy*. The potential energy is proportional to the mass of the object, the strength of the gravitational field, and the object's height above the ground:

$$E_p = mgh$$

where m is the mass of the object in kg, g is the acceleration due to gravity ($9.8 m/s^2$ on Earth), and h is the height above the ground in meters.

Example: What is the gravitational potential energy of a 5.0 kilogram bowling ball in the hand of a bowler (approximately 1.2 meters high)?

Solution: The mass is 5.0 kg, the height is 1.2 m, and we assume the bowler is on Earth, so g is 9.8 m/s^2:

$$E_p = (5.0\text{kg})\left(9.8\text{m}/\text{s}^2\right)(1.2\text{m}) = 59\text{J}$$

Here is a new unit: *joules*, represented by a capital J. A joule is a unit of energy, and it is the same as $\frac{\text{kg}\cdot\text{m}^2}{\text{s}^2}$.

If the bowler were to drop that ball, it would lose potential energy and gain *kinetic energy*. The kinetic energy of an object is proportional to its mass and the square of its speed:

$$E_k = \frac{1}{2}mv^2$$

where mass is in kg and speed (v) is in m/s. You can quickly check that the units still come out to a joule! If there is no air resistance, then as an object falls all of its potential energy is converted to kinetic energy. (If you'd like to explore what happens when friction is accounted for, you can play with this PhET simulation: https://phet.colorado.edu/en/simulations/energy-skate-park.)

Example: If the bowling ball were dropped, what would its speed be right before it hits the ground? (Neglect air resistance.)

Solution: Ignoring air resistance, according to the Law of Conservation of Energy, the kinetic energy of the ball right before it hits the ground must be equal to the gravitational potential energy of the ball right before its release. Therefore:

$$mgh = \frac{1}{2}mv^2$$

We can eliminate an m from both sides and rearrange to solve for v:

$$v = \sqrt{2gh} = \sqrt{2\left(9.8\text{m}/\text{s}^2\right)(1.2\text{m})} = \sqrt{23.52\frac{\text{m}^2}{\text{s}^2}} = 4.8\frac{\text{m}}{\text{s}}$$

The ball will have a speed of 4.8 m/s just before it hits the ground.

Exercise 3 Kinetic and Potential Energy

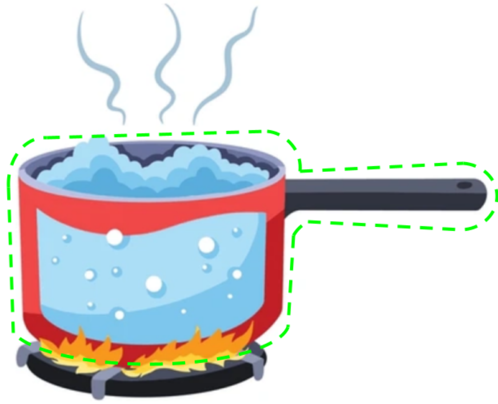
Working Space

1. How fast should you toss a ball straight up if you want it to reach your friend on the second floor (3.7 meters above you)?
2. Your little brother is teasing you from a treehouse 5.0 meters off the ground. If your slingshot can shoot a pebble at 15 m/s, can you hit your little brother?
3. A 63-kg roller-blader rolls down a hill. If the hill is 8.0 meters high and she reaches the bottom of the hill with a speed of 11.2 m/s, how much energy was converted to heat through friction?

Answer on Page 65

3.3 Types of Systems

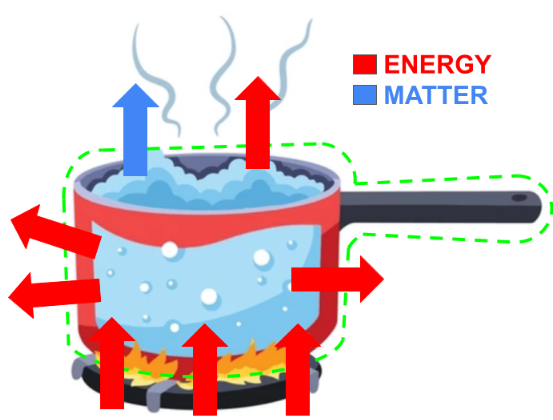
We classify systems based on the flow of matter and energy. A *system* is a set of interconnected elements. Your body is a system, as is a television or a boiling pot of water. Scientists define systems by separating the parts of the system from the rest of the universe, usually called "the surroundings". You can represent this separation with a dashed line. Here is a diagram defining a pot of boiling water as a system:



Everything inside the dashed line is the system: the pot and the water boiling in it. Everything outside the dashed line is the surroundings: the stove, the air around the pot, etc. Defining a system is *arbitrary*: there isn't one hard and fast definition of a system. Think of your school: you could look at the system of a classroom, the system of a hallway and all the classes connected to it, or the entire school building. How you define a system depends on what you're studying.

3.3.1 Open Systems

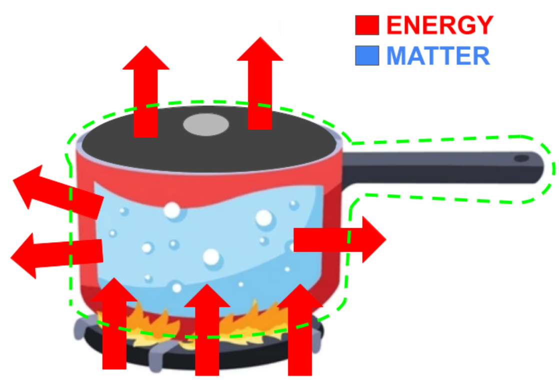
An *open system* allows for matter and energy to cross the imaginary boundary between the system and its surroundings. The uncovered pot of boiling water is an example of an open system. Energy enters the system as heat from the stove, and leaves the system as heat in the steam rising from the pot. Notice that steam rising: see how it crosses the imaginary boundary? The steam is matter *leaving the system*. Since matter and energy can cross the boundary between the system and its surroundings, the uncovered boiling pot is an open system.



The system also loses energy through the sides of the pot: if you touched the pot, it would feel hot, which means heat energy can also leave the system through the sides of the pot. This is due to the collision between air particles (the surroundings) and the outside of the pot (the system). With every collision, a little heat is transferred from the pot to the air. This is why your hot drink gets cold if you leave it out, even if you don't add any ice.

3.3.2 Closed Systems

A *closed system* allows for the transfer of energy but not the transfer of matter. If we put a lid on this pot, it would become a closed system.



Notice that the difference between an open and closed system is the flow of *matter*: open systems allow for the movement of matter, while closed systems do not. However, energy can still enter and leave closed systems. Sealed containers that aren't insulated are good examples of closed systems - a car with the windows up and doors closed.

3.3.3 Isolated Systems

An *isolated system* does not allow for the flow of matter *or* energy in or out of the system. There is no such thing as a truly isolated system: in reality, a small amount of energy can be transferred even through the best thermal insulators. However, for well-insulated systems, it can be a good approximation to model that system as isolated. A simple example would be a sealed, well-insulated coffee thermos. The transfer of heat energy between the coffee in the thermos and the thermos' surroundings is so slow that we can ignore that small amount of transfer and approximate the thermos as an isolated system.

3.3.4 Classifying Systems

To quickly categorize a system as open, closed, or isolated, ask yourself two questions:

1. Can matter enter or leave the system?
2. Can energy enter or leave the system?

If your answer to the first question is yes, you know automatically the system is open. If no, move to the second question. If energy can enter or leave, the system is closed. If not, the system is isolated. Sometimes, textbooks and exams will describe a system as ”well-insulated”. This is directing you to assume any transfer of energy between the system and surroundings is negligible, and that you should treat the system as an isolated system.

Exercise 4 Open, Closed, and Isolated Systems

Classify each system as open, closed, or isolated. Justify your answer.

1. The human body
2. Earth
3. Your cell phone
4. A well-insulated cooler with the lid sealed
5. A well-insulated cooler with the lid open
6. A bottle of soda before it is opened to be drunk

Working Space

Answer on Page 65

Chapter 4

Mass, Weight, and Gravity

Mass is a measure of the amount of matter in an object. Weight is the force of gravity on that object. An object's mass is the same no matter where it is in the universe: the amount of "stuff" in an object does not depend on its location. However, an object's weight *does* change: the same object has a different weight on the Moon than it does on Earth or Venus. Since the force of gravity on the Moon is about $1/6^{th}$ as on Earth, you would weigh $1/6^{th}$ as much on the Moon, but your mass would be the same.

4.1 Kilograms versus Pounds

In elementary school, you probably learned that 1 kilogram is about 2.2 pounds. This is only true *on Earth*. In everyday language, we may use kilograms and pounds, or mass and weight, interchangeably; scientists do not.

