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Chapter 1

Introduction to the Kontinua Sequence

The purpose of this book is to help you along the long and difficult trek to becoming a modern problem solver. As you explore this path, you will learn how to use the tools of math, computers, and science.

If this path is so arduous, it is only fair to ask why you should bother in the first place. There are big problems out there that will require expert problem solvers. Those people will make the world a better place, while also enjoying interesting and lucrative careers. We are talking about engineers, scientists, doctors, computer programmers, architects, actuaries, and mathematicians. Right now, those occupations represent about 6% of all the jobs in the United States. Soon, that number is expected to rise above 10%. On average, people in that 10% of the population are expected to have salaries twice that of their non-technical counterparts.

Solving problems is difficult. At some point on this journey, you will see people who are better at solving problems than you are. You, like every other person who has gone on this journey, may think "I have worked so hard on this, but that person is better at it than I am. I should quit." *Don't*.

Instead, remember these two important facts. First, solving problems is like a muscle. The more you do, the better you get at it. It is OK to say "I am not good at this yet." That just means you need more practice.

Second, you don't need to be the best in the world. 10 million people your age can be better at solving problems than you, *and you can still be in the top 10% of the world*. If you complete this journey, there will be problems for you to solve and a job where your problem-solving skills will be appreciated.

Where do we start?

The famous physicist Richard Feynman once asked, "If, in some cataclysm, all of scientific knowledge were to be destroyed, and only one sentence was passed on to the next generation of creatures, what statement would contain the most information in the fewest words?"

His answer was "All things are made of atoms—little particles that move around in perpetual motion, attracting each other when they are a little distance apart, but repelling upon being squeezed into one another."

That seems like a good place to start.

Chapter 2

Matter and Energy

The universe is made of matter and energy. Current models posit that the universe is approximately 68% dark energy, 27% dark matter, and 5% ordinary matter. Everything you can see and touch is part of the small part of the universe made of ordinary matter. Most science deals with ordinary matter and its interactions; highly trained theoretical physicists are currently debating the nature and effects of dark matter and dark energy.

What is this ordinary matter made of? All things (including the air around you) are made of atoms. Atoms are incredibly tiny — there are more atoms in a drop of water than there are drops of water in all the oceans.

Every atom has a nucleus that contains protons and neutrons. Orbiting around the nucleus is a cloud of electrons. However, the mass of the atom comes mainly from the protons and neutrons, since they are about 2000 times as massive as an electron! These three particles, protons, neutrons, and electrons, are called *subatomic particles*. (See figure 2.1.)

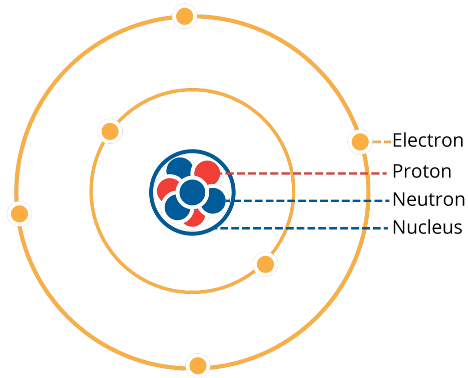


Figure 2.1

2.1 Atoms and Their Models

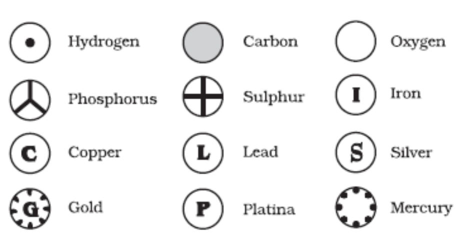


Figure 2.2

Over the history of science, there have been many ideas about the structure of atoms. This history is a good example of how science develops: unexpected results drive scientists to update their models, moving us closer and closer to a true model of the atom.

During his investigations into the behavior of gases, John Dalton noted that different elements combine in strict ratios. For example, he noted that nitrogen and oxygen combine in a 1:1 and 1:2 fashion, but no ratio in between.

This first model of the atom is rudimentary; each element is a unique atom, and those atoms cannot be subdivided. The atom is modeled as one large, solid, uniform, and neutral object. British physicist J.J. Thomson discovered that atoms could be split into a light, negatively charged particle and a heavier, positively charged particle (we now know this is the nucleus, the dense grouping of protons and neutrons in the center of an atom).

Suddenly, the atom went from neutral and indivisible to made of different types of charged particles. Further experiments by Ernest Rutherford showed the atom to be mainly empty space, further updating scientists' model of the atom. Subsequently, Bohr explained the phenomena of spectral lines (we will discuss this further in Sequence 2) by modeling electrons as being restricted to orbiting specific distances from the nucleus.

Figure 2.3

This is likely the model you are most familiar with seeing, and it is the one we will use most often in this text. The first figure shown in this chapter is a Bohr model: it shows the protons and neutrons in the nucleus, and models the electrons as moving in discrete orbits around the nucleus.

However, the Bohr model is slightly inaccurate. While it is a convenient model for thinking about atoms, in reality, electrons don't neatly orbit the nucleus. Scientists don't know exactly where an electron will be in relation to the nucleus, but they do know where it is most likely to be. They use a cloud that is thicker in the center but fades out at the edges to represent an electron's position (see figure 2.3). This cloud represents the probability distribution of finding an electron in a particular region around the nucleus. The figure is only in two dimensions, but the actual electron cloud is three-dimensional. The shape of the cloud depends on the energy level of the electron, and different energy levels have different shapes and sizes of clouds.

While the cloud model is more accurate, we will use the Bohr model as it allows the viewer to easily and quickly assess the number and arrangement of electrons.

2.1.1 Classifying Atoms

We classify atoms by the numbers of protons they have. An atom with one proton is a hydrogen atom, an atom with two protons is a helium atom, and so forth (refer to Table 8.1 on page 55)). We say that hydrogen and helium are *elements* because the classification of elements is based on the proton number. And we give each element an atomic symbol. Hydrogen gets H, helium gets He,

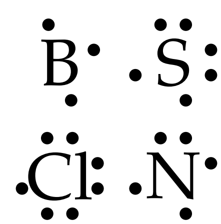


Figure 2.4

oxygen gets O, carbon gets C, and so on. You can see an element's symbol on the periodic table.

An atom will typically have the same number of electrons as protons, but when it gains or loses electrons, it becomes an ion with a net electric charge. Atoms can gain or lose electrons through various processes, such as chemical reactions or ionizing radiation. An atom likes to gain or lose electrons to achieve a full outer electron shell, typically with 8 electrons (the octet rule). When an atom gains electrons, it becomes a negatively charged ion, or *anion*. When it loses electrons, it becomes a positively charged ion, or *cation*. (I remember it as "Anions are anti, cations are cat-ions" because cats are positive!)

2.1.2 When Atoms Combine

When atoms of different elements combine, they make *compounds*. Compounds are substances made up of more than one element. Compounds can be *molecules* or *crystal lattices*. In the next section you'll learn *why* these different structures form.

There are many kinds of compounds. You know a few:

- Table salt is crystals made of Na^+ and Cl^- ions: a sodium atom that as lost an electron and a chlorine atom that gained an electron
- Baking soda, or sodium bicarbonate, is $NaHCO_3$.
- O_2 is the oxygen molecules that you breathe out of the air (air, a blend of gases, is mostly N_2.).
- Common quartz is SiO_2: silicon dioxide

The subscripts indicate what quantities of the elements are present in the compound. Each number indicates the quantity of the preceding element. A drop of water, H_2O, has two hydrogen atoms and one oxygen atom. Often times we'll see these numbers used to determine the ratio of elements in a compound. To do this, we compare the quantities of each element to find their simplest whole-number ratio. This is called the *empirical formula* of the compound.

Example: What is the ratio of elements present in Epsom salt?

Solution: Epsom salt, chemical name magnesium sulfate, has the chemical formula $MgSO_4$. Therefore, the ratio of Mg:S:O is 1:1:4.

Exercise 1 Ratios of Atoms in Molecules

Give the elemental ratio for each compound.

1. methane, CH_4
2. copper (II) sulfate, $CuSO_4$
3. glucose, $C_6H_{12}O_6$

Working Space

Answer on Page 71

2.2 Types of Matter

One way to classify matter is by the types of chemical bonds that hold a material's atoms together. The nature of these bonds, in turn, affects the properties of the material. For now, all you need to know is there are three types of chemical bonds: metallic, covalent, and ionic. Materials held together with the same type of bonds tend to have similar properties. For example, copper, bronze, iron, and steel (all containing metallic bonds) are all shiny, ductile, malleable, and good conductors of heat and electricity. On the other hand, Epsom salt and table salt form large crystals, have very high melting points, and are poor conductors of electricity in their pure form. These two substances (Epsom and table salt) both contain ionic bonds.

2.2.1 Covalent Compounds

Water is an example of a covalent compound: it is made of two hydrogen atoms covalently bonded one oxygen atom (see figure 2.5). The result is a water molecule. The different atoms cluster together because they share electrons in their clouds. This is the nature of a *covalent bond*: it is formed when atoms share electrons. Sometimes, electrons are shared evenly, but in water, they are shared unevenly. Oxygen is better at attracting electrons to itself than hydrogen, and so the shared electrons so they spend more time on the oxygen atom than the hydrogen atoms. As a result, the oxygen side of a water molecule has a slight negative charge, while the hydrogen atoms have a slight positive charge. These slight charges are represented with a lower case Greek letter delta, δ. When electrons are shared unevenly, we call this a *polar* covalent bond, because there are positive and negative poles at either end of the bond.

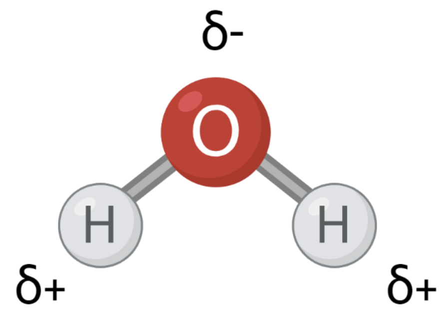


Figure 2.5

Whether electrons are shared evenly or unevenly is based on the elements' relative *electronegativities*. Electronegativity is simply a measure of how strongly an atom can attract electrons to itself. In general, elements on the right side of the periodic table are more electronegative than elements on the left side. When covalent bonds form between two elements of similar electronegativities, the electrons are shared evenly. We call this a *non-polar* covalent bond. Oil is an example of a non-polar covalent substance. Different oils have different ratios and structures, but all oils are made mainly of carbon and hydrogen, which have similar electronegativities. What happens when you try to mix oil and water? They don't mix well! This is due to the difference between their bond types. Polar substances, like water, mix best with other polar substances, while non-polar substances, like oils, mix best with non-polar substances. You'll learn more about why this is in Sequence 2.

For both polar and non-polar covalent bonds, the electrons are held tightly to the nuclei, even if they are shared among atoms. Those electrons don't move to another molecule: they move around within the molecule they are already a part of. Since electrons don't flow freely in covalent substances, they are also poor conductors of electricity. Covalent compounds also tend to have lower melting and boiling points compared to ionic compounds.

2.2.2 Ionic Compounds

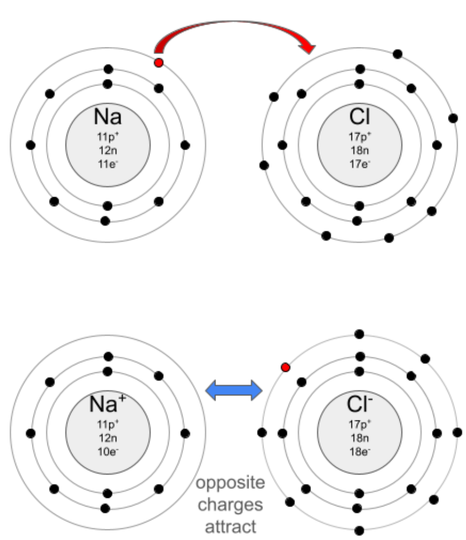


Figure 2.6

Ionic bonds are the electrical attraction between opposite-charged ions. When a neutral atom gains or loses an electron it becomes an *ion* (a charged atom), and oppositely-charged ions are attracted to each other. Which atom gets the electron and which loses it is based on their electronegativities: the more electronegative atom steals one or more electrons from the less electronegative atom. There are also polyatomic ions: groups of atoms held together with covalent bonds that have an overall charge (figure ... shows the Lewis dot structure of a phosphate polyatomic ion, as an example). For now, we'll focus just on ionic bonds between monoatomic ions, like in table salt. You'll learn more about polyatomic ions and the compounds they form in Sequence 2.