Mass	**Weight**
measure of *amount*	measure of *force*
units include grams, kilograms, milligrams	units include pounds, Newtons, dynes
a *scalar* measurement	a *vector* measurement
does *not* depend on location	does depend on location

On Earth, a 1 kilogram mass weighs 2.2 pounds. That same mass weighs 0.37 pounds on the Moon. You learned to "convert" between pounds in kilograms in elementary school because everywhere on Earth, a 1-kg mass weighs 2.2 lbs. The conversion doesn't apply to other locations in the universe because mass and weight are different.

4.1.1 Scalars and Vectors

One of the fundamental differences between mass and weight is the type of measurement: scalar versus vector. *Scalar* measurements are just a counting number: they tell the amount of magnitude. Mass is a scalar measurement: it tells how much matter is in an object. Energy is also a scalar measurement, as well as temperature and density. *Vectors*, on the other hand, also have a *direction*. A complete measure of a force gives the magnitude and direction. So, your weight isn't just 185 pounds: it's 185 pounds *downward*. Velocity is a speed plus a direction, such as 35 mph east. We will expand on this later in the vectors chapter!

4.2 Mass and Gravity

When we discuss weight, we usually mean the gravitational force between an object and the largest nearby object. My weight on Earth, a spacecraft's weight on the Moon, or a rover's weight on Mars are all examples. But planets and moons don't have some special property that makes them emit gravity. Rather, *all objects with mass are gravitationally attracted to each other*. But, compared to the mass of the Earth, the mass of all the other objects around you (a table, your family, your house or apartment complex) is very, very small.

The force of the gravitational attraction between two objects is proportional to the product of their masses, and inversely proportional to their distance squared. This means that as objects get farther away, the force decreases. If you double the distance, the force quarters. Quadruple the distance, the force is $1/16^{th}$ as much. This is why you are more attracted to the Earth than you are to distant stars, even though they have much more mass than the Earth.

Newton's Law of Universal Gravitation

Two masses (m_1 and m_2) that are a distance of r from each other are attracted toward each other with a force of magnitude:

$$F = G\frac{m_1 m_2}{r^2} \tag{4.1}$$

where G is the universal gravitational constant. If you measure the mass in kilograms and the distance in meters, G is about 6.674×10^{-11}. That will get you the force of the attraction in newtons.

Exercise 5 Gravitation between two planets

A small moon orbits a planet at a distance of 4.0×10^6 m from the planet's center. The mass of the planet is 6.0×10^{24} kg, and the mass of the moon is 8.0×10^{22} kg.

1. Using Newton's Law of Universal Gravitation, calculate the gravitational force between the planet and the moon.
2. If the moon were moved to a distance of 8.0×10^6 m from the planet's center (twice the original distance), determine the new gravitational force.

Working Space

Answer on Page 66

Exercise 6 Gravity

Working Space

The Earth's mass is about 6×10^{24} kilograms.

Your spacecraft's mass is 6,800 kilograms.

Your spacecraft is also about 100,000 km from the center of the Earth. (For reference, the moon is about 400,000 km from the center of the Earth.)

What is the magnitude of the force of gravity that is pulling your spacecraft and the Earth toward each other?

Answer on Page 67

4.3 Mass and Weight

Gravity pulls on things proportional to their mass, so we often ignore the difference between mass and weight.

The weight of an object is the force due to the object's mass and gravity. When we say, "This potato weighs 1 pound," we actually mean "This potato weighs 1 pound on Earth." That same potato would weigh about one-fifth of a pound on the moon (see figure 4.1).

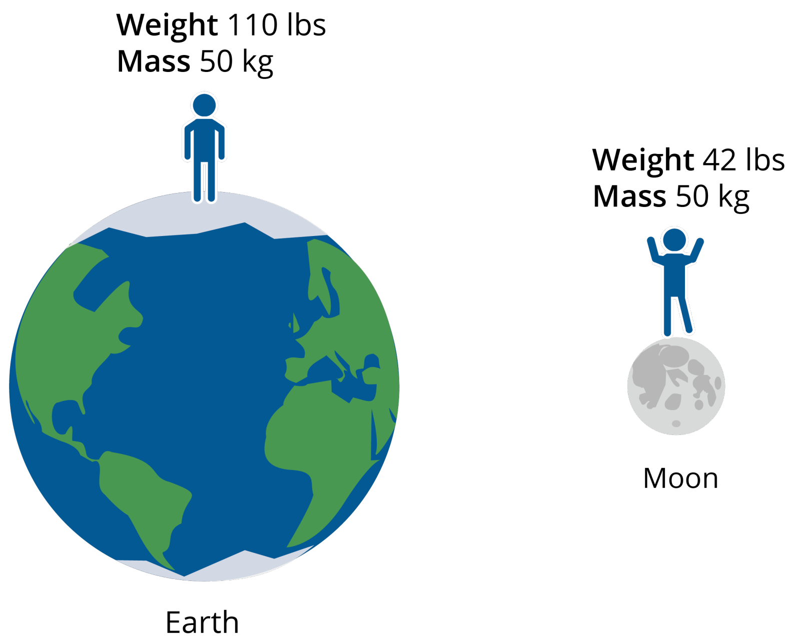


Figure 4.1: Mass is a measure of all the matter in an object. Weight is a measure of the force of gravity on that object. Mass is not location-dependent, while weight is.

However, that potato has a mass of 0.45 kg no matter where it is in the universe.

We talked about the Law of Universal Gravitation. We can simplify it for our calculations to derive the weight of objects near to the surface of the earth.

Starting with Equation 4.1, for objects near Earth's surface, the radius is essentially constant and equal to Earth's radius R_E. Substituting $r = R_E$ gives us:

$$F = G\frac{Mm}{R_E^2} \tag{4.2}$$

The only thing variable in this equation is m, so the object's mass can be factored out, leaving:

$$F = m\left(G\frac{M}{R_E^2}\right) \tag{4.3}$$

The quantity in parentheses involves only constants: the gravitational constant G, Earth's mass M, and Earth's radius R_E. Since this value stays the same for all objects near the surface, we define it as the gravitational field strength:

$$g = G\frac{M}{R_E^2} \tag{4.4}$$

Substituting this definition back into the force equation gives:

$$F = mg \tag{4.5}$$

where m is the mass of some object, and $g = 9.8\ \text{m/s}^2$ is the gravitational constant, a constant found by $G\frac{M}{R_E^2}$.

The steps in sequnce are shown below:

$$\begin{aligned} F &= G\frac{Mm}{r^2} \\ &= G\frac{Mm}{R_E^2} \\ &= m\left(G\frac{M}{R_E^2}\right) \\ g &= G\frac{M}{R_E^2} \\ F &= mg \end{aligned}$$

Equation 4.5 is the equation we will use a standard for the force of gravity, F_g, for most objects on Earth. Objects undering projectile motion, objects at rest, and objects hung by tension all have a force of gravity acting on them, [1]

Exercise 7 May the Force be with You 1

A hammer is falling along the side of a building, close to the surface of the Earth. What is it's acceleration?

Working Space

Answer on Page 67

[1] You may occasionally use 10 for g to simplify expressions if the problem says to do so, but use 9.8 otherwise.

Exercise 8 May the Force be with You 2

A coin is dropped. It is measured that the force of gravity on the coin is .0198 N. What is the mass of the coin?

Working Space

Answer on Page 67

4.4 Center of Mass

The *center of mass* of a body is the point where its mass is evenly balanced in all directions, as if all the mass were concentrated at that single point. When we talk about *Free Body Diagrams* in a future chapter, we will assume the entire body's center of mass is located at a point that we use to analyze forces from. For an object with different densities at different points — for example, a hammer with a dense metal head and a lighter wooden handle — the center of mass is found by averaging the positions of all parts, giving more weight to the heavier regions. You will see the term *center of mass* throughout this sequence; we will assume you understand how center of mass is calculated, but typically will not fully determine it.

For a one-dimensional object on some line, the center of mass can be found using the formula:

1-Dimensional Center of Mass

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2 + \cdots + m_n x_n}{m_1 + m_2 + m_n}$$

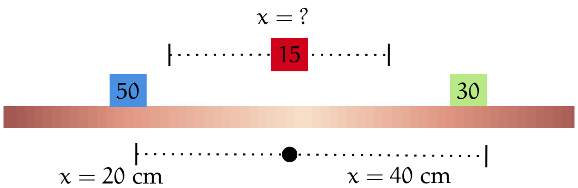
where x_{CM} and $x_1 \ldots x_n$ are the distance from *a chosen origin* location. Notice how this is similar the formula for averages.

We can rewrite this using summation notation:

$$x_{CM} = \frac{\Sigma m_i x_i}{\Sigma m_i}$$

Exercise 9 Balancing a seesaw

Three masses are placed on a seesaw. M_1 has a mass of 30 lbs and is placed 40 cm to the right the equilibrium point of the seesaw. M_2 has a mass of 50 lbs and is placed 20 cm to the left of the equilibrium point. Where does M_3, which weighs 15 lbs, need to go in order for the seesaw to balanced? Assume the seesaw plank itself is uniformly distributed.



Working Space

Answer on Page 67

In higher dimensions, it is easier to use integration to calculate the coordinates of the center of mass of some body, ecspecially when it is not uniform (equally distributed). Although we haven't introduced integration formally, it is important to begin thinking about these topics:

2D and 3D Center of Mass

$$\mathbf{r} = x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}} + z\,\hat{\mathbf{k}}$$

$$dm = \rho(x, y, z)\, dV$$

$$2\text{D}: \quad M = \iint_V \rho(\mathbf{r})\, dV, \qquad 3\text{D}: \quad M = \iiint_V \rho(\mathbf{r})\, dV$$

where M is the net mass, dm is some infinitesimal weight at point $\mathbf{r}$, the position vector within V. V is the dimensions of some the 3d boundary, and $\rho(x, y, z)$ is the density at the infinitesimal point (x, y, z)

The individual components can be found by:

$$x_{CM} = \frac{1}{M} \iiint_V x\, \rho(x, y, z)\, dV$$
$$y_{CM} = \frac{1}{M} \iiint_V y\, \rho(x, y, z)\, dV$$
$$z_{CM} = \frac{1}{M} \iiint_V z\, \rho(x, y, z)\, dV$$

Chapter 5

Introductory Motion Concepts

5.1 Position, Velocity, Acceleration

When studying motion, it is important to have ways to describe various aspects of an object.

Position is a location of an object relative to a coordinate system. The coordinate system, reference points, and *origin* is typically decided by you, the viewer. An object's position at one point in time can be n-dimensions, but typically we talk about 1D, 2D, and 3D.

- 1D can be thought of moving along a straight line (x)
- 2D can be represented on coordinate plane (x, y)
- 3D can be represented by 3 components (x, y, z)

The total distance traveled in a 1D plane is a total amount traveled. The *displacement* is net distance from the *starting location* or *origin*. Typically, a *change in position* is represented as a Δx (in horizontal direction) and Δy (in vertical direction). This distance can be thought of as a vector, with a directional component and magnitude. You may also see displacement as $\mathbf{s}$.

Average *Velocity*, usually represented as $\vec{\mathbf{v}}$ or $\mathbf{v}$ is defined as the net displacement over a certain period of time. Because displacement is a vector quantity, so is average velocity.

Average speed, however, does not contain directional information. Like the speedometer in a car, speed only contains information about change in displacement over time, $\frac{\Delta x}{\Delta t}$, making it a *scalar*. The magnitude of velocity is speed, and velocity is speed with a given direction (north, 48° south-east, etc).

The speedometer, however, tells us *instantaneous* speed, such that $\vec{\mathbf{v}} = \frac{d\mathbf{s}}{dt}$, the *derivative* of position. This gives us an instantaneous change in position, and as the dt, or difference in time, decreases, we get closer to the instantaneous speed.

Similarly, acceleration, $\mathbf{a}$, is the *instantaneous* change in velocity per difference in time, dt. This allows us to determine the instantaneous velocity using calculus: $\mathbf{a} = \frac{d\mathbf{v}}{dt}$ or equivalently, $\mathbf{a} = \frac{d^2\mathbf{s}}{dt^2}$ using second derivatives. This book will reintroduce these concepts later, but for now, know that all three quantities are related.

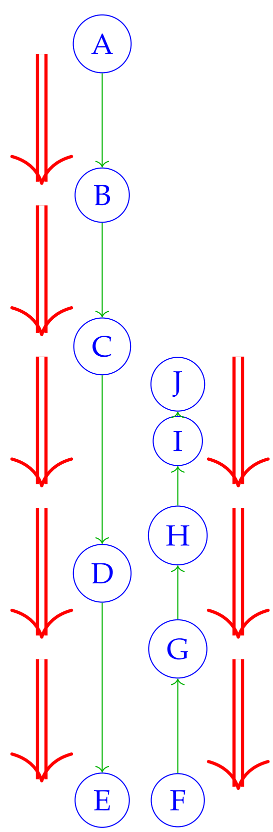
Typically, people will talk about velocity and acceleration at a *specific instant in time*, such as "at 3 seconds, the velocity is 5 m/s with a change of 2 m/s every second".

5.2 1D Motion Diagrams

Motion Diagrams are diagrams used to represent motion in a 1D plane, such as a ball being dropped from a height and the bouncing up to a fractional height. Imagine taking evenly spaced snapshots or blinking once every second as the ball changes position. A motion diagram captures this sequence, showing how the ball changes position over time, and whether it speeds up or slows down. A motion diagram contains usually 3 types of symbols:

- $\cdot$ for position representation
- $\rightarrow$ for velocity vectors
- $\Longrightarrow$ for acceleration vectors

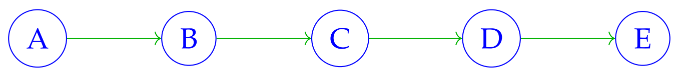
Below is the scenario of the ball being dropped and bouncing back up. Note that the upwards motion is shifted just for clarity. Here, red represents acceleration, green represents velocity, and blue represents equally spaced position dots.



Notice that in the above example, acceleration is constant throughout the motion, represented by the equally sized red arrows.

If there is no change in velocity, there is no acceleration vector. In this case, the would be no red arrow vectors.

Here is another motion diagram, representing an object moving forwards with constant velocity.



Chapter 6

Force, Mass, and Acceleration

To talk about forces, we first need to understand the concept of forces. A force is a push or pull on an object which (normally) influences its direction or speed. Forces are *vectors*, which means they have a direction and a magnitude, and it means that forces can be added together, even when in opposite directions. This leads us to Newton's Laws of Motion, which describe the relationship between forces and motion.

6.1 Newton's First Law

Newton's First Law of Motion states that an object at rest will stay at rest, and an object in motion will stay in motion with a constant velocity (same speed and direction) unless acted upon by a *net external force*. This means that if there is no net force on an object, it will not *accelerate*.

For example, a car that stops pressing the gas on a flat road will eventually stop due to friction, which is a force that acts opposite to the direction of motion. If there were no friction, the car would continue moving at a constant speed forever.

This law is sometimes also referred to as the law of *inertia*, which is the tendency of objects to resist changes in their motion.

6.2 Mass, Acceleration, and Newton's Second Law

Each atom has a mass, which means everything made up of those atoms has mass as well (and that's pretty much everything!). We measure mass in grams. A paper clip is about 1 gram of steel. An adult human can have a mass of 70,000 grams, so for larger things, we often talk about kilograms, which is 1000 grams.

The first interesting thing about mass is that objects with more mass require more force to accelerate. For example, pushing a bicycle so that it accelerates from a standstill to jogging speed in 2 seconds requires much less force than pushing a train so that it accelerates at the same rate.

Newton's Second Law of Motion

The force necessary to accelerate an object of mass m at an acceleration of a is given by:

$$F = ma$$

This is said as "the force is equal to the mass times the acceleration."

This is known as Newton's Second Law of Motion.[1]

What are the units here? We already know that mass is measured in kilograms. We can measure velocity in meters per second, but that is different from acceleration. So if we want to go from 0 to 5 meters per second (that's jogging speed) in two seconds, that is a change in velocity of 2.5 meters per second every second. We would say this acceleration is $2.5m/s^2$.

6.2.1 Velocity versus Acceleration

Acceleration is the change in velocity. If an object is speeding up or slowing down, it is accelerating. In everyday language, we often use *decelerate* to indicate slowing down, but in physics you can use the word accelerate (slowing down is just negative acceleration). Since velocity is a vector (it has a magnitude and direction), changing direction is *also acceleration*. On the other hand, an object moving at a constant velocity (same speed, same direction) is *not accelerating*!

[1] This is a simplified version of Newton's Law of Gravitation. The formula $F = G\frac{m_1 m_2}{r^2}$ simplifies very close to $F = ma$ for any object within the Earth's atmosphere.

Exercise 10 Is it accelerating?

State whether the described object is accelerating or not.

1. A satellite orbiting the Earth at a constant speed.
2. A car moving due west at a constant 30 mile per hour.
3. A child coming to a stop on their bicycle.
4. A roller coaster going around a loop at a constant speed.
5. A roller coaster speeding up as it goes down the initial hill.
6. A book sitting on a table.

Working Space

Answer on Page 68

It's a common misconception that all objects in motion are accelerating. If an object is moving with a constant velocity (same speed, same direction), then it is not accelerating.

6.2.2 Calculating Acceleration

When an object is speeding up or slowing down, we can calculate the acceleration by dividing the change in velocity by the time it takes to make that change.

Calculating Acceleration

The acceleration of an object from an initial velocity, v_i, to a final velocity, v_f, over a period of time, t, is given by:

$$a = \frac{v_f - v_i}{t}$$

Notice that if the velocity does not change, then $v_f - v_i = 0$ and the acceleration is also zero.

Example: Your car can go from zero to 60 mph in 3 seconds. What is the acceleration in m/s^2?