Let's examine how a simple ionic compound is formed: sodium chloride, also known as table salt, is made up of sodium and chlorine atoms (see figure 2.6). When sodium and chlorine come in contact with each other, electrons move from the sodium to the chlorine, making a sodium *cation* (positively-charged ion) and a chloride *anion* (negatively-charged ion). Yes — chlor*ide* is correct! When naming an anion, the ending of the element name changes to *-ide*. Once the sodium cation and chloride anion are formed, their opposite charges attract them to each other, like north and south magnet poles.

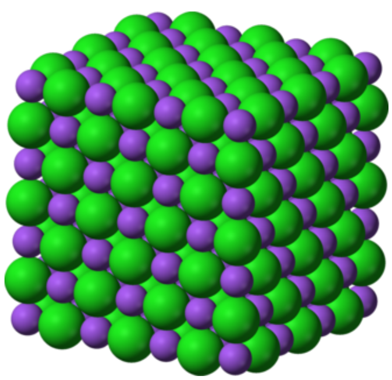


Figure 2.7

When there are many, many sodium and chloride ions around, they spontaneously arrange themselves in a pattern, giving ionic compounds their characteristic crystal structure (see figure 2.7). Because the electrons are tightly held be each ion, ionic substances don't conduct electricity well as solids. The atomic crystal lattice also determines the shape of the macroscopic crystals. Salt crystals are generally cubic, while other crystals (like quartz) form hexagonal prisms. You'll learn how to predict the atomic and macroscopic crystal structure of different compounds in Sequence 2.

Modeling Ionic Compounds

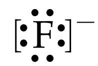
You can represent ions with Lewis dot diagrams by adding or subtracting electrons from the model. Since electrons are negative, anions have gained electrons while cations have lost them. Additionally, the ion is in brackets and the overall charge is indicated outside the top right corner of the brackets.

Example: Create Lewis dot diagrams for Na^+, F^-, O^{2+}, and Mg^-.

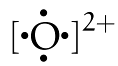
Solution: Sodium is in column 1, so a neutral sodium atom has 1 valence electron. A +1 charge means it has lost 1 electron, leaving zero.

$$[\mathrm{Na}]^+$$

Fluorine is in column 17 and has 7 valence electrons when neutral. The anion F^- has gained one electron, for a total of 8.



Neutral oxygen has 6 valence electrons, therefore O^{2+} has 4.



Neutral magnesium has 2 valence electrons, so Mg^{-} has 3 valence electrons.

To make Lewis dot diagrams of ionic compounds, you show both ions in the ratio given in the formula. For example, a Lewis dot diagram of $MgCl_2$ would show one magnesium cation and two chloride anions.

2.2.3 Metallic Compounds

You may already know that metals (both pure and alloyed) are excellent conductors of electricity and heat. This is a consequence of their metallic bonds. In pure metals and alloys, the outermost layer of electrons can move freely from one atom to the next. As a result, at the atomic level, metals are best characterized as a lattice of cations surrounded by a ”sea of electrons” (see figure fixme metallic bonding figure)

The free-flowing sea of electrons in pure and alloyed metal means the cation lattice can be rearranged without breaking the metallic bond. As a result, metals are ductile (able to be drawn out into a wire) and malleable (able to be hammered into a new shape without cracking or breaking). (fixme figure showing deformation of cation lattice in bending metal)

Metals can be pure, like copper or iron, or *alloys*, like bronze or steel. *Alloys* are mixtures between two or more elements where at least one element is a metal. Steel is an alloy of iron and carbon; bronze is an alloy of copper and tin. Alloying a metal changes its properties because of the change in the cation lattice (see figure fixme pure vs alloyed lattice and structural changes).

2.3 Energy and Work

Energy is defined as the ability to do work, but what does this mean? First, we need to understand what *work* is. When you lift an object into the air, you are doing work on that object. When water turns a turbine in a hydroelectric plant, the water is doing work on the turbine. And when you hit the brakes on your car, the brake pads are doing work on the tires (albeit, negative work). *Energy* is being transferred between these pairs of objects when work is done.

Some everyday examples of energy include:

1. The Calories in your food
2. The light from the Sun
3. Heat in the Earth's mantle
4. The motion of a spinning wheel

Some types of energy are easy to visualize, while others are not. Energy is what moves from one object to another when work is being done. When you lift something, the energy stored in your body (in the form of sugar and fat) is transferred to the object, accelerating it upwards. Your body continues to transfer energy as you lift the object against gravity. When you've lifted it as high as you can, most of the energy your body lost (we call this "burning Calories" colloquially) is stored as *potential energy*, due to the object's height.

Another example is a simple circuit connecting a battery and a light bulb. The battery has stored potential energy. When the circuit is complete, the potential energy in the battery is transferred to electrons in the light bulb, causing them to move and gain kinetic energy. In the light bulb's filament (we are referencing old, non-LED light bulbs here!), the electrons encounter resistance, which slows them down. The energy the electrons lose in this process is being transformed into light and heat, lighting your room.

The Work-Energy theorem explains the relationship between work and energy, and we will introduce the theorem and use it to explain energy transfer in a subsequent chapter.

2.4 Mass-Energy Equivalence

You've probably seen the equation

$$E = mc^2$$

E is energy, m is mass, and c is the speed of light in a vacuum (about $3 \times 10^8 \frac{m}{s}$). So far, we've been discussing matter and energy separately. This equation shows that matter can be converted to energy, and vice-versa. This is the source of the light energy emitted by the sun.

The Sun is mostly hydrogen. At the very center, it is so hot and dense that the nuclei of hydrogen atoms are fused together to form helium atoms, a process called the proton-proton chain reaction. The actual reaction involves several steps and is more complicated, but the overall process can be summarized as:

$$4\text{H} \rightarrow \text{He}$$

If every hydrogen atom has a mass of approximately $1.6735575 \times 10^{-27}$ kg and every helium atom has a mass of approximately $6.6464731 \times 10^{-27}$ kg, how much energy is released when one atom of helium is created? First, notice that one helium has less mass than 4 hydrogen atoms:

$$4 \times \left(1.6735575 \times 1{-}^{27}\right) - 6.6464731 \times 10^{-27} = 4.77569 \times 10^{-29}$$

Now, we can use $E = mc^2$ to find out how much energy is equivalent to 4.77569×10^{-29} kg:

$$E = \left(4.77569 \times 10^{-29}\text{kg}\right)\left(2.99792458 \times 10^{8}\frac{\text{m}}{\text{s}}\right)^2$$
$$E \approx 4.292 \times 10^{-12}\text{joules}$$

All of these numbers are very small and hard to visualize. We could ask this: if 1 kilogram of hydrogen (about enough to fill a standard beach ball) were fused to make helium, how much energy would be released? For every kilogram of hydrogen that enters the proton-proton chain reaction, about 0.02854 kilograms of mass are converted to energy (the mass of about 5 quarters).

$$E = (0.02854 \text{ kg})\left(2.99792458 \times 10^{8}\frac{\text{m}}{\text{s}}\right)^2 \approx 2.5647 \times 10^{15} \text{ joules}$$

This is more than 700,000,000 kWh (kilowatt hours); the average US household uses only 30 kWh per day. Fusing one kilogram of hydrogen releases enough energy to power an average US home every day for *over 65,000 years*! This relatively huge release of energy is why scientists continue to research nuclear fusion energy sources. The nuclear power plants currently running around the world today rely on *nuclear fission*: the splitting apart of atoms, the opposite process of *nuclear fusion*. Nuclear fission releases much, much less energy per kilogram of input material than nuclear fusion, and thus stable, affordable nuclear fusion power plants remain a ”holy grail” of scientific research.

2.5 Conclusion

We have seen that the universe is made of dark energy, dark matter, and ordinary matter. Ordinary matter is made of atoms, which can be classified based on their number of protons. Atoms combine in different ways to make compounds, and the manner of combination (ionic, covalent, or metallic bonding) affects the macroscopic properties of the substance. Energy allows matter to do work, and work is the transfer of energy.

Matter and energy do share a fundamental property: they are both conserved. Neither matter nor energy can be created or destroyed. This means the total amount of ordinary matter and energy in the universe is constant. This great scientific truth *something about its application* In the next chapter,

Chapter 3

Conservation of Mass and Energy

One of the most fundamental laws in science is the conservation of mass and energy. This law states that mass (matter) and energy cannot be created or destroyed. This means the total matter and energy in the universe always stays the same.

3.1 Conservation of Mass

Since matter cannot be created or destroyed, a chemical reaction does not change the mass of the reactants as they form products. Consider the reaction between vinegar and baking soda, which produces carbon dioxide, water, and sodium acetate:

acetic acid
common name: vinegar

dihydrogen monoxide
common name: water

$$NaHCO_3 + CH_3COOH \rightarrow NaCH_3COO + H_2O + CO_2$$

sodium bicarbonate
common name: baking soda

sodium acetate

carbon dioxide

The reactants are labeled with red, and the products with blue. This equation is *balanced*: that is, it shows the same number of each element on each side of the arrow. This shows that the atoms are not created or destroyed during a chemical reaction; they are only rearranged. Take a minute to count up each element on each side. You should find there are 5 hydrogens, 1 sodium, 3 carbons, and 5 oxygens on each side.

Now, let's look at an *unbalanced* chemical reaction. As you know, hydrogen and oxygen combine to form water. Additionally: hydrogen and oxygen both exist as *diatomic gases*. When we say "oxygen gas" or "hydrogen gas", we mean the diatomic molecules, O_2 and H_2, respectively. Here is an unbalanced chemical reaction between hydrogen gas and oxygen gas to form water:

$$H_2 + O_2 \rightarrow H_2O$$

How do we know this equation is unbalanced? Count up the elements: there are two oxygen atoms on the reactant side, but only one on the product side. This violates the conservation of matter: that oxygen atom cannot just disappear!

3.1.1 Balancing Chemical Reactions

We solve this by *balancing* the chemical reaction: adjusting the number of products and reactants to comply with the Law of Conservation of Matter. You'll learn strategies for balancing chemical reactions in Sequence 2, but for now we'll briefly balance this chemical reaction so that it complies with the Law of Conservation of Matter. You may be tempted to simply add a lone oxygen atom to the products side:

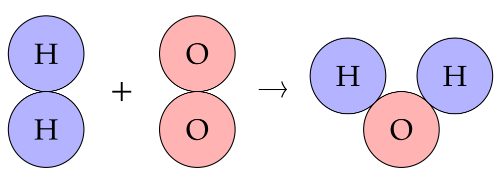
$$H_2 + O_2 \rightarrow H_2O + O$$

The major reason this is incorrect is that oxygen does not exist as a lone atom - as discussed above - so it doesn't make sense to have a lone oxygen as a product. So maybe we should add a molecule of oxygen gas to both sides?

$$H_2 + O_2 \rightarrow H_2O + O_2$$

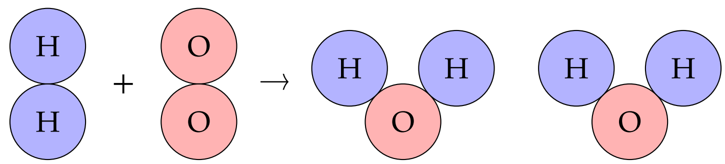
Well now we have the same problem we started with: the oxygens are unbalanced. When balancing chemical reactions, we can only add *whole molecules* that are already in the reaction. Let's take another look at our unbalanced reaction with some molecular models for visualization:

$$H_2 + O_2 \rightarrow H_2O$$



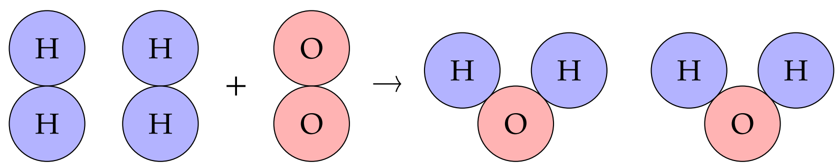
You can clearly see we need more oxygens on the product side. Since we can only add *whole molecules*, our only option is to add another water. We do this by adding a coefficient of 2 in front of H_2O in our equation, which indicates 2 water molecules (just like $2x$ means two x's):

$$H_2 + O_2 \rightarrow 2H_2O$$



We've fixed our oxygen problem: now there are two oxygen atoms on both sides. But now we have a hydrogen problem: there are 2 on the reactant side and 4 on the product side. We can address this by adding another hydrogen gas molecule to the reactant side:

$$2H_2 + O_2 \rightarrow 2H_2O$$



And now we have the same number of hydrogens and oxygens on each side! Notice we have all the same reactants and products that we started with, but now in ratios that reflect the conservation of matter.