Solution: First, let's convert from the imperial units of miles per hour to the SI units of meters per second. You can do this using a search engine, but we will show how to do it by hand below. (You will learn more about this method in the Units chapter).

$$\frac{60 \text{ miles}}{1 \text{ hour}} \cdot \frac{1.61 \text{ km}}{1 \text{ mile}} \cdot \frac{1000 \text{ m}}{1 \text{ km}} \cdot \frac{1 \text{ hour}}{3600 \text{ seconds}} \approx \frac{26.82 \text{ m}}{\text{s}}$$

Now we have the starting velocity (0 m/s), the ending velocity (26.82 m/s), and the time (3 s), and we can find the acceleration:

$$a = \frac{v_f - v_i}{t} = \frac{26.82\frac{m}{s} - 0\frac{m}{s}}{3s} \approx 8.94\frac{m}{s^2}$$

6.2.3 Determining Force

What about measuring force? Newton decided to name the unit after himself: The force necessary to accelerate one kilogram at $1m/s^2$ is known as *a newton*. It is often denoted by the symbol N.

$$1N = 1\frac{kg \cdot m}{s^2}$$

Example: If the car in the above example has a mass of 1500 kg, how much force does the engine use to accelerate the car?

Solution: We have already found the car's acceleration: 8.94 m/s^2. With the mass and acceleration, we can use Newton's Second Law to find the force needed to accelerate the car:

$$F = m \cdot a = 1500 \text{ kg} \cdot 8.94\frac{m}{s^2} = 13410 \text{ N}$$

Exercise 11 Acceleration

Working Space

While driving a bulldozer, you come across a train car (with no brakes and no locomotive) sitting on a track in the middle of a city. The train car has a label telling you that it has a mass of 2,400 kg. There is a time-bomb welded to the interior of the train car, and the timer tells you that you can safely push the train car for 120 seconds. To get the train car to where it can explode safely, you need to accelerate it to 20 meters per second. Fortunately, the track is level and the train car's wheels have almost no rolling resistance.

With what force, in newtons, do you need to push the train for those 120 seconds?

Answer on Page 68

6.3 Net Force

So far, we've looked at examples where only one force is acting on an object. In reality, there are usually multiple forces acting on an object. For example, the engine pushes your car forward while friction pulls it backwards. Or your chair is pushing up on you while gravity pulls you down. How then can we describe the motion of an object if more than one force is acting on it?

We can rearrange Newton's Law:

$$a = \frac{F_{net}}{m}$$

This means that an object's acceleration is directly proportional to the *net force* acting on the object and inversely proportional to the object's mass. The *net force* is the vector sum of all the forces acting on an object. The vector sum just means we have to take the direction of the force into account. Usually, up and right are positive while down and left are negative. You can see some examples in figure 6.1.

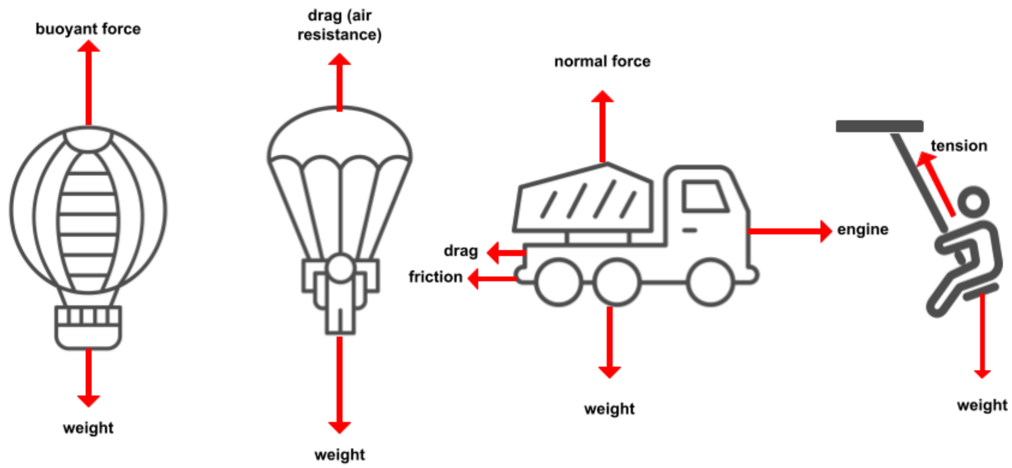
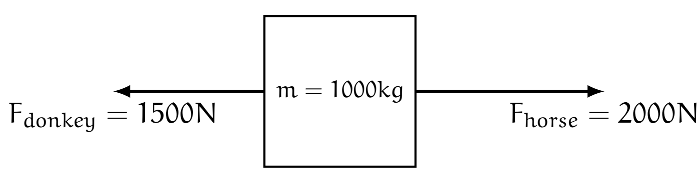


Figure 6.1: The hot air balloon has an upwards net force. The parachuter has a downwards net force. The truck has a rightward net force. The swinger has a leftward net force.

For now, we'll only look at parallel forces (up/down or left/right). You'll learn to use vector addition to combine orthogonal (at right angles) and skew (at angles other than parallel or right) forces in a later chapter.

Example: A donkey and a horse are each pulling a cart. The donkey pulls to the left with a force of 1500 N, while the horse pulls to the right with a force of 2000 N. What is the net force on the cart? If the cart has a mass of 1000 kg, in what direction and with what magnitude will the cart accelerate?

Solution: We begin by drawing a diagram (this is good practice - while a diagram may not be necessary for such a simple question, it will be very useful as we examine more complex scenarios in future chapters).



It is customary to take right as positive, so the horse applies a positive force while the donkey applies a negative force to the cart. Therefore, the net force is:

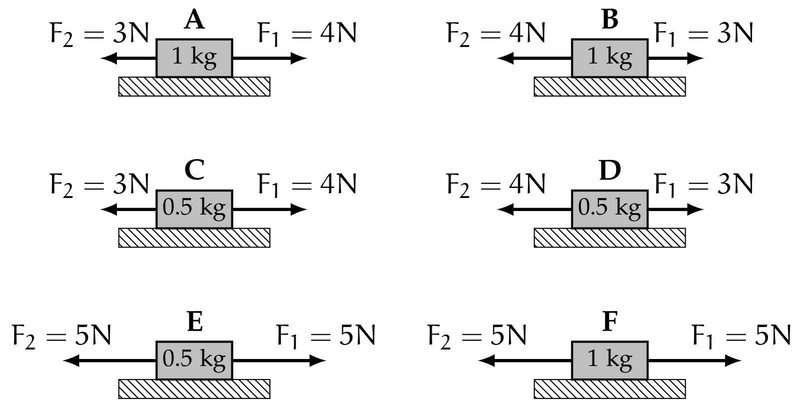
$$F_{net} = F_{horse} - F_{donkey} = 2000N - 1500N = 500N$$

Since the net force is positive, it points to the right and the cart will accelerate to the right at a rate of:

$$a = \frac{F_{net}}{m} = \frac{500\text{N}}{1000\text{kg}} = 0.5\frac{\text{m}}{\text{s}^2}$$

Exercise 12 Net Force and Acceleration

Rank the acceleration of the boxes shown below from greatest to least. All surfaces are frictionless and each box starts at rest. Take left as negative, and negative accelerations are less than a zero acceleration.



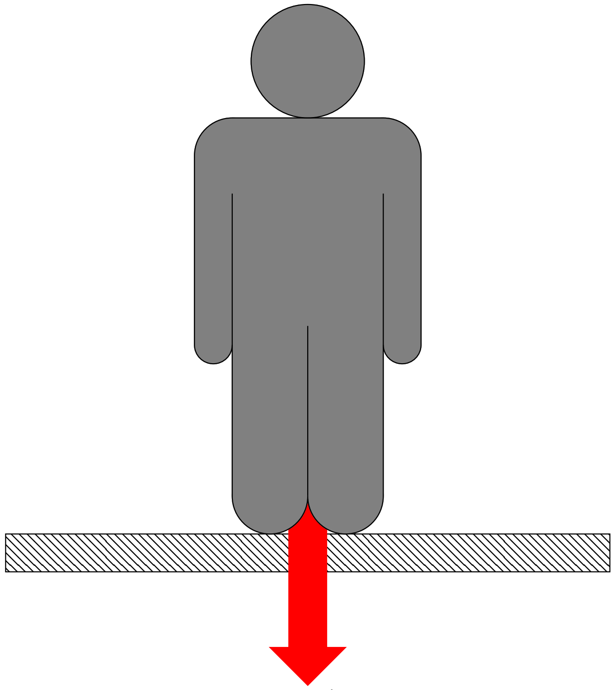
Working Space

Answer on Page 68

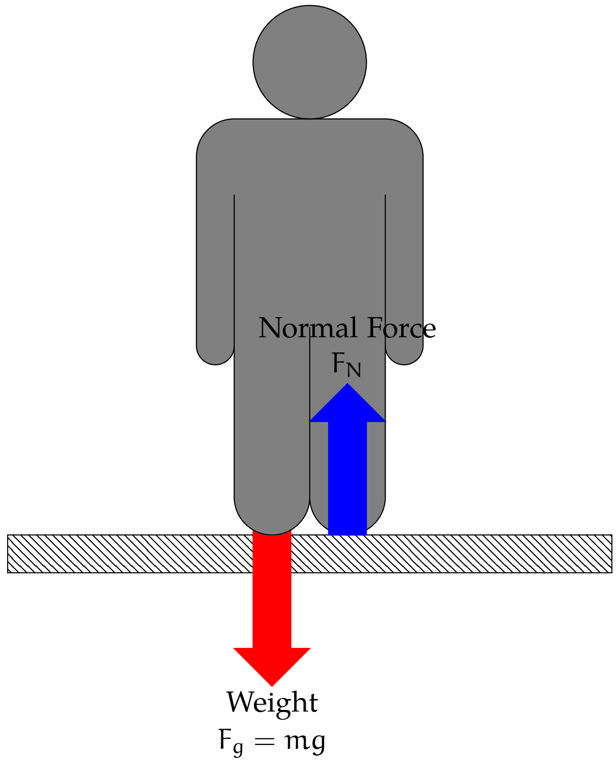
6.4 Normal Force and Apparent Weight

If you were to stand on a scale while riding an elevator, you would see the scale fluctuate as the elevator accelerates up and down (if you have a bathroom scale and live in a building with an elevator, you can try this yourself!). The reading on the scale is your *apparent weight* and is equal to the normal force between you and the floor.

What is a *normal force*? First, the word "normal" doesn't have the colloquial meaning of average or usual. In mathematics, "normal" means perpendicular. A normal force is perpendicular to the *contact* surface between two objects. Let's look at a person standing at rest. We know that gravity is pulling down on the person:



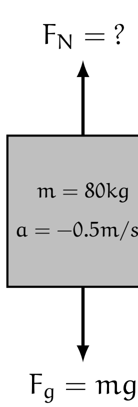
Since the person is not accelerating, there must be some other force acting on them in the upwards direction that balances out the person's weight (recall that if $a = 0m/s^2$, then it must be true that $F_{net} = 0N$). In this case, the balancing force is the *normal force* of the floor pushing up on the person's feet:



When you step on a bathroom scale, it is actually measuring the normal force! When you are not accelerating, your weight and the normal force between you and the scale are equal, so the scale gives you an accurate measure of your weight. Let's look at what happens if you are accelerating, as in an elevator.

Example: Maria has a mass of 80. kg. If the elevator in her apartment building initially accelerates at 0.50 m/s^2, what is her apparent weight as the elevator begins to move down?

Solution: We begin with a diagram:



Her apparent weight is the normal force. Seeing that the net force acting on Maria is given by $F_N - F_g$, we can use Newton's Second Law to find F_N:

$$F_{net} = F_N - F_g = ma$$
$$F_N - (80\text{kg})\left(9.8\frac{m}{s^2}\right) = (80\text{kg})\left(-0.5\frac{m}{s^2}\right)$$
$$F_N = (80\text{kg})\left(9.8\frac{m}{s^2} - 0.5\frac{m}{s^2}\right) = 744\text{N}$$

Maria's apparent weight is 744 newtons when the elevator is accelerating downwards.

Exercise 13 Moving Elevators

A 700-N person is standing on a scale in an elevator. Rank the reading on the scale from greatest to least for the following scenarios:

1. The elevator has an initial upward velocity of 2 m/s and an upward acceleration of 3 m/s^2
2. The elevator is initially at rest and has an upward acceleration of 3 m/s^2
3. The elevator has an initial downward velocity of 2 m/s and an upward acceleration of 5 m/s^2
4. The elevator is initially at rest and has a downward acceleration of 9.8 m/s^2
5. The elevator has an initial upward velocity of 2 m/s and a downward acceleration of 5 m/s^2
6. The elevator has an initial downward velocity of 2 m/s and is not accelerating

Working Space

Answer on Page 69

Let's talk about the effect of normal force on the net force. On standard flat ground, the Normal Force will be equal to the gravitational force (on Earth), assuming the object is stationary. This normal force also influences another contact force, friction, which we will talk about in the next chapter. If an object is on an incline such as a ramp, the normal force is *perpendicular* to the ramp.

6.5 Newtons and Joules

You now know the fundamental units for newtons and joules, which hints at the relationship between force and energy:

$$N = \frac{kg \cdot m}{s^2} \quad J = \frac{kg \cdot m^2}{s^2} = N \cdot m$$

A joule is equivalent to a newton times a meter. So, multiplying a force (newtons) by a displacement (meters) tells you how the energy (joules) of the object changes. This relationship is described by the Work-Kinetic Energy Theorem, which we will explore in the next chapter.

6.6 Newton's Third Law

Now we can discuss Newton's Third Law, which states that every action has an equal but opposite reaction. In other words, if body A exerts a force on body B, then body B simultaneously exerts a force of equal magnitude and opposite direction on body A. Mathematically, we can express this as:

$$\mathbf{F}_{AB} = -\mathbf{F}_{BA}$$

Any time we use Newton's Second Law, we can combine this with the Third Law to state that *the sum of the forces equals zero* in some action-reaction pair, assuming that the system is isolated. For example, if two boxes (1 and 2) are applying a force on each other, we can say that (no matter their mass values), $F_{\text{Box 1 on 2}} = -F_{\text{Box 2 on 1}}$. This applies to physics concepts like air resistance and drag, tension forces of conjoined boxes or train cars, and gravitation of planets, all of which we will discuss in future Sequences.

Note the following:

- Action-reaction pairs do *not* cancel when applying Newton's Second Law to a single body.
- The forces are always of the same type (both contact, both gravitational, etc).
- They occur simultaneously; there is no time delay.

Like we said above, gravity and the normal force are usually action-reaction pairs. Similarly, a force applied on a box always has an equal but opposite force on the root cause of the force. Two objects connected by a massless rope have the same tension force on each other ($F_{1 \text{ on } 2} = -F_{2 \text{ on } 1}$) **Example:** Consider a swimmer pushing the wall of a pool. What forces are in play?

Solution: While the swimmer exerts a force $\mathbf{F}_{\text{wall}}$ on the wall, the wall exerts the reaction $-\mathbf{F}_{\text{wall}}$ on the swimmer, propelling the swimmer forward. Although the wall does not move due to mass and acceleration differences, the forces are equal and opposite.

Chapter 7

Friction

Imagine there is a large and heavy steel box resting in the middle of a floor, and you push it hard enough to get it moving. If you stop pushing, will it continue to glide gracefully across the floor?

Probably not. Unless the floor is very slippery for some reason, the box will come to a halt immediately after you stop pushing. We would say that it is stopped by the force of *friction*.

What is really happening? The kinetic energy of the box is being converted into heat between the bottom of the box and floor. As the bottom of the box and the floor get warmer, the speed of the box decreases.

The amount of friction is proportional to the force with which the box is pressing against the floor — so you should expect a box that is twice as heavy to experience twice as much frictional force.

In other words, the frictional force is proportional to the normal force. On a level floor, the normal force is parallel and equal to the force of gravity acting on the box. On a slope, the normal force points perpendicular to the surface of the slope. We will be talking about sloped surfaces in a later chapter, so focus on level surfaces for now.

(FIXME: picture here)

The amount of friction is also determined by the materials that are sliding against each other. For example, if the floor is ice, the frictional force will be less than if the floor is made of wood. Written mathmatically, we can express the force of friction as

$$F_f = \mu N$$

where F_f is the force of friction, N is the normal force (you may see written as R , F_n , or just N), and μ is a coefficient that depends on the materials in contact with each other.

If you are pushing the box with a force of F and it is moving but neither accelerating nor decelerating, then the force you are applying is exactly balanced by the frictional force. If the box is pressing against the floor with a force of N, then we say the *coefficient of friction* is given by the ratio between the weight of the steel box and the floor:

$$\mu = \frac{F_f}{N}$$

Exercise 14 Bicycle Stopping

Working Space

You are riding your bicycle on a flat, horizontal at 11 meters per second when you suddenly slam on the brakes and lock up the wheels.

You weigh 55 kg.

When any piece of rubber is skidding across a dry road, the coefficient of friction will be about 0.7.

Answer the following questions:

- How much kinetic energy do you have when you engage the brakes?
- As you skid, how much frictional force is decelerating you?
- For how many meters will you slide?

Answer on Page 69

Notice that the force of friction is not determined by how much of the tire is touching the ground. The coefficient of friction of the two materials and the normal force is all you need to compute the friction.

7.1 Static vs Kinetic Friction Coefficients

Let's return to the box on the floor we discussed earlier. As you start to push it, it will sit still until your force is greater than the force of friction. However, once it starts moving, the force of friction seems to be less.