A final note: if atoms are in parentheses followed by a subscript, the subscript applies to every atom in the parentheses. For example, zinc nitrate, $Zn\,(NO_3)_2$ is made of 1 zinc, 2 nitrogens, and 6 oxygens.

Exercise 2 Balanced and Unbalanced Reactions

Classify the following chemical reactions as balanced or unbalanced. If it is unbalanced, state what element(s) are not conserved.

1. $NiCl_2 + 2NaOH \rightarrow Ni\,(OH)_2 + 2NaCl$
2. $HgO \rightarrow Hg + O_2$
3. $BaSO_4 + 2C \rightarrow 2BaS + CO$
4. $Cd\,(NO_3)_2 + H_2S \rightarrow CdS + 2HNO_3$

Working Space

Answer on Page 71

3.2 Conservation of Energy

Just like matter, energy is also conserved: it cannot be created or destroyed, only change forms. You'll learn more about the types of energy in a subsequent chapter, Work and Energy. The transformation of energy from one type to another drives our modern world: your phone transforms electrical potential energy into light and sound energy, a nuclear power plant transforms nuclear energy to electrical energy, and your car transforms chemical potential energy (in the gasoline) into kinetic energy (motion).

3.2.1 Friction, Heat, and Energy "Loss"

Imagine rolling a ball across a flat surface: you have given the ball some kinetic energy in its motion. If the kinetic energy were conserved, the ball would keep rolling at the same speed forever, as long as it was on a flat surface. Experience tells us this isn't what happens: the ball will eventually come to a stop. Why doesn't this violate the Conservation of Energy?

Friction is the force that opposes motion: whenever you slide two objects past each other, friction transforms kinetic energy into heat. Rub the palms of your hands together. You should feel warmth, a product of the friction between your hands. As the ball in the example above rolls, it also experiences friction between itself and the ground. The friction slowly transforms the kinetic energy of the ball into heat, causing the ball to lose kinetic energy. When all of the ball's kinetic energy is transformed to heat, the ball comes to a rest. So, the kinetic energy of the ball wasn't destroyed and didn't disappear: it became heat.

In fact, nearly all energy in the universe will eventually be transformed to heat, resulting in the inevitable "heat death" of the universe. Here is a short video about heat, entropy, and the heat death of the universe: https://www.youtube.com/watch?v=gOWt_Hq3yrE/.

3.2.2 Energy Conservation in Falling Objects

When an object is positioned above the ground, it has *gravitational potential energy*. The potential energy is proportional to the mass of the object, the strength of the gravitational field, and the object's height above the ground:

$$E_p = mgh$$

where m is the mass of the object in kg, g is the acceleration due to gravity ($9.8 m/s^2$ on Earth), and h is the height above the ground in meters.

Example: What is the gravitational potential energy of a 5.0 kilogram bowling ball in the hand of a bowler (approximately 1.2 meters high)?

Solution: The mass is 5.0 kg, the height is 1.2 m, and we assume the bowler is on Earth, so g is 9.8 m/s^2:

$$E_p = (5.0 kg) \left(9.8 m/s^2\right) (1.2 m) = 59 J$$

Here is a new unit: *joules*, represented by a capital J. A joule is a unit of energy, and it is the same as $\frac{kg \cdot m^2}{s^2}$.

If the bowler were to drop that ball, it would lose potential energy and gain *kinetic energy*. The kinetic energy of an object is proportional to its mass and the square of its speed:

$$E_k = \frac{1}{2} m v^2$$

where mass is in kg and speed (v) is in m/s. You can quickly check that the units still come out to a joule! If there is no air resistance, then as an object falls all of its potential energy is converted to kinetic energy. (If you'd like to explore what happens when friction is accounted for, you can play with this PhET simulation: https://phet.colorado.edu/en/simulations/energy-skate-park.)

Example: If the bowling ball were dropped, what would it's speed be right before it hits the ground? (Neglect air resistance.)

Solution: Ignoring air resistance, according to the Law of Conservation of Energy, the kinetic energy of the ball right before it hits the ground must be equal to the gravitational potential energy of the ball right before its release. Therefore:

$$mgh = \frac{1}{2} m v^2$$

We can eliminate an m from both sides and rearrange to solve for v:

$$v = \sqrt{2gh} = \sqrt{2 (9.8 m/s^2)(1.2 m)} = \sqrt{23.52 \frac{m^2}{s^2}} = 4.8 \frac{m}{s}$$

The ball will have a speed of 4.8 m/s just before it hits the ground.

Exercise 3 Kinetic and Potential Energy

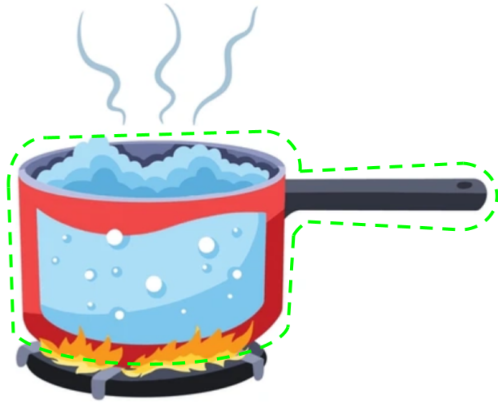
Working Space

1. How fast should you toss a ball straight up if you want it to reach your friend on the second floor (3.7 meters above you)?
2. Your little brother is teasing you from a treehouse 5.0 meters off the ground. If your slingshot can shoot a pebble at 15 m/s, can you hit your little brother?
3. A 63-kg roller-blader rolls down a hill. If the hill is 8.0 meters high and she reaches the bottom of the hill with a speed of 11.2 m/s, how much energy was converted to heat through friction?

Answer on Page 71

3.3 Types of Systems

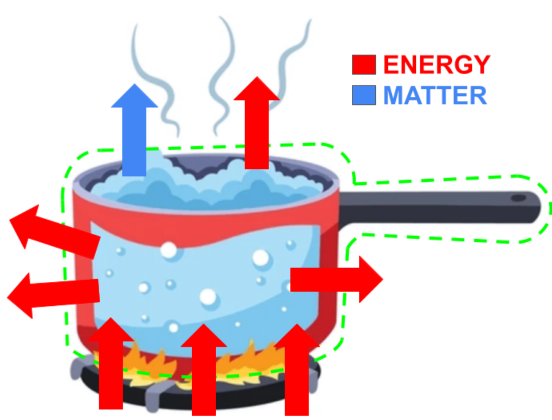
We classify systems based on the flow of matter and energy. A *system* is a set of interconnected elements. Your body is a system, as is a television or a boiling pot of water. Scientists define systems by separating the parts of the system from the rest of the universe, usually called ”the surroundings”. You can represent this separation with a dashed line. Here is a diagram defining a pot of boiling water as a system:



Everything inside the dashed line is the system: the pot and the water boiling in it. Everything outside the dashed line is the surroundings: the stove, the air around the pot, etc. Defining a system is *arbitrary*: there isn't one hard and fast definition of a system. Think of your school: you could look at the system of a classroom, the system of a hallway and all the classes connected to it, or the entire school building. How you define a system depends on what you're studying.

3.3.1 Open Systems

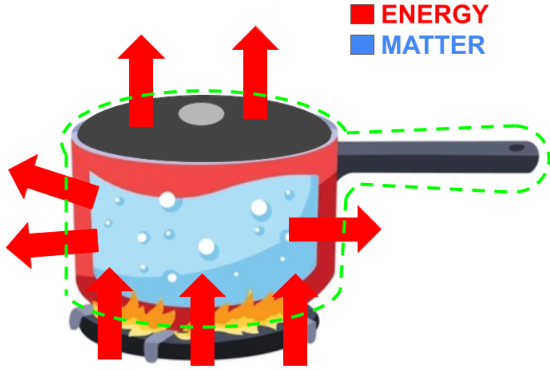
An *open system* allows for matter and energy to cross the imaginary boundary between the system and its surroundings. The uncovered pot of boiling water is an example of an open system. Energy enters the system as heat from the stove, and leaves the system as heat in the steam rising from the pot. Notice that steam rising: see how it crosses the imaginary boundary? The steam is matter *leaving the system*. Since matter and energy can cross the boundary between the system and its surroundings, the uncovered boiling pot is an open system.



The system also loses energy through the sides of the pot: if you touched the pot, it would feel hot, which means heat energy can also leave the system through the sides of the pot. This is due to the collision between air particles (the surroundings) and the outside of the pot (the system). With every collision, a little heat is transferred from the pot to the air. This is why your hot drink gets cold if you leave it out, even if you don't add any ice.

3.3.2 Closed Systems

A *closed system* allows for the transfer of energy but not the transfer of matter. If we put a lid on this pot, it would become a closed system.



Notice that the difference between an open and closed system is the flow of *matter*: open systems allow for the movement of matter, while closed systems do not. However, energy can still enter and leave closed systems. Sealed containers that aren't insulated are good examples of closed systems - a car with the windows up and doors closed.

3.3.3 Isolated Systems

An *isolated system* does not allow for the flow of matter *or* energy in or out of the system. There is no such thing as a truly isolated system - in reality a small amount of energy can be transferred even through the best thermal insulators. However, for well-insulated systems, it can be a good approximation to model that system as isolated. A simple example would be a sealed, well-insulated coffee thermos. The transfer of heat energy between the coffee in the thermos and the thermos' surroundings is so slow that we can ignore that small amount of transfer and approximate the thermos as an isolated system.

3.3.4 Classifying Systems

To quickly categorize a system as open, closed, or isolated, ask yourself two questions:

1. Can matter enter or leave the system?
2. Can energy enter or leave the system?

If your answer to the first question is yes, you know automatically the system is open. If no, move to the second question. If energy can enter or leave, the system is closed. If not, the system is isolated. Sometimes, textbooks and exams will describe a system as ”well-insulated”. This is directing you to assume any transfer of energy between the system and surroundings is negligible, and that you should treat the system as an isolated system.

Exercise 4 Open, Closed, and Isolated Systems

Classify each system as open, closed, or isolated. Justify your answer.

1. The human body
2. Earth
3. Your cell phone
4. A well-insulated cooler with the lid sealed
5. A well-insulated cooler with the lid open
6. A bottle of soda before it is opened to be drunk

Working Space

Answer on Page 71

Chapter 4

Mass, Weight, and Gravity

Mass is a measure of the amount of matter in an object. Weight is the force of gravity on that object. An object's mass is the same no matter where it is in the universe: the amount of "stuff" in an object does not depend on its location. However, an object's weight *does* change: the same object has a different weight on the Moon than it does on Earth or Venus. Since the force of gravity on the Moon is about $1/6^{\text{th}}$ as on Earth, you would weigh $1/6^{\text{th}}$ as much on the Moon, but your mass would be the same.

4.1 Kilograms versus Pounds

In elementary school, you probably learned that 1 kilogram is about 2.2 pounds. This is only true *on Earth*. In everyday language, we may use kilograms and pounds, or mass and weight, interchangeably; scientists do not.

Mass	**Weight**
measure of *amount*	measure of *force*
units include grams, kilograms, milligrams	units include pounds, Newtons, dynes
a *scalar* measurement	a *vector* measurement
does *not* depend on location	does depend on location

On Earth, a 1 kilogram mass weighs 2.2 pounds. That same mass weighs 0.37 pounds on the Moon. You learned to "convert" between pounds in kilograms in elementary school because everywhere on Earth, a 1-kg mass weighs 2.2 lbs. The conversion doesn't apply to other locations in the universe because mass and weight are different.

4.1.1 Scalars and Vectors

One of the fundamental differences between mass and weight is the type of measurement: scalar versus vector. *Scalar* measurements are just a counting number: they tell the amount of magnitude. Mass is a scalar measurement: it tells how much matter is in an object. Energy is also a scalar measurement, as well as temperature and density. *Vectors*, on the other hand, also have a *direction*. A complete measure of a force gives the magnitude and direction. So, your weight isn't just 185 pounds: it's 185 pounds *downward*. Velocity is a speed plus a direction, such as 35 mph east. We will expand on this later in the vectors chapter!

4.2 Mass and Gravity

When we discuss weight, we usually mean the gravitational force between an object and the largest nearby object. My weight on Earth, a spacecraft's weight on the Moon, or a rover's weight on Mars are all examples. But planets and moons don't have some special property that makes them emit gravity. Rather, *all objects with mass are gravitationally attracted to each other*. But, compared to the mass of the Earth, the mass of all the other objects around you (a table, your family, your house or apartment complex) is very very small.

The force of the gravitational attraction between two objects is proportional to the product of their masses, and inversely proportional to their distance squared. This means that as objects get farther away, the force decreases. If you double the distance, the force quarters. Quadruple the distance, the force is $1/16^{th}$ as much. This is why you are more attracted to the earth than you are to distant stars, even though they have much more mass than the earth.

Newton's Law of Universal Gravitation

Two masses (m_1 and m_2) that are a distance of r from each other are attracted toward each other with a force of magnitude:

$$F = G\frac{m_1 m_2}{r^2}$$

where G is the universal gravitational constant. If you measure the mass in kilograms and the distance in meters, G is about 6.674×10^{-11}. That will get you the force of the attraction in newtons.

Example:

Exercise 5 Gravity

Working Space

The earth's mass is about 6×10^{24} kilograms.

Your spacecraft's mass is 6,800 kilograms.

Your spacecraft is also about 100,000 km from the center of the earth. (For reference, the moon is about 400,000 km from the center of the earth.)

What is the force of gravity that is pulling your spacecraft and the earth toward each other?

Answer on Page 72

4.3 Mass and Weight

Gravity pulls on things proportional to their mass, so we often ignore the difference between mass and weight.

The weight of an object is the force due to the object's mass and gravity. When we say, "This potato weighs 1 pound," we actually mean "This potato weighs 1 pound on earth." That same potato would weigh about one-fifth of a pound on the moon (see figure 4.1).

However, that potato has a mass of 0.45 kg no matter where it is in the universe.

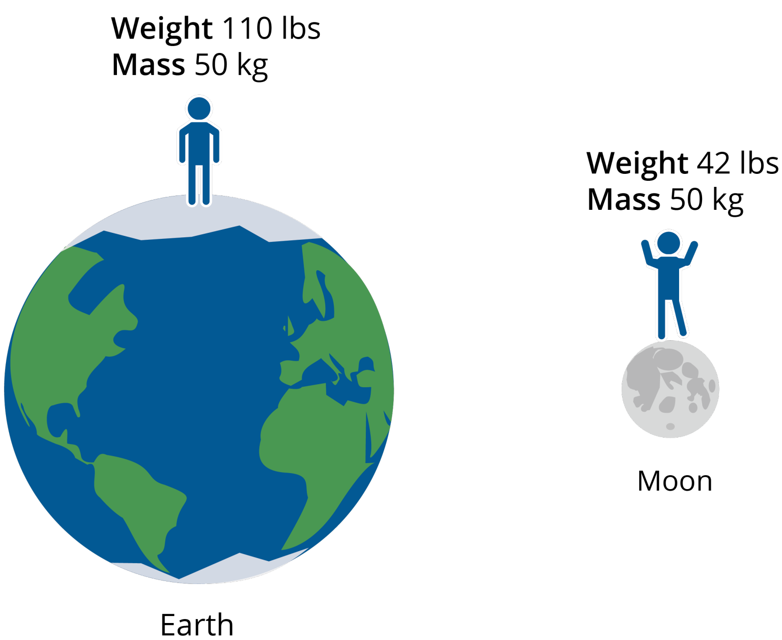


Figure 4.1: Mass is a measure of all the matter in an object. Weight is a measure of the force of gravity on that object. Mass is not location-dependent, while weight is.

Chapter 5

Force, Mass, and Acceleration

5.1 Mass and Acceleration

Each atom has a mass, which means everything made up of those atoms has mass as well (and that's pretty much everything!). We measure mass in grams. A paper clip is about 1 gram of steel. An adult human can have a mass of 70,000 grams, so for larger things, we often talk about kilograms, which is 1000 grams.

The first interesting thing about mass is that objects with more mass require more force to accelerate. For example, pushing a bicycle so that it accelerates from a standstill to jogging speed in 2 seconds requires much less force than pushing a train so that it accelerates at the same rate.

> **Newton's Second Law of Motion**
>
> The force necessary to accelerate an object of mass m at an acceleration of a *on Earth* is given by:
>
> $$F = ma$$
>
> This means the force is equal to the mass times the acceleration.

This is known as Newton's Second Law of Motion.[1]

What are the units here? We already know that mass is measured in kilograms. We can measure velocity in meters per second, but that is different from acceleration. Acceleration is the rate of change in velocity. So if we want to go from 0 to 5 meters per second (that's jogging speed) in two seconds, that is a change in velocity of 2.5 meters per second every second. We would say this acceleration is $2.5 m/s^2$.

5.1.1 Velocity versus Acceleration

Acceleration is the change in velocity. If an object is speeding up or slowing down, it is accelerating. In everyday language, we often use *decelerate* to indicate slowing down, but in physics you can use the word accelerate (slowing down is just negative acceleration). Since velocity is a vector (it has a magnitude and direction), changing direction is *also*

[1] This is a simplified version of Newton's Law of Gravitation. The formula $F = G\frac{m_1 m_2}{r^2}$ simplifies very close to $F = ma$ for any object within the Earth's atmosphere.

acceleration. On the other hand, an object moving at a constant velocity (same speed, same direction) is *not accelerating*!

Exercise 6 Is it accelerating?

State whether the described object is accelerating or not.

1. A satellite orbiting the Earth at a constant speed.
2. A car moving due west at a constant 30 mile per hour.
3. A child coming to a stop on their bicycle.
4. A roller coaster going around a loop at a constant speed.
5. A roller coaster speeding up as it goes down the initial hill.
6. A book sitting on a table.

Working Space

Answer on Page 72

It's a common misconception that all objects in motion are accelerating. If an object is moving with a constant velocity (same speed, same direction), then it is not accelerating.

5.1.2 Calculating Acceleration

When an object is speeding up or slowing down, we can calculate the acceleration by dividing the change in velocity by the time it takes to make that change.

Calculating Acceleration

The acceleration of an object from an initial velocity, v_i, to a final velocity, v_f, over a period of time, t, is given by:

$$a = \frac{v_f - v_i}{t}$$

Notice that if the velocity does not change, then $v_f - v_i = 0$ and the acceleration is also zero.

Example: Your car can go from zero to 60 mph in 3 seconds. What is the acceleration in m/s^2?

Solution: First, let's convert from the imperial units of miles per hour to the SI units of meters per second. You can do this using a search engine, but we will show how to do it by hand below. (You will learn more about this method in the Units chapter).

$$\frac{60 \text{ miles}}{1 \text{ hour}} \cdot \frac{1.61 \text{ km}}{1 \text{ mile}} \cdot \frac{1000 \text{ m}}{1 \text{ km}} \cdot \frac{1 \text{ hour}}{3600 \text{ seconds}} \approx \frac{26.82 \text{ m}}{\text{s}}$$

Now we have the starting velocity (0 m/s), the ending velocity (26.82 m/s), and the time (3 s), and we can find the acceleration:

$$a = \frac{v_f - v_i}{t} = \frac{26.82\frac{m}{s} - 0\frac{m}{s}}{3s} \approx 8.94\frac{m}{s^2}$$

5.1.3 Determining Force

What about measuring force? Newton decided to name the unit after himself: The force necessary to accelerate one kilogram at $1m/s^2$ is known as *a newton*. It is often denoted by the symbol N.

$$1\text{N} = 1\frac{kg \cdot m}{s^2}$$

Example: If the car in the above example has a mass of 1500 kg, how much force does the engine use to accelerate the car?

Solution: We have already found the car's acceleration: 8.94 m/s^2. With the mass and acceleration, we can use Newton's Second Law to find the force needed to accelerate the car:

$$F = m \cdot a = 1500 \text{ kg} \cdot 8.94\frac{m}{s^2} = 13410 \text{ N}$$

Exercise 7 Acceleration

Working Space

While driving a bulldozer, you come across a train car (with no brakes and no locomotive) sitting on a track in the middle of a city. The train car has a label telling you that it has a mass of 2,400 kg. There is a time-bomb welded to the interior of the train car, and the timer tells you that you can safely push the train car for 120 seconds. To get the train car to where it can explode safely, you need to accelerate it to 20 meters per second. Fortunately, the track is level and the train car's wheels have almost no rolling resistance.

With what force, in newtons, do you need to push the train for those 120 seconds?

Answer on Page 73

5.2 Net Force

So far, we've looked at examples where only one force is acting on an object. In reality, there are usually multiple forces acting on an object. For example, the engine pushes your car forward while friction pulls it backwards. Or your chair is pushing up on you while gravity pulls you down. How then can we describe the motion of an object if more than one force is acting on it?

We can rearrange Newton's Law:

$$a = \frac{F_{net}}{m}$$

This means that an object's acceleration is directly proportional to the *net force* acting on the object and inversely proportional to the object's mass. The *net force* is the vector sum of all the forces acting on an object. The vector sum just means we have to take the direction of the force into account. Usually, up and right are positive while down and left are negative. You can see some examples in figure 5.1.

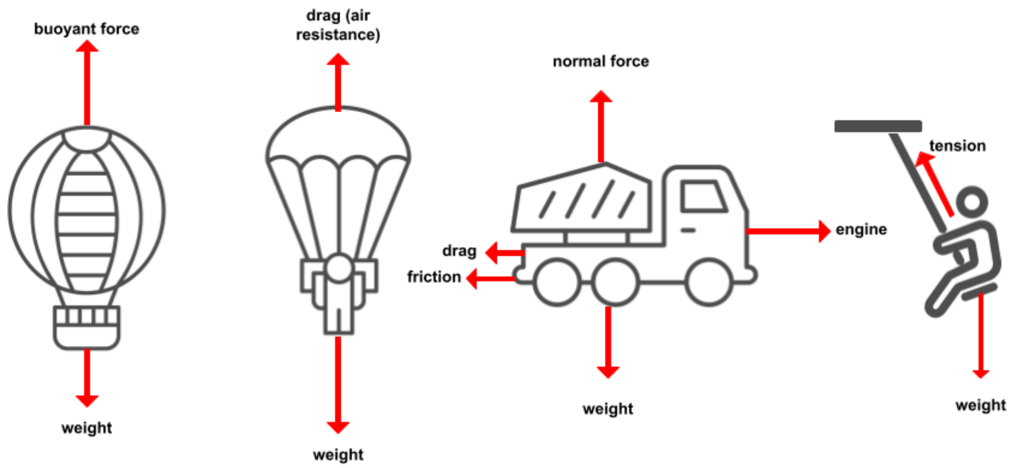
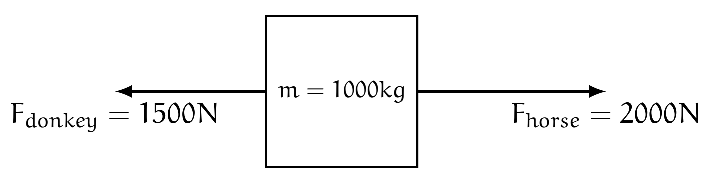


Figure 5.1: The hot air balloon has an upwards net force. The parachuter has a downwards net force. The truck has a rightward net force. The swinger has a leftward net force.

For now, we'll only look at parallel forces (up/down or left/right). You'll learn to use vector addition to combine orthogonal (at right angles) and skew (at angles other than parallel or right) forces in a later chapter.

Example: A donkey and a horse are each pulling a cart. The donkey pulls to the left with a force of 1500 N, while the horse pulls to the right with a force of 2000 N. What is the net force on the cart? If the cart has a mass of 1000 kg, in what direction and with what magnitude will the cart accelerate?

Solution: We begin by drawing a diagram (this is good practice - while a diagram may not be necessary for such a simple question, it will be very useful as we examine more complex scenarios in future chapters).



It is customary to take right as positive, so the horse applies a positive force while the donkey applies a negative force to the cart. Therefore, the net force is:

$$F_{net} = F_{horse} - F_{donkey} = 2000N - 1500N = 500N$$

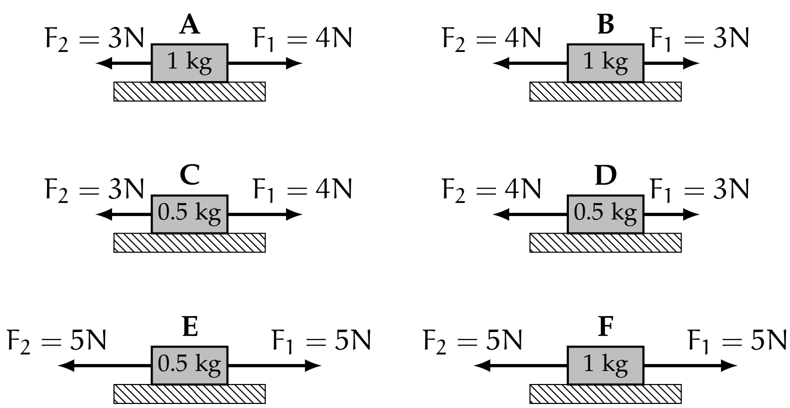
Since the net force is positive, it points to the right and the cart will accelerate to the right at a rate of:

$$a = \frac{F_{net}}{m} = \frac{500\text{N}}{1000\text{kg}} = 0.5\frac{\text{m}}{\text{s}^2}$$

Exercise 8 Net Force and Acceleration

Working Space

Rank the acceleration of the boxes shown below from greatest to least. All surfaces are frictionless and each box starts at rest. Take left as negative, and negative accelerations are less than a zero acceleration.