Between the two materials, there are actually two different friction coefficients:

- Static friction coefficient: The coefficient you use to figure out how much force you need to get the box to start to move.
- Kinetic friction coefficient: The coefficient you use once the box is sliding against the floor.

The kinetic friction coefficient is always less than the static friction coefficient:

- *Static*, μ_s: When the car is parked with its brakes on, it has a friction coefficient of about 1.0.
- *Kinetic*, μ_k: For a car skidding on a dry road, the friction coefficient is about 0.7.

It is important to note once the force of static friction is overcome, there will be a force of kinetic friction less than the static friction force.

Exercise 15 Rocket Sled

Working Space

You built a rocket sled with steel runners that is resting on a flat, level wooden floor. The sled weighs 50 kg and you weigh 55 kg.

Before you get on the sled, you try pushing it around the floor. You find that you can get it to move from a standstill if you push it with a force of 270 N. Once it is moving, you can keep it moving at the same speed using a force of 220 N.

What are μ_s and μ_k of your sled's runners on your wooden floor?

Next, you get on the sled and gradually increase the thrust of the rocket mounted on the sled until it starts to move. You then keep the thrust constant.

How much force was the rocket exerting on you and the sled when it started to move?

How how fast do you accelerate now that the sled is moving?

Answer on Page 69

7.2 Skidding and Anti-Lock Braking Systems

When a car goes through a curve, the friction of the tire on the road is what changes the direction of the car's travel. Even though the wheel is turning, this is the *static friction coefficient* because the surface of the tire is not sliding across the road. The part of the tire touching the road at any instant is at rest relative to the road. The force of friction is the so-called *centripetal force* keeping the object on the road.

If you go into the curve too fast, the tire may not have enough friction to turn the car. In this case the car will start to slide sideways. Now, the friction between the tire and road uses the kinetic coefficient. In other words, you have significantly less friction than you had before you started to skid.

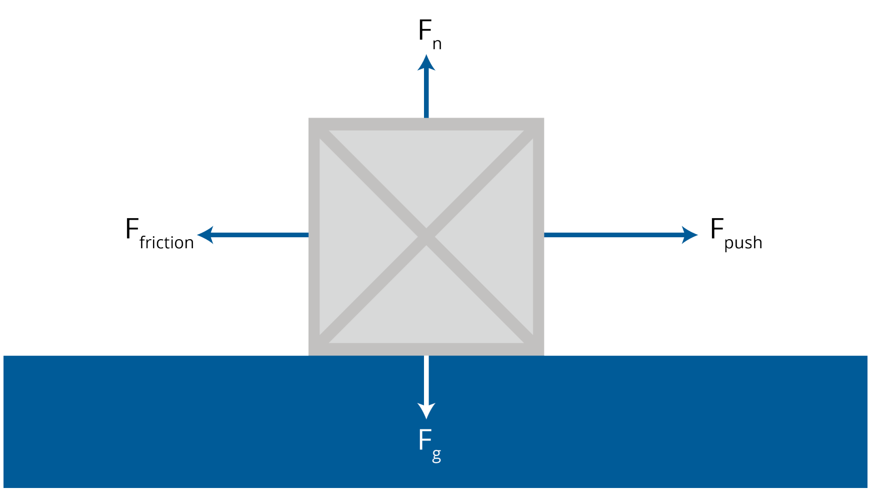
When you are driving a car, the force of friction that your tires create is your friend. It lets you steer, accelerate, and stop.

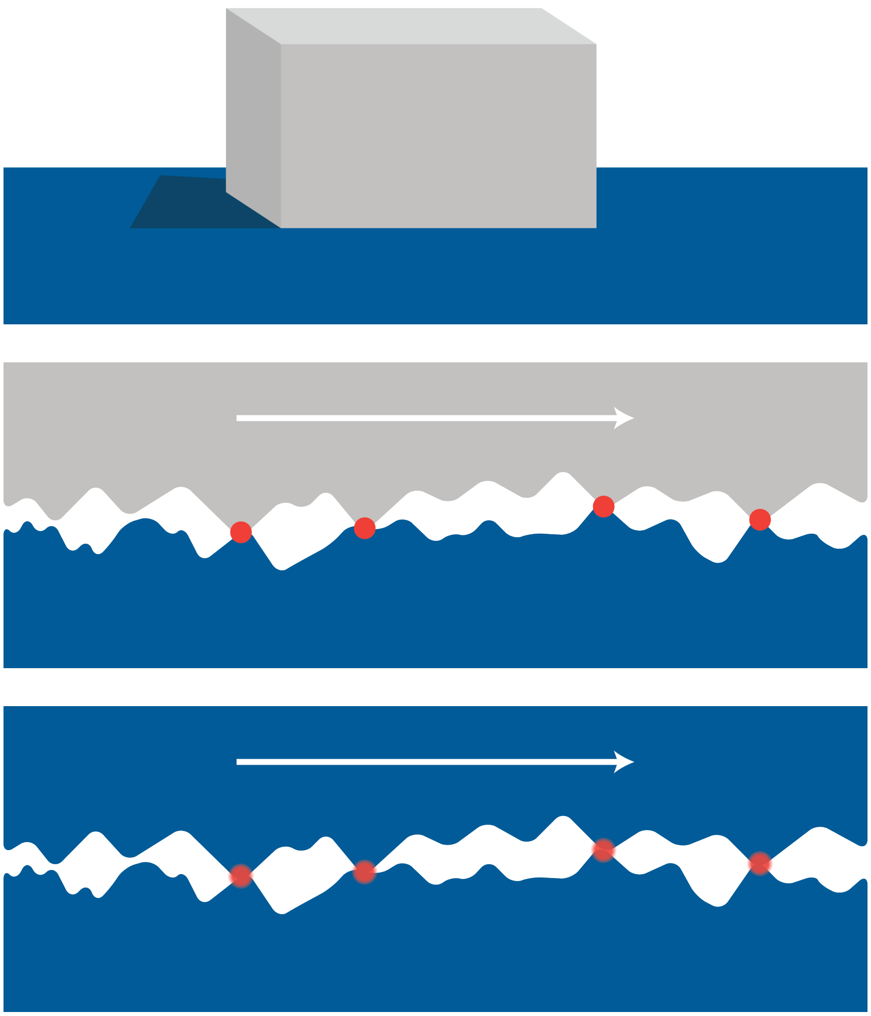
In older cars, if you panicked and slammed on the brakes, you would probably lock up the wheels: they would stop turning suddenly. And the surface of the tire would begin to slide across the pavement. At that moment, two problems occurred:

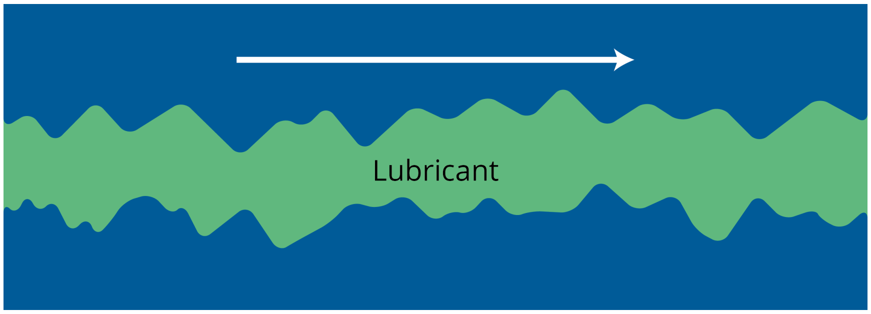
- You don't stop as quickly, because now the friction between your tires and the road is based on the kinetic friction coefficient instead of the static friction coefficient.
- You can't steer the car. Steering only happens because the wheels are turning in a particular direction.

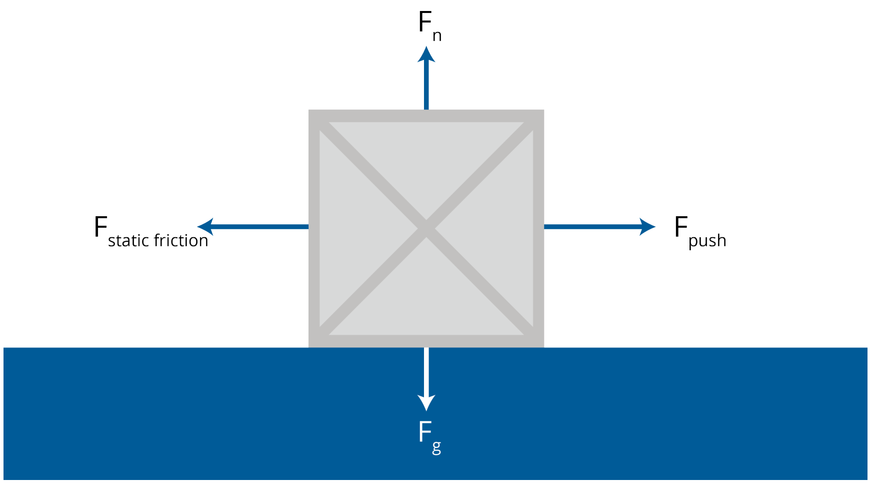
To prevent this problem, car companies developed the anti-lock brake system, or ABS.