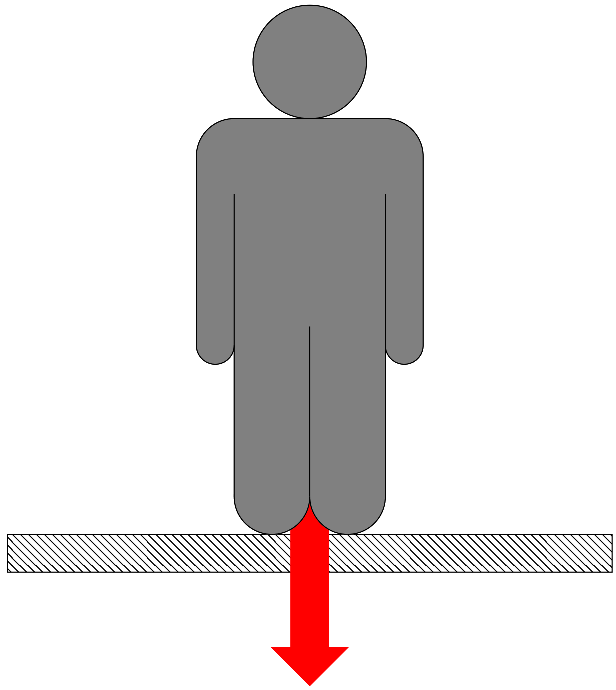


Answer on Page 73

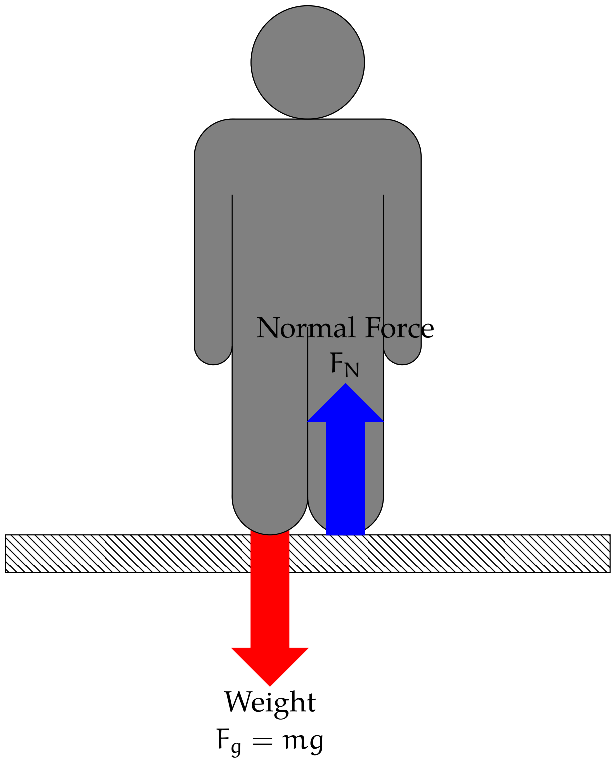
5.3 Normal Force and Apparent Weight

If you were to stand on a scale while riding an elevator, you would see the scale fluctuate as the elevator accelerates up and down (if you have a bathroom scale and live in a building with an elevator, you can try this yourself!). The reading on the scale is your *apparent weight* and is equal to the normal force between you and the floor.

What is a *normal force*? First, the word "normal" doesn't have the colloquial meaning of average or usual. In mathematics, "normal" means perpendicular. A normal force is perpendicular to the contact surface between two objects. Let's look at a person standing at rest. We know that gravity is pulling down on the person:



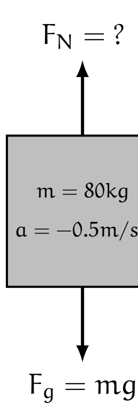
Since the person is not accelerating, there must be some other force acting on them in the upwards direction that balances out the person's weight (recall that if $a = 0 m/s^2$, then it must be true that $F_{net} = 0N$). In this case, the balancing force is the *normal force* of the floor pushing up on the person's feet:



When you step on a bathroom scale, it is actually measuring the normal force! When you are not accelerating, your weight and the normal force between you and the scale are equal, so the scale gives you an accurate measure of your weight. Let's look at what happens if you are accelerating, as in an elevator.

Example: Maria has a mass of 80. kg. If the elevator in her apartment building initially accelerates at 0.50 m/s^2, what is her apparent weight as the elevator begins to move down?

Solution: We begin with a diagram:



Her apparent weight is the normal force. Seeing that the net force acting on Maria is given by $F_N - F_g$, we can use Newton's Second Law to find F_N:

$$F_{net} = F_N - F_g = ma$$
$$F_N - (80\text{kg})\left(9.8\frac{m}{s^2}\right) = (80\text{kg})\left(-0.5\frac{m}{s^2}\right)$$
$$F_N = (80\text{kg})\left(9.8\frac{m}{s^2} - 0.5\frac{m}{s^2}\right) = 744\text{N}$$

Maria's apparent weight is 744 newtons when the elevator is accelerating downwards.

Exercise 9 Moving Elevators

A 700-N person is standing on a scale in an elevator. Rank the reading on the scale from greatest to least for the following scenarios:

1. The elevator has an initial upward velocity of 2 m/s and an upward acceleration of 3 m/s^2
2. The elevator is initially at rest and has an upward acceleration of 3 m/s^2
3. The elevator has an initial downward velocity of 2 m/s and an upward acceleration of 5 m/s^2
4. The elevator is initially at rest and has a downward acceleration of 9.8 m/s^2
5. The elevator has an initial upward velocity of 2 m/s and a downward acceleration of 5 m/s^2
6. The elevator has an initial downward velocity of 2 m/s and is not accelerating

Working Space

Answer on Page 73

5.4 Newtons and Joules

You now know the fundamental units for newtons and joules, which hints at the relationship between force and energy:

$$N = \frac{kg \cdot m}{s^2} \quad J = \frac{kg \cdot m^2}{s^2} = N \cdot m$$

A joule is equivalent to a newton times a meter. So, multiplying a force (newtons) by a displacement (meters) tells you how the energy (joules) of the object changes. This relationship is described by the Work-Kinetic Energy Theorem, which we will explore in the next chapter.

5.5 Newton's Third Law

Now we can discuss Newton's Third Law, which states that every action has an equal but opposite reaction. In other words, if body A exerts a force on body B, then body B simultaneously exerts a force of equal magnitude and opposite direction on body A. Mathematically, we can express this as:

$$\mathbf{F}_{AB} = -\mathbf{F}_{BA}$$

Any time we use Newton's Second Law, we can combine this with the third law to state that *the sum of the forces equals zero* in some action-reaction pair, assuming that the system is isolated. For example, if two boxes (1 and 2) are applying a force on each other, we can say that (no matter their mass values), $F_{\text{Box 1 on 2}} = -F_{\text{Box 2 on 1}}$. This applies to physics concepts like air resistance and drag, tension forces of conjoined boxes or train cars, and gravitation of planets, all of which we will discuss in future workbooks.

Note the following:

- Action-reaction pairs do *not* cancel when applying Newton's Second Law to a single body.
- The forces are always of the same type (both contact, both gravitational, etc).
- They occur simultaneously; there is no time delay.

Example: Consider a swimmer pushing the wall of a pool. What forces are in play?

Solution: While the swimmer exerts a force $\mathbf{F}_{\text{wall}}$ on the wall, the wall exerts the reaction $-\mathbf{F}_{\text{wall}}$ on the swimmer, propelling the swimmer forward. Although the wall does not move due to mass and acceleration differences, the forces are equal and opposite.

Chapter 6

Units and Conversions

Accurate measurements are at the heart of good data and good problem solving. Engineers need to be able to describe many different types of phenomena, such as distance, sound, light, force, and more.

At this point, you are working with a lot of units: grams for weight, joules for energy, newtons for force, meters for distance, seconds for time, and so on. For each type of measurement, there are several different units. For example, distance can be measured in feet, miles, and light-years.

Some Equalencies

Distance	
1 mile	1.6093 kilometers
1 foot	0.3048 meters
1 inch	2.54 centimeters
1 light-year	9.461×10^{12} kilometers
Volume	
1 milliliter	1 cubic centimeter
1 quart	0.9461 liters
1 gallon	3.7854 liters
1 fluid ounce	29.6 milliliters
Mass	
1 pound	0.4535924 kilograms
1 ounce	0.4535924 grams
1 metric ton	1000 kilograms
Force	
1 newton	1 kilogram meter per sec^2
Pressure	
1 pascal	1 newton per square meter
1 bar	0.98692 atmosphere
1 pound per square inch	6897 pascals
Energy	
1 joule	1 newton meter
1 calorie	4.184 joules
1 kilowatt-hour	3.6×10^6 joules

(You don't need to memorize these! Just remember that this page is here.)

In the metric system, prefixes are often used to express a multiple. Here are the common prefixes:

Common Prefixes for Metric Units

giga	$\times 10^9$
mega	$\times 10^6$
kilo	$\times 10^3$
milli	$\div 10^3$
micro	$\div 10^6$
nano	$\div 10^9$

(These are worth memorizing. Here's a mnemonic for the most common ones: "King Henry Doesn't Usually Drink Chocolate Milk" Or Kilo ($\times 10^3$), Hecto ($\times 10^2$), Deca ($\times 10^1$), Unit (for example: gram) ($\times 10^0$), Deci ($\times 10^{-1}$), Centi (1×0^{-2}), Mili ($\times 10^{-3}$).)

6.1 Conversion Factors

Here is a really handy trick to remembering how to do conversions between units.

Often, you will be given a table like the one above, and someone will ask you "How many miles are in 0.23 light-years?" You know that 1 mile = 1.6093 kilometers and that 1 light-year is 9.461×10^{12} kilometers. How do you do the conversion?

The trick is to treat the two parts of the equality as a fraction that equals 1. In other words, you think:

$$\frac{1 \text{ miles}}{1.6093 \text{ km}} = \frac{1.6093 \text{ km}}{1 \text{ miles}} = 1$$

and

$$\frac{1 \text{ light-years}}{9.461 \times 10^{12} \text{ km}} = \frac{9.461 \times 10^{12} \text{ km}}{1 \text{ light-years}} = 1$$

We call these fractions *conversion factors*.

Now, your problem is

$$0.23 \text{ light-years} \times \textit{ Some conversion factors} = ? \text{ miles}$$

Note that when you multiply fractions together, things in the numerators can cancel with things in the denominator:

$$\left(\frac{31\pi}{47}\right)\left(\frac{11}{37\pi}\right) = \left(\frac{31\not{\pi}}{47}\right)\left(\frac{11}{37\not{\pi}}\right) = \left(\frac{31}{47}\right)\left(\frac{11}{37}\right)$$

When working with conversion factors, you will do the same with the units:

$$0.23 \text{ light-years}\left(\frac{9.461\times10^{12}\text{ km}}{1\text{ light-years}}\right)\left(\frac{1\text{ miles}}{1.6093\text{ km}}\right) =$$
$$0.23 \text{ \sout{light-years}}\left(\times\frac{9.461\times10^{12}\text{ \sout{km}}}{1\text{ \sout{light-years}}}\right)\left(\frac{1\text{ miles}}{1.6093\text{ \sout{km}}}\right) = \frac{(0.23)(9.461\times10^{12})}{1.6093}\text{ miles}$$

Exercise 10 Simple Conversion Factors

Working Space

How many calories are in 4.5 kilowatt-hours?

Answer on Page 74

6.2 Conversion Factors and Ratios

Conversion factors also work on ratios. For example, if you are told that a bug is moving 0.5 feet every 120 milliseconds, what is that in meters per second?

The problem then is

$$\frac{0.5\text{ feet}}{120\text{ milliseconds}} = \frac{?\text{ m}}{\text{second}}$$

So you will need conversion factors to replace the “feet” with “meters” and to replace “milliseconds” with “seconds”:

$$\left(\frac{0.5\ \cancel{\text{feet}}}{120\ \cancel{\text{milliseconds}}}\right)\left(\frac{0.3048\ \text{meters}}{1\ \cancel{\text{feet}}}\right)\left(\frac{1000\ \cancel{\text{milliseconds}}}{1\ \text{second}}\right) = \frac{(0.5)(0.3048)(1000)}{120}\ \text{m/second}$$

Exercise 11 Conversion Factors

The hole in the bottom of the boat lets in 0.1 gallons every 2 minutes. How many milliliters per second is that?