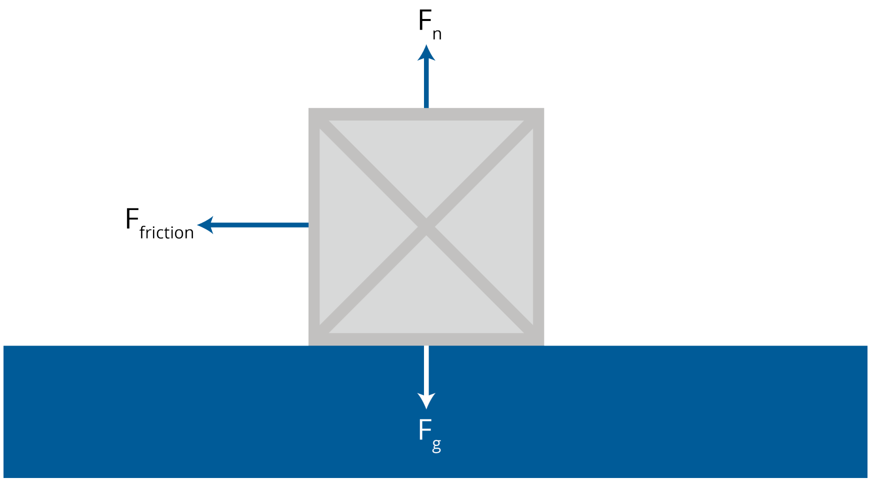
FIXME: More here.






Lubricant


F n
F static friction
F push
F g



F n
F friction
F g

Chapter 8

Units and Conversions

Accurate measurements are at the heart of good data and good problem solving. Engineers need to be able to describe many different types of phenomena, such as distance, sound, light, force, and more.

At this point, you are working with a lot of units: grams for weight, joules for energy, newtons for force, meters for distance, seconds for time, and so on. For each type of measurement, there are several different units. For example, distance can be measured in feet, miles, and light-years.

Some Equalencies

Distance	
1 mile	1.6093 kilometers
1 foot	0.3048 meters
1 inch	2.54 centimeters
1 light-year	9.461×10^{12} kilometers
Volume	
1 milliliter	1 cubic centimeter
1 quart	0.9461 liters
1 gallon	3.7854 liters
1 fluid ounce	29.6 milliliters
Mass	
1 pound	0.4535924 kilograms
1 ounce	0.4535924 grams
1 metric ton	1000 kilograms
Force	
1 newton	1 kilogram meter per sec^2
Pressure	
1 pascal	1 newton per square meter
1 bar	0.98692 atmosphere
1 pound per square inch	6897 pascals
Energy	
1 joule	1 newton meter
1 calorie	4.184 joules
1 kilowatt-hour	3.6×10^6 joules

(You don't need to memorize these! Just remember that this page is here.)

In the metric system, prefixes are often used to express a multiple. Here are the common prefixes:

Common Prefixes for Metric Units

giga	$\times 10^9$
mega	$\times 10^6$
kilo	$\times 10^3$
milli	$\div 10^3$
micro	$\div 10^6$
nano	$\div 10^9$

(These are worth memorizing. Here's a mnemonic for the most common ones: "King Henry Doesn't Usually Drink Chocolate Milk" Or Kilo ($\times 10^3$), Hecto ($\times 10^2$), Deca ($\times 10^1$), Unit (for example: gram) ($\times 10^0$), Deci ($\times 10^{-1}$), Centi (1×0^{-2}), Mili ($\times 10^{-3}$).)

8.1 Conversion Factors

Here is a really handy trick to remembering how to do conversions between units.

Often, you will be given a table like the one above, and someone will ask you "How many miles are in 0.23 light-years?" You know that 1 mile = 1.6093 kilometers and that 1 light-year is 9.461×10^{12} kilometers. How do you do the conversion?

The trick is to treat the two parts of the equality as a fraction that equals 1. In other words, you think:

$$\frac{1 \text{ miles}}{1.6093 \text{ km}} = \frac{1.6093 \text{ km}}{1 \text{ miles}} = 1$$

and

$$\frac{1 \text{ light-years}}{9.461 \times 10^{12} \text{ km}} = \frac{9.461 \times 10^{12} \text{ km}}{1 \text{ light-years}} = 1$$

We call these fractions *conversion factors*.

Now, your problem is

$$0.23 \text{ light-years} \times \textit{ Some conversion factors} = ? \text{ miles}$$

Note that when you multiply fractions together, things in the numerators can cancel with things in the denominator:

$$\left(\frac{31\pi}{47}\right)\left(\frac{11}{37\pi}\right) = \left(\frac{31\cancel{\pi}}{47}\right)\left(\frac{11}{37\cancel{\pi}}\right) = \left(\frac{31}{47}\right)\left(\frac{11}{37}\right)$$

When working with conversion factors, you will do the same with the units:

$$0.23 \text{ light-years}\left(\frac{9.461 \times 10^{12} \text{ km}}{1 \text{ light-years}}\right)\left(\frac{1 \text{ miles}}{1.6093 \text{ km}}\right) =$$

$$0.23 \cancel{\text{light-years}}\left(\times\frac{9.461 \times 10^{12} \cancel{\text{km}}}{1 \cancel{\text{light-years}}}\right)\left(\frac{1 \text{ miles}}{1.6093 \cancel{\text{km}}}\right) = \frac{(0.23)(9.461 \times 10^{12})}{1.6093} \text{ miles}$$

Exercise 16 Simple Conversion Factors

Working Space

How many calories are in 4.5 kilowatt-hours?

Answer on Page 70

8.2 Conversion Factors and Ratios

Conversion factors also work on ratios. For example, if you are told that a bug is moving 0.5 feet every 120 milliseconds, what is that in meters per second?

The problem then is

$$\frac{0.5 \text{ feet}}{120 \text{ milliseconds}} = \frac{? \text{ m}}{\text{second}}$$

So you will need conversion factors to replace the “feet” with “meters” and to replace “milliseconds” with “seconds”:

$$\left(\frac{0.5 \cancel{\text{feet}}}{120 \cancel{\text{milliseconds}}}\right)\left(\frac{0.3048 \text{ meters}}{1 \cancel{\text{feet}}}\right)\left(\frac{1000 \cancel{\text{milliseconds}}}{1 \text{ second}}\right) = \frac{(0.5)(0.3048)(1000)}{120} \text{ m/second}$$

Exercise 17 Conversion Factors

The hole in the bottom of the boat lets in 0.1 gallons every 2 minutes. How many milliliters per second is that?

Working Space

Answer on Page 70

8.3 When Conversion Factors Don't Work

Conversion factors only work when the units being converted are proportional to each other. Gallons and liters, for example, are proportional to each other: If you have n gallons, you have $n \times 3.7854$ liters.

Degrees celsius and degrees farenheit are *not* proportional to each other. If your food is n degrees celsius, it is $n \times \frac{9}{5} + 32$ degrees farenheit. You can't use conversion factors to convert celsius to farenheit.

Chapter 9

The Greek Alphabet

If you do anything involving math or physics, you will be encountering Greek letters on a regular basis. Here is a table for your reference:

Capital	Lower	Pronounced	Capital	Lower	Pronounced
A	α	Alpha	N	ν	Nu
B	β	Beta	Ξ	ξ	Xi ("ku-ZY")
Γ	γ	Gamma	O	o	Omicron
Δ	δ	Delta	Π	π	Pi
E	ϵ	Epsilon	P	ρ	Rho
Z	ζ	Zeta	Σ	σ	Sigma
H	η	Eta	T	τ	Tau
Θ	θ	Theta	Υ	υ	Upsilon
I	ι	Iota	Φ	ϕ	Phi
K	κ	Kappa	X	χ	Chi ("Kai")
Λ	λ	Lambda	Ψ	ψ	Psi ("Sigh")
M	μ	Mu	Ω	ω	Omega

Appendix A

Answers to Exercises

Answer to Exercise 1 (on page 8)

1. C:H = 1:4
2. Cu:S:O = 1:1:4
3. C:H:O = 6:12:6 = 1:2:1

Answer to Exercise 2 (on page 17)

1. balanced
2. unbalanced; oxygen
3. unbalanced; barium, sulfur, oxygen, and carbon
4. balanced

Answer to Exercise 3 (on page 20)

1. $v = \sqrt{2gh} = \sqrt{2\,(9.8 m/s^2)\,(3.7 m)} = 8.5 m/s$
2. $h = \frac{v^2}{2g} = \frac{\left(15\frac{m}{s}\right)^2}{2\left(9.8\frac{m}{s^2}\right)} = 11.5 m$. You can hit your little brother, since you could shoot the pebble as high as 11.5 m and he is only 5.0 m above you.
3. We know that $E_{p,initial} = E_{k,final} + E_{losttofriction}$. Therefore, $E_{losttofriction} = E_{p,initial} - E_{k,final} = mgh - \frac{1}{2}mv^2 = (63kg)\left(9.8\frac{m}{s^2}\right)(8m) - \frac{1}{2}(63kg)\left(11.2\frac{m}{s}\right)^2 = 4939.2J - 3951.4J = 988J$.

Answer to Exercise 4 (on page 23)

1. The human body is an open system because matter can enter (food, water, oxygen) and leave (waste, carbon dioxide, sweat) your body.
2. The Earth is an open system because matter can enter (asteroids falling, spaceships returning) and leave (space vehicles and astronauts). On the other hand, the Earth can be well-approximated as a closed system. Before the 20th Century, humans had no way to deliberately expel matter from the Earth, and the mass of asteroids that are pulled in by the Earth's gravity is negligible compared to the Earth. Therefore, in the right circumstances, it would be appropriate to model the Earth as a closed system. (It is closed because energy in the form of sunlight is constantly entering the system.)
3. A cell phone is a closed system - you don't put any matter in or take it out of your phone, but it constantly uses battery and then is recharged, showing that energy enters and leaves your phone.
4. Since the cooler is described as well-insulated and the lid is closed, it can be approximated as an isolated system. Scientific equipment, like bomb calorimeters, rely on this approximation.
5. With the lid open, matter can enter and leave and therefore the cooler is an open system (yes, even though it is well-insulated).
6. A sealed bottle of soda is a closed system - the soda and carbon dioxide can't escape, but energy in the form of heat can be transferred in and out of the system (the contents of the bottle will lose heat if you put it in the fridge and gain heat if you leave it in the sun).

Answer to Exercise 5 (on page 27)

(a)

$$\begin{aligned} F &= G\frac{m_1 m_2}{r^2} \\ &= (6.674 \times 10^{-11})\frac{(6.0 \times 10^{24})(8.0 \times 10^{22})}{(4.0 \times 10^6)^2} \\ &= 2.0022 \times 10^{24} \end{aligned}$$

(b) Now we have a new distance, $r' = 8.0 \times 10^6$ m, double the original. Notice that gravitational force is inversely proportional to the square of the distance:

$$F \propto \frac{1}{r^2}$$

So if distance doubles,

$$r' = 2r \quad \Rightarrow \quad F' = \frac{F}{4}$$

Which results in 5.0055×10^{23} N

Answer to Exercise 6 (on page 28)

$$F = G\frac{m_1 m_2}{r^2} = (6.674 \times 10^{-11})\frac{(6.8 \times 10^3)(6 \times 10^{24})}{(10^5)^2} = 6.1 \times 10^6$$

About 6 million newtons.

Answer to Exercise 7 (on page 30)

The gravitational acceleration is 9.8 $\mathrm{m/s^2}$, no matter the mass of the object.

Answer to Exercise 8 (on page 31)

The mass of the coin is $\frac{.0198\ \mathrm{N}}{9.8\ \mathrm{m/s^2}} = 0.002$ grams

Answer to Exercise 9 (on page 32)

We can set up a balanced center of mass equation for this problem. Our problem is only in one dimension, so there is only one distance to be checked. If we set our $x_{CM} = 0$ where the fulcrum or pivot of a seesaw would be by assuming the balance scenario, we can solve our problem:

$$\begin{aligned}
x_{CM} &= \frac{\Sigma m_i x_i}{\Sigma m_i} \\
0 &= \frac{(30)(40) + (50)(-20) + (15)\,d}{30 + 50 + 15} \\
&= \frac{1200 - 1000 + (15)\,d}{85} \\
&= \frac{200 + (15)\,d}{85} \\
d &= -13\tfrac{1}{3}\mathrm{m}
\end{aligned}$$

Since our answer is negative, it means to the left of the fulcrum (given we chose left to be negative). Thus, M_3 should be placed $13\frac{1}{3}$ m to the left.

Answer to Exercise 10 (on page 41)

1. Acceleration: the satellite is moving in a circle, therefore changing direction and accelerating.
2. Not acceleration: the car isn't changing speed or direction.
3. Acceleration: the child is changing speed.
4. Acceleration: the roller coaster is changing direction.
5. Acceleration: the roller coaster is changing speed.
6. Not acceleration: the book isn't changing speed or direction.

Answer to Exercise 11 (on page 43)

If you accelerate to 20 m/s in 120 s, the acceleration is:

$$a = \frac{v_f - v_i}{t} = \frac{20 \text{ m/s} - 0 \text{ m/s}}{120 \text{ s}} = \frac{1}{6}\frac{\text{m}}{\text{s}^2}$$

To achieve this acceleration, you will need to apply a force of:

$$F = m \cdot a = 2400 \text{ kg} \cdot \frac{1}{6}\frac{\text{m}}{\text{s}^2} = 400 \text{ N}$$

Answer to Exercise 12 (on page 45)

C, A, E/F, B, D

$$a_A = \frac{F_{net,A}}{m_A} = \frac{4N - 3N}{1kg} = \frac{1N}{1kg} = 1\frac{m}{s^2}$$

$$a_B = \frac{F_{net,B}}{m_B} = \frac{3N - 4N}{1kg} = \frac{-1N}{1kg} = -1\frac{m}{s^2}$$

$$a_C = \frac{F_{net,C}}{m_C} = \frac{4N - 3N}{0.5kg} = \frac{1N}{0.5kg} = 2\frac{m}{s^2}$$

$$a_D = \frac{F_{net,D}}{m_D} = \frac{3N - 4N}{0.5kg} = \frac{-1N}{0.5kg} = -2\frac{m}{s^2}$$

$$a_E = \frac{F_{net,E}}{m_E} = \frac{5N - 5N}{0.5kg} = \frac{0N}{0.5kg} = 0\frac{m}{s^2}$$

$$a_F = \frac{F_{net,F}}{m_F} = \frac{5N - 5N}{1kg} = \frac{0N}{1kg} = 0\frac{m}{s^2}$$

Answer to Exercise 13 (on page 48)

3, 1/2, 6, 5, 4 The velocity does not affect the apparent weight, only the acceleration. If the elevator is accelerating upwards, the apparent weight increases. If the elevator is accelerating downwards, the apparent weight decreases. In fact, for scenario 4, the elevator is in free-fall and the person has no apparent weight. When the elevator is not accelerating, the person's apparent weight is their true weight.

Answer to Exercise 14 (on page 52)

Kinetic energy? $E = \frac{1}{2}mv^2 = (.5)(55)(11^2) = 3,327.5\frac{kgm^2}{s^2} = 3,327.5$ joules.

Frictional force? $F = \mu N = (0.7)(55)(9.8) = 377.3$ newtons.

Distance? $D = \frac{3,327.5}{377.3} \approx 8.8$ seconds. (found by using $W = F_f D$ and $W = \Delta E$)

Answer to Exercise 15 (on page 54)

The empty sled is pushing directly down on the floor with a force of $(50)(9.8) = 490$ N.

The force to overcome the static friction is:

$$270 = 490\mu_s$$

Thus, $\mu_s = 0.551$

The force to match kinetic friction is:

$$220 = 490\mu_k$$

Thus, $\mu_k = 0.449$

Once you are on the sled, it is pressing directly down on the floor with a force of $(50 + 55)(9.8) = 1,029$ N.

The force to overcome the static friction is:

$$F = (1,029)(0.551) = 567 \text{ N}$$

Once the sled is moving, friction is counteracting some of your force. How much?

$$F_f = (1,029)(0.449) = 462 \text{ N}$$

All of your acceleration is due to the remaining $567 - 492 = 75$ N.

We know that $F = ma$. In this case $F = 75$ N and $m = 105$ kg. So

$$a = \frac{75}{105} = 0.714 \text{ meters per second per second}$$

Answer to Exercise 16 (on page 61)

$$4.5 \cancel{\text{kWh}} \left(\frac{3.6 \times 10^6 \cancel{\text{joules}}}{1 \cancel{\text{kWh}}}\right)\left(\frac{1 \text{ calories}}{4.184 \cancel{\text{joules}}}\right) = \frac{(4.5)(3.6 \times 10^6)}{4.184} = 1.08 \times 10^6 \text{calories}$$

Answer to Exercise 17 (on page 62)

$$\frac{0.1 \cancel{\text{gallons}}}{2 \cancel{\text{minutes}}}\left(\frac{3.7854 \cancel{\text{liters}}}{1 \cancel{\text{gallons}}}\right)\left(\frac{1000 \text{ milliliters}}{1 \cancel{\text{liters}}}\right)\left(\frac{1 \cancel{\text{minutes}}}{60 \text{ seconds}}\right) = \frac{(0.1)(3.7854)(1000)}{(2)(60)} \text{ ml/second} = 3.1545 \text{ ml/second}$$

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