Working Space

Answer on Page 74

6.3 When Conversion Factors Don't Work

Conversion factors only work when the units being converted are proportional to each other. Gallons and liters, for example, are proportional to each other: If you have n gallons, you have $n \times 3.7854$ liters.

Degrees celsius and degrees farenheit are *not* proportional to each other. If your food is n degrees celsius, it is $n \times \frac{9}{5} + 32$ degrees farenheit. You can't use conversion factors to convert celsius to farenheit.

Chapter 7

The Greek Alphabet

If you do anything involving math or physics, you will be encountering Greek letters on a regular basis. Here is a table for your reference:

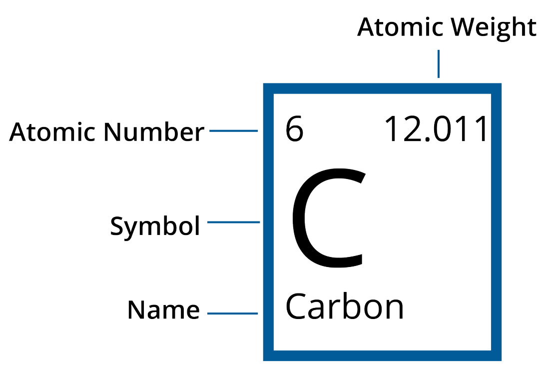
Capital	Lower	Pronounced	Capital	Lower	Pronounced
A	α	Alpha	N	ν	Nu
B	β	Beta	Ξ	ξ	Xi ("ku-ZY")
Γ	γ	Gamma	O	o	Omicron
Δ	δ	Delta	Π	π	Pi
E	ϵ	Epsilon	P	ρ	Rho
Z	ζ	Zeta	Σ	σ	Sigma
H	η	Eta	T	τ	Tau
Θ	θ	Theta	Υ	υ	Upsilon
I	ι	Iota	Φ	ϕ	Phi
K	κ	Kappa	X	χ	Chi ("Kai")
Λ	λ	Lambda	Ψ	ψ	Psi ("Sigh")
M	μ	Mu	Ω	ω	Omega

CHAPTER 8

Atomic and Molecular Mass

8.1 Reading a Periodic Tile

Let's look at the different information shown on a periodic tile:



The four things we learn from a periodic tile are:

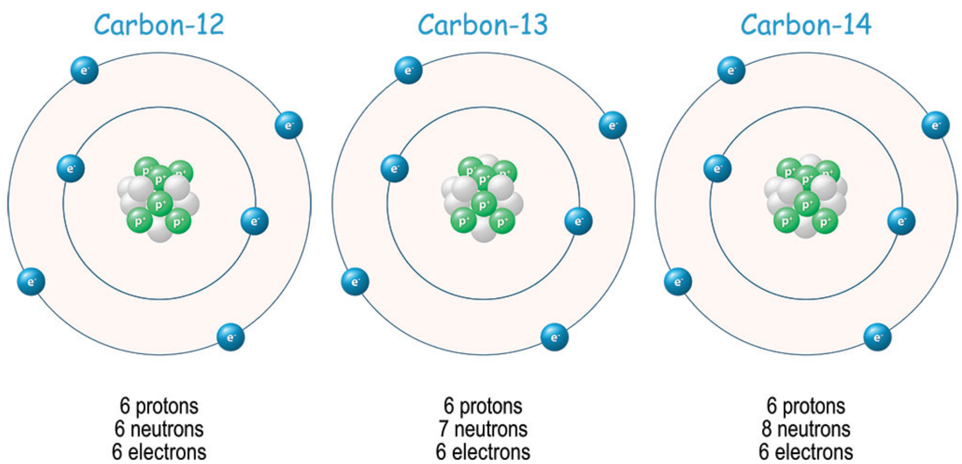
1. the symbol: as discussed in the previous chapter, each element has a unique symbol. Element symbols are used when showing the structure of a molecule and modeling chemical reactions.

2. the atomic number: this is also unique for each element. Take a look at the periodic table a few pages forward. Every tile has a unique atomic number, and the tiles are laid out in a generally increasing atomic number (you'll learn why the periodic table is arranged this way in Sequence 2).

3. the atomic weight: this is the average mass of all the atoms of that element in existence. Just like your overall grade in a class is the weighted average of all the individual grades you earned, atomic weight is the weighted average of the masses of all the individual atoms of that element. This is also sometimes referred to as atomic mass.

4. the name: not all periodic tables show the name of an element on its tile. This is why it is useful to know the symbols of common elements.

Recall from the previous chapter that we classify atoms by the number of protons they have. What this means is that if we want to know what element an atom is, we have to look at the number of protons. Take a look at the three carbon atoms below and note what is the same and what is different among them:



These different versions of carbon all have 6 protons, which is also carbon's atomic number. This isn't a coincidence: the atomic number *is* the number of protons in every atom of an element. If I tell you an atom has 4 protons, you would find atomic number 4 and see that the element is beryllium. To know how many protons an oxygen atom has, you would find its tile and see that it has atomic number 8.

Ok, so now we know atoms of the same element have the same number of protons, and that number is given by the element's atomic number. The difference between these carbon atoms explains the other number on a periodic tile: the atomic mass.

A proton and a neutron have about the same mass. An electron, on the other hand, has much less mass: One neutron weighs about the same amount as 2000 electrons. This means that the mass of any object comes mostly from the protons and neutrons in the nucleus of its atoms.

We know how many protons an atom has by what element it is, but how do we know the number of neutrons?

8.2 Mass of Atoms and Molecules

As you've seen, a periodic tile for an element tells us the average mass of an atom of that element in Daltons or amus (atomic mass units). The average mass of a carbon atom is 12.011 amu, and the average mass of an iron atom is 55.845 amu. Using the periodic table, determining the average mass of an atom is straightforward. What about molecules?

Consider water: H_2O. It is made of 2 hydrogen atoms, each with an average mass of 1.008 amu, and one oxygen atom, with an average mass of 15.999 amu. To find the mass of the molecule, called *molecular mass*, you simply add the masses of each of the atoms in the molecule. So, the molecular mass of water is 1.008 amu + 1.008 amu + 15.999 amu = 18.015 amu.

Exercise 12 Determining Molecular Mass

Find the molecular mass, in amu, of the following substances:

1. CH_4
2. $CuSO_4$
3. $C_6H_{12}O_6$

Working Space

Answer on Page 74

8.3 Mole Concept

An atomic mass unit is a very, very, very small unit; we would much rather work in grams. Grams are a large enough unit that you can develop a natural sense for how much a gram is. Additionally, while you can't see a single carbon atom with your eyes, you can see 10 grams of carbon (about enough to fill a pen cap). To convert between the very, very, very small unit of amus to the tangible unit of grams, we use *Avogadro's Number* (sometimes called *Avogadro's Constant*).

Since 1 amu is defined as $1/12^{th}$ of the mass of a carbon-12 atom, carbon-12 by definition has a mass of 12 amu. Additionally, Avogadro's number is the number of carbon atoms in 12.000 grams of pure carbon-12. This amount is called *a mole*. If you have 12 doughnuts, that's a dozen doughnuts. If you have 20 donuts, you have a score of donuts. 500 donuts: a ream of donuts. If you have $6.02214076 \times 10^{23}$ doughnuts, you have a *mole* of doughnuts.

This isn't really a practical measurement, as a mole of doughnuts would be about the size of the earth. We use moles for small things like molecules. However, a mole is a not an abbreviation for a molecule. For a better idea about how large of a number Avogadro's number is, you can watch this video: https://www.youtube.com/watch?v=TEl4jeETVmg. A mole of carbon-12 has a mass of 12.000 g, but a mole of natural carbon (which includes all the isotopes of carbon) has a mass of 12.011 g. The mole is defined such that one mole of an element is the same mass in grams as one atom is in amus. Let's say you want to know how much a mole of $NaCl$ weighs. From the periodic table, you see that Na has an atomic mass of 22.990 atomic mass units, and Cl has 35.453 atomic mass units. One atom of $NaCl$ has a mass of $22.990 + 35.453 = 58.443$ atomic mass units. This means a mole of $NaCl$ has a mass of 58.443 grams. Handy, right? This is called the *molar mass*. It is the mass of one mole of a substance, and is given in units of g/mol (grams per mole). The molar mass of NaCl is 58.443 g/mol. The molar mass of carbon is 12.011 g/mol. Using dimensional analysis and the molar mass, you can determine the mass of a given number of moles of a substance.

Example: What is the mass of 2 moles of copper?

Solution: The conversion we will use is 1 mol Cu = 63.546 g Cu.

$$\frac{2 \text{ mol Cu}}{} \times \frac{63.546 \text{ g Cu}}{1 \text{ mol Cu}} = 127.092 \text{ g Cu}$$

Therefore, 2 moles of copper has a mass of 127.092 grams.

You can also find the molar mass of a molecule, like methane. Just like with elements, a mole of a molecule has the same mass in grams as a single molecule has in amus.

Example: What is the mass of 3.5 moles of methane?

Solution: Methane (CH_4) has a molecular mass of 16.043 amu, which means 1 mole of methane has a mass of 16.043 grams.

$$\frac{3.5 \text{ mol } CH_4}{} \times \frac{16.043 g CH_4}{1 \text{ mol } CH_4} = 56.151 \text{ g } CH_4$$

You can also use the molar mass to determine how many moles of a substance there are in a given mass of that substance.

Example: A standard AAA battery contains about 7.00 g of zinc. How many moles of zinc are in a AAA battery?

Solution: Zinc's molar mass is 65.38 g/mol.

$$\frac{7.00 \text{ g Zn}}{} \times \frac{1 \text{ mol Zn}}{65.38 \text{ g Zn}} \approx 0.107 \text{ g Zn}$$

In summary, a mole of a substance contains approximately 6.02×10^{23} particles (atoms or molecules) of that substance and has a mass equal to the molecular mass in grams.

The Mole Concept

For a substance, X, with a molar mass of x g/mol,

$$1 \text{ mol X } = 6.02 \times 10^{23} \text{ particles of X } = x \text{ g of X}$$

Exercise 13 Grams, Moles, Molecules, and Atoms

Working Space

Complete the table.

Substance	num. of particles	num. of moles	grams
$NaHCO_3$			35
HCl		1.2	
KH_2PO_4	12.5×10^{24}		

Answer on Page 74

Exercise 14 Burning Methane

Working Space

Natural gas is mostly methane (CH_4). When one molecule of methane burns, two oxygen molecules (O_2) are consumed. One molecule of H_2O and one molecule of CO_2 are produced.

If you need 200 grams of water, how many grams of methane do you need to burn?

(This is how the hero in "The Martian" made water for his garden.)

Answer on Page 75

If you fill a balloon with helium, it will have two different kinds of helium atoms. Most of the helium atoms will have 2 neutrons, but a few will have only 1 neutron. We say that these are two different *isotopes* of helium. We call them helium-4 (or ^4He) and helium-3 (or ^3He). Isotopes are named for the sum of protons and neutrons the atom has: helium-3 has 2 protons and 1 neutron.

A hydrogen atom nearly always has just 1 proton and no neutrons. A helium atom nearly always has 2 protons and 2 neutrons. So, if you have a 100 hydrogen atoms and 100 helium atoms, the helium will have about 4 times more mass than the hydrogen. We say "Hydrogen is about 1 atomic mass unit (amu), and helium-4 is about 4 atomic mass units."

What, precisely, is an atomic mass unit? It is defined as 1/12 of the mass of a carbon-12 atom. Scientists have measured the mass of helium-4, and it is about 4.0026 atomic mass units. (By the way, an atomic mass unit is also called a *dalton*.)

Now you are ready to take a good look at the periodic table of elements. Here is the version from Wikipedia:

Periodic Table of Elements

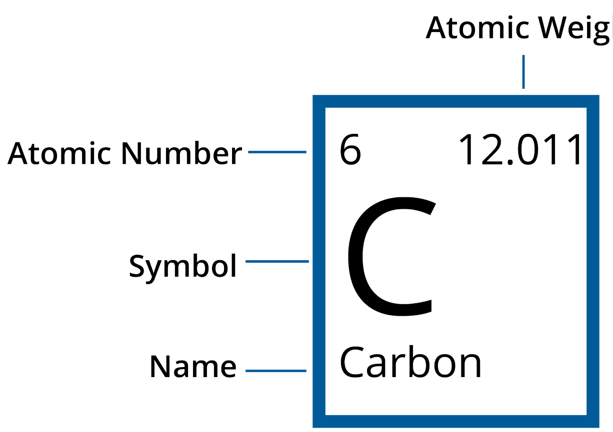
IA	IIA	IIIB	IVB	VB	VIB	VIIB	VIIIB	VIIIB	VIIIB	IB	IIB	IIIA	IVA	VA	VIA	VIIA	VIIIA
1 **H** Hydrogen 1.01																	2 **He** Helium 4.00
3 **Li** Lithium 6.94	4 **Be** Beryllium 9.01											5 **B** Boron 10.81	6 **C** Carbon 12.01	7 **N** Nitrogen 14.01	8 **O** Oxygen 16.00	9 **F** Fluorine 19.00	10 **Ne** Neon 20.18
11 **Na** Sodium 22.99	12 **Mg** Magnesium 24.31											13 **Al** Aluminum 26.98	14 **Si** Silicon 28.09	15 **P** Phosphorus 30.97	16 **S** Sulfur 32.06	17 **Cl** Chlorine 35.45	18 **Ar** Argon 39.95
19 **K** Potassium 39.10	20 **Ca** Calcium 40.08	21 **Sc** Scandium 44.96	22 **Ti** Titanium 47.87	23 **V** Vanadium 50.94	24 **Cr** Chromium 52.00	25 **Mn** Manganese 54.94	26 **Fe** Iron 55.85	27 **Co** Cobalt 58.93	28 **Ni** Nickel 58.69	29 **Cu** Copper 63.55	30 **Zn** Zinc 65.38	31 **Ga** Gallium 69.72	32 **Ge** Germanium 72.63	33 **As** Arsenic 74.92	34 **Se** Selenium 78.97	35 **Br** Bromine 79.90	36 **Kr** Krypton 83.80
37 **Rb** Rubidium 85.47	38 **Sr** Strontium 87.62	39 **Y** Yttrium 88.91	40 **Zr** Zirconium 91.22	41 **Nb** Niobium 92.91	42 **Mo** Molybdenum 95.95	43 **Tc** Technetium (98)	44 **Ru** Ruthenium 101.07	45 **Rh** Rhodium 102.91	46 **Pd** Paladium 106.42	47 **Ag** Silver 107.87	48 **Cd** Cadmium 112.41	49 **In** Indium 114.82	50 **Sn** Tin 118.71	51 **Sb** Antimony 121.76	52 **Te** Tellurium 127.60	53 **I** Iodine 126.90	54 **Xe** Xenon 131.29
55 **Cs** Cesium 132.91	56 **Ba** Barium 137.33	57 - 71 **Lanthanides**	72 **Hf** Hafnium 178.49	73 **Ta** Tantalum 180.95	74 **W** Tungsten 183.84	75 **Re** Rhenium 186.21	76 **Os** Osmium 190.23	77 **Ir** Iridium 192.22	78 **Pt** Platinum 195.08	79 **Au** Gold 196.97	80 **Hg** Mercury 200.59	81 **Tl** Thallium 204.38	82 **Pb** Lead 207.20	83 **Bi** Bismuth 208.98	84 **Po** Polonium (209)	85 **At** Astatine (210)	86 **Rn** Radon (222)
87 **Fr** Francium (223)	88 **Ra** Radium (226)	89 - 103 **Actinides**	104 **Rf** Rutherfordium (265)	105 **Db** Dubnium (268)	106 **Sg** Seaborgium (271)	107 **Bh** Bohrium (270)	108 **Hs** Hassium (277)	109 **Mt** Meitnerium (276)	110 **Ds** Darmstadtium (281)	111 **Rg** Roentgenium (280)	112 **Cn** Copernicium (285)	113 **Nh** Nihonium (284)	114 **Fl** Flerovium 289	115 **Mc** Moscovium (288)	116 **Lv** Livermorium (293)	117 **Ts** Tennessine (294)	118 **Og** Oganesson (294)

57 **La** Lanthanum 138.91	58 **Ce** Cerium 140.12	59 **Pr** Praseodymium 140.91	60 **Nd** Neodymium 144.24	61 **Pm** Promethium (145)	62 **Sm** Samarium 150.36	63 **Eu** Europium 151.96	64 **Gd** Gadolium 157.25	65 **Tb** Terbium 158.93	66 **Dy** Dysprosium 162.50	67 **Ho** Holmium 164.93	68 **Er** Erbium 167.26	69 **Tm** Thulium 168.93	70 **Yb** Ytterbium 173.05	71 **Lu** Lutetium 174.97
89 **Ac** Actinium (227)	90 **Th** Thorium 232.04	91 **Pa** Protactinium 231.04	92 **U** Uranium 238.03	93 **Np** Neptunium (237)	94 **Pu** Plutonium (244)	95 **Am** Americium (243)	96 **Cm** Curium (247)	97 **Bk** Berkelium (247)	98 **Cf** Californium (251)	99 **Es** Einsteinium (252)	100 **Fm** Fermium (257)	101 **Md** Mendelevium (258)	102 **No** Nobelium (259)	103 **Lr** Lawrencium (262)

Figure 8.1: Periodic table from Wikipedia.

There is a square for each element. In the middle, you can see the atomic symbol and the name of the element. In the upper-right corner is the atomic number — the number of protons in the atom.

In the upper-left corner is the atomic mass in atomic mass units.



Look at the atomic mass of boron. About 80% of all boron atoms have six neutrons. The other 20% have only 5 neutrons. This difference is why most boron atoms have a mass of about 11 atomic mass units, but some have a mass of about 10 atomic mass units. The atomic mass of boron is equivalent to the average mass of a boron atom: 10.811.

Exercise 15 Mass of a Water Molecule

Working Space

Using the periodic table, what is the average mass of one water molecule in atomic mass units?

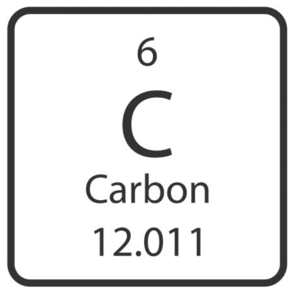
Answer on Page 75

8.4 xfer from intro chapter

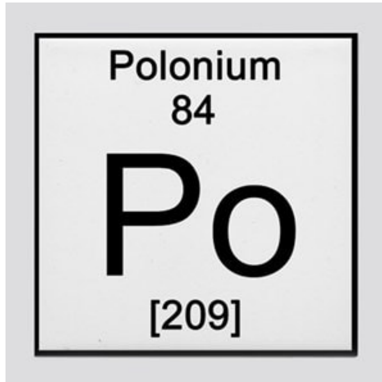
fixme integrate into this chapter

8.4.1 Reading the Periodic Table

The Periodic Table organizes what we know about the structure of different elements. Each element has its own block or tile on the Periodic Table, and the information on the tile tells us about the structure of that atom. Take a look at the tile for carbon:



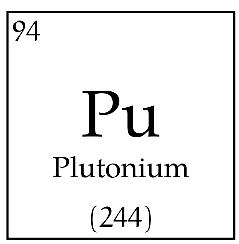
The letter (or letters, as is the case for other elements) is the atomic symbol for the element. There are two key numbers: the atomic number and the average atomic mass. For carbon, the atomic number is 6 and the average atomic mass is 12.011. The atomic number tells us how many protons there are in the nucleus of any atom of carbon. Since every element has a unique number of protons, every element has a unique atomic number. All carbon atoms have 6 protons. The other number is the average atomic mass - it tells us the weighted average of the mass of all the carbons in the universe. When the average atomic mass is in a whole number, as it is for polonium, it means that the element is very unstable. As a result, the mass given is the mass of the most stable isotope (we'll talk more about stability and isotopes below). On some periodic tables, the mass number of the most stable isotope will be in parentheses or brackets. In summary, if the larger number is a whole number, it is the mass number; if it is a decimal (even if the decimal ends in .00), it is the average atomic mass, which we will discuss further below.



The Royal Society of Chemistry has a very useful interactive periodic table: periodic-table.rsc.org. We can use the periodic tile for an element to determine the number of protons, electrons, and most common number of neutrons for a neutral atom of that element (we'll explain why the periodic tile tells us the "most common number of neutrons" below).

Example: State the atomic symbol for and the number of protons, neutrons, and electrons in a neutral atom of plutonium.

Solution: The plutonium tile on your periodic table should look something like this:



[The information may be arranged differently, but you should at least see the symbol and two numbers.] As you can see, the atomic symbol for plutonium is Pu. Since its atomic number is 94, we know every atom of plutonium has 94 protons. To know the number of electrons, we will take advantage of the fact that the question is asking about a *neutral* atom. This means there are the same number of positive charges as negative charges. So, since there are 94 protons, a neutral atom of plutonium must have 94 electrons (each proton has a +1 charge and each electron has a -1 charge). Lastly, let's determine the number of neutrons. The other number, 244, is the mass number. It represents the total number of protons and neutrons in the nucleus. Since we know plutonium has 94 protons, we can find the number of neutrons by subtracting the atomic number from the mass number:

$$244 - 94 = 150$$

mass number (protons + neutrons) — atomic number (protons) = number of neutrons

Therefore, an atom of plutonium has 150 neutrons. Now let's address how to find the number of neutrons when the periodic tile shows an average atomic mass, instead of a mass number. This occurs when there is more than one "version" of an element. In the case of plutonium, there is only one version, which is why the periodic tile shows a mass number instead of an average atomic mass. To learn about average atomic mass, we will use carbon as an example.

Have you heard of carbon-14 dating? The phrase "carbon-14" refers to a rare type of carbon that decays radioactively. By seeing how much carbon-14 has decayed, scientists can estimate the age of organic materials, such as bone or ash. Carbon-14 is a radioactive isotope (or version) of carbon. The 14 refers to the mass number - the total amount of protons and neutrons in the nucleus (sometimes, we shorten the isotope name by just using the atomic symbol, in this case C-14). Isotopes are versions of an element with different numbers of neutrons. The atomic number is the same for them all - they all have the same number of protons. But the different number of neutrons causes different isotopes to have different masses. Examine the models of carbon-12, carbon-13, and carbon-14 below. What is different between them? What is the same?

You should have noticed that all three atoms have 6 protons and 6 electrons, while they have differing numbers of neutrons. The most common isotope of carbon is carbon-12, with 6 protons and 6 neutrons in its nucleus. Carbon-14, on the other hand, has 8 neutrons, which makes the nucleus unstable, leading to radioactive decay. The average atomic mass is the weighted average of all the carbon atoms in existence. Since the vast majority of carbon is carbon-12, the average atomic mass is very close to 12. You cannot determine the mass number of an individual atom from the periodic table; it only tells you the average of all the isotopes. However, especially for light atoms (atoms in the first two rows of the periodic table), you can usually determine the mass number of the most common isotope by rounding the average atomic mass to the nearest whole number.

Example: Germanium has atomic symbol Ge. State the number of protons, number of electrons, and most common number of neutrons in a neutral atom of germanium.

Solution: Examining the periodic table, we see that germanium has an atomic number of 32, which means a neutral atom of germanium has 32 protons and 32 electrons. The average atomic mass is 72.630, which rounds up to 73. So, the most common isotope of germanium is Ge-73, which has $73 - 32 = 41$ neutrons.

Exercise 16 Determining Numbers of Subatomic Particles

Working Space

Use a periodic table to complete the table below (assume neutral atoms):

Element Name	Atomic Symbol	Protons	Most Common Number of Neutrons	Electrons
	Fr			
				33
Erbium				
		48		

Answer on Page 76

8.5 Heavy atoms aren't stable

When you look at the periodic table, there are a surprisingly large number of elements. You might be told to "Drink milk so that you can get the calcium you need." However, no one has told you "You should eat kale so that you get enough copernicium in your diet."

Copernicium, with 112 protons and 173 neutrons, has only been observed in a lab. It is highly radioactive and unstable (meaning it decays). A copernicium atom usually lives for less than a minute before decaying.

The largest stable element is lead, which has 82 protons and between 122 and 126 neutrons. Elements with lower atomic numbers than lead, have at least one stable isotope, while elements with higher atomic numbers than lead don't.

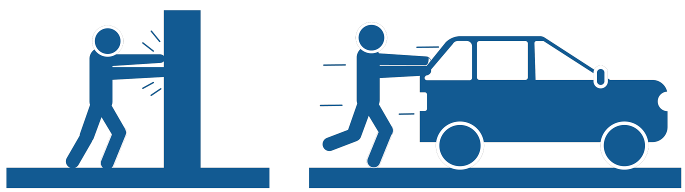
Bismuth, with an atomic number of 83, is *almost* stable. In fact, most bismuth atoms will live for billions of years before decaying!

Chapter 9

Work and Energy

In this chapter, we are going to talk about how engineers define work and energy. It frequently takes force to get work done. Let's start with thinking about the relationship between force and energy. As we learned earlier, Force is measured in newtons, and one newton is equal to the force necessary to accelerate one kilogram at a rate of $1m/s^2$.

When you lean on a wall, you are exerting a force on the wall, but you aren't doing any work. On the other hand, if you push a car for a mile, you are clearly doing work. Work, to an engineer, is the force you apply to something, as well as the distance that something moves, in the direction of the applied force. We measure work in *joules*. A joule is one newton of force applied over one meter.



For example, if you push a car with a force of 10 newtons for 12 meters, you have done 120 joules of work.

Formula for Work

$$W = F \cdot d$$

where W is the work in joules, F is the *force* in newtons, and d is the distance in meters.

If the force is not in the same direction as the distance, we can use the cosine of the angle between the force and the distance:

$$W = F \cdot d \cdot \cos(\theta)$$

where θ is the angle between the force and the distance.

The work-energy theorem (or work-energy principle) states that the net work done on an object is equal to the **change in its kinetic energy**. In other words, if you do work on an object, you are changing its kinetic energy. This is derived from Newton's second law of motion, covered in the previous chapter.

$$W = \Delta E = \Delta KE (\text{with units of Joules (J) or Newton-meters (Nm)})$$

Work is how energy is transferred from one thing to another. When you push the car, you also burn sugars (energy of the body) in your blood. That energy is then transferred to the car after it has been pushed uphill.

Thus, we measure the energy something consumes or generates in units of work: joules, kilowatt-hours, horsepower-hours, foot-pounds, BTUs (British Thermal Unit), and calories.

Let's go over a few different forms that energy can take.

9.1 Forms of Energy

In this section we are going to learn about several different types of energy:

- Heat
- Electricity
- Chemical Energy
- Kinetic Energy
- Gravitational Potential Energy

9.1.1 Heat

When you heat something, you are transferring energy to it. The BTU is a common unit for heat. One BTU is the amount of heat required to raise the temperature of one pound of water by one degree. One BTU is about 1,055 joules. In fact, when you buy and sell natural gas as fuel, it is priced by the BTU.

9.1.2 Electricity

Electricity is the movement of electrons. When you push electrons through a space that resists their passage (like a light bulb), energy is transferred from the power source (like a battery) into the source of the resistance.

Let's say your lightbulb consumes 60 watts of electricity, and you leave it on for 24 hours. We would say that you have consumed 1.44 kilowatt hours, or 3,600,000 joules.

9.1.3 Chemical Energy

As mentioned earlier, some chemical reactions consume energy and some produce energy. This means energy can be stored in the structure of a molecule. When a plant uses photosynthesis to rearrange water and carbon dioxide into a sugar molecule, it converts the energy in the sunlight (solar energy) into chemical energy. Remember that photosynthesis is a process that consumes energy. Therefore, the sugar molecule has more chemical energy than the carbon dioxide and water molecules that were used in its creation.

In our diet, we measure this energy in *kilocalories*. A calorie is the energy necessary to raise one gram of water one degree Celsius, and is about 4.19 joules. This is a very small unit. An apple has about 100,000 calories (100 kilocalories), so people working with food started measuring everything in kilocalories.

Here is where things get tricky: People who work with food got tired of saying "kilocalories", so they just started using "Calorie" to mean 1,000 calories. This has created a great deal of confusion over the years. So if the C is capitalized, "Calorie" probably means kilocalorie.

9.1.4 Kinetic Energy

A mass in motion has energy. For example, if you are in a moving car and you slam on the breaks, the energy from the motion of the car will be converted into heat in the breaks and under the tires.

How much energy does the car have?

$$E = \frac{1}{2}mv^2$$

Formula for Kinetic Energy

$$E = \frac{1}{2}mv^2$$

where E is the energy in joules, m is the mass in kilograms, and v is the speed in meters per second.

9.1.5 Gravitational Potential Energy

When you lift something heavy onto a shelf, you are giving it *potential energy*. The amount of energy that you transferred to it is proportional to its weight and the height that you lifted it.

$$E = mgh$$

a rate of $9.8 m/s^2$.

Formula for Gravitational Potential Energy

The formula for gravitational potentional energy is

$$E = mgh$$

where E is the energy in joules, m is the mass of the object you lifted, g is acceleration due to gravity, and h is the height that you lifted it.

On earth, then, gravitational potential energy is given by

$$E = (9.8)mh$$

since objects in free-fall near Earth's surface accelerate at $9.8 m/s^2$.

There are other kinds of potential energy. For example, when you draw a bow in order to fire an arrow, you have given that bow potential energy. When you release it, the potential energy is transferred to the arrow, which expresses it as kinetic energy.

9.2 Conservation of Energy

The first law of thermodynamics says "Energy is neither created nor destroyed."

Energy can change forms. Your cells consume chemical energy to give gravitational potential energy to a car you push up a hill. However, the total amount of energy in a closed system stays constant.

Exercise 17 The Energy of Falling

Working Space

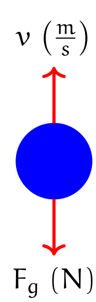
A 5 kg cannonball falls off the top of a 3 meter ladder. As it falls, its gravitational potential energy is converted into kinetic energy. How fast is the cannonball traveling just before it hits the floor?

Answer on Page 76

9.3 Work and Kinetic Energy

As stated above, the work-energy theorem tells us that the change in an object's kinetic energy is equal to the work done on that object. For now, we will only consider examples where the force and the direction of motion are parallel or perpendicular. When you learn about vectors, we will expand this to include forces that are skew to the direction of motion.

Consider what happens when you toss a ball in the air: once the ball leaves your hand, the only force acting on it is gravity. Initially, the ball is moving upwards while gravity points downwards:



Intuitively, we know that the ball will slow down (lose kinetic energy) as it moves upwards:

$$\Delta KE < 0$$

Since $W = \Delta KE$, we also know that gravity must be doing *negative work*. Whenever the direction of the force is opposite the direction of the motion, the work done by that force is negative.

Example: if the ball has a mass of 0.5 kg, how much kinetic energy does it lose as it moves upwards by 1 m?

Solution: The force acting on the ball is its weight, $F_w = mg$, and we will designate this as negative since weight points downwards. Using the work-energy theorem,

$$\Delta KE = F \cdot d = (mg) \cdot d = (0.5kg)\left(-9.8\frac{m}{s^2}\right)(1m) = -4.9J$$

Therefore, the ball loses 4.9 joules of kinetic energy for every 1 meter it moves upwards (the fact it is *losing kinetic energy* is represented by the result being negative).

Exercise 18 How far will you slide?

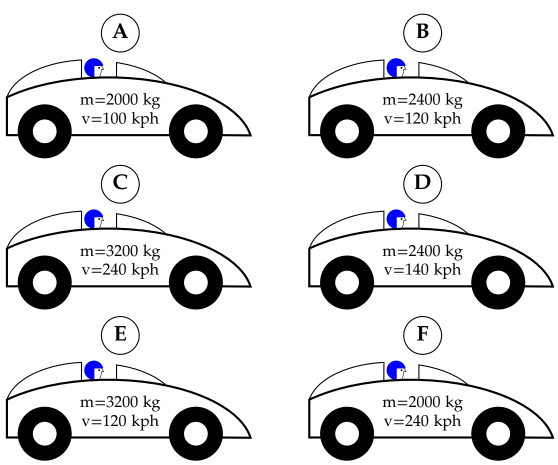
You are playing softball and have to slide into home. If you sprint at a maximum of 10 m/s and the force of friction between you and the ground is 0.3 times your weight, how far from the base can you start your slide and still reach home?

Working Space

Answer on Page 76

Exercise 19 Ranking Stopping Force

In drag racing, cars can reach speeds of 150 miles per hour (approximately 240 kilometers per hour). In order to be able to stop quickly and safely, drag racing cars are built with parachutes that deploy at the end of the race. Consider a drag race where cars of different masses reach different maximum speeds. There is 100 meters between the finish line and the fence surrounding the race track. If all the race cars deploy their parachutes at the finish line while going their maximum speed, rank the force needed from the parachute to stop each car in the required distance from least to greatest:



Working Space

Answer on Page 77

9.3.1 Forces that do no work

If the object you are pushing doesn't move. or the applied force is perpendicular to the direction of motion, that force does no work. Let's look at a few examples:

Pushing Against an Immobile Object

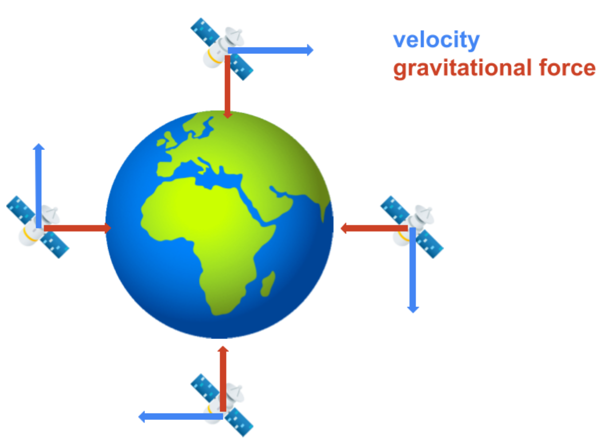
At the beginning of the chapter, we said that when you push on a wall, you don't do any work. Why is this? Well, if the wall is a good wall (that is, strong enough to not be pushed over by a person), the wall won't move while you push on it. This means the distance over which your push is applied is zero, and therefore the work done ($F \cdot d = F \cdot 0 = 0$) is zero joules.

Walking Across a Room with a Book

Imagine holding a book flat on your hands and walking at a constant velocity. Your hand is applying an upwards force to the book, but the book is moving horizontally. This means the force and direction of motion are *perpendicular*. Recall from the beginning of the chapter than if the force and distance are not parallel, then the work is given by $W = F \cdot d\cos(\theta)$. (When the vectors are parallel, $\theta = 0$ and $\cos(\theta) = 1$, while when the vectors point in opposite directions, $\theta = 180°$ and $\cos(\theta) = -1$.) When the vectors are perpendicular, then $\theta = 90°$ and $\cos(\theta) = 0$. Therefore, $W = 0$ as well and the upward force of your hands does no net work.

Circular Motion

We will discuss circular motion further in a subsequent chapter. For now, know that constant-speed circular motion is caused by a constant-magnitude force that always points to the center of the circle the object is moving in. For example, you can take a weight on the end of a string and spin it. The tension in the string spins the weight, and the string always points from the object to your hand (the center of the weight's circular path). For a satellite, that force is gravitational attraction to the Earth.



As a result, the force changes the *direction*, but not the *magnitude* of the satellite's velocity. Let's re-examine the equation for kinetic energy:

$$\mathrm{KE} = \frac{1}{2}mv^2$$

Since the velocity is squared, the direction of motion doesn't affect the kinetic energy (a ball moving at 5 m/s upwards has the same kinetic energy as if the ball were moving at 5 m/s downwards). So, a force that causes circular motion doesn't change a circling object's kinetic energy, and therefore does no work (as expected when force and direction of motion are perpendicular)!

9.4 Efficiency

Although energy is always conserved as it moves through different forms, scientists aren't always that good at controlling it.

In terms of an equation, efficiency is the ratio of the useful energy output to the total energy input. It is usually expressed as a percentage.

Formula for Efficiency

$$\text{Efficiency} = \frac{\text{Useful Energy Output}}{\text{Total Energy Input}} \times 100\%$$

where the useful energy output is the energy that is actually used to do work or complete a task, and the total energy input is the total energy consumed by the system.

A machine is considered 100% efficient only if all the input work is converted into useful output work, with no energy lost to heat, friction, or sound. 100% efficient process don't exist in real-life: every process loses some useful energy to heat.

For example, when a car engine consumes the chemical energy in gasoline, only about 20% of the energy consumed is used to turn the wheels. Most of the energy is actually lost as heat. If you run a car for a while, the engine gets very hot, as does the exhaust coming from the tailpipe.

A human is about 25% efficient. Most of the loss is in the heat produced during the chemical reactions that turns food into motion.

In general, if you are trying to increase efficiency in any system, the solution is usually easy to identify by the heat that is produced. Reduce the heat, increase the efficiency.

Light bulbs are an interesting case. To get the same amount of light of a 60 watt incandescent bulb, you can use an 8 watt LED or a 16 watt fluorescent light. This is why we say that the LED light is much more efficient. If you run both, the incandescent bulb will consume 1.44 kilowatt-hours; the LED will consume only 0.192 kilowatt-hours.

In addition to light, the incandescent bulb is producing a lot of heat. If it is inside your house, what happens to the heat? It warms your house.

In the winter, when you want light and heat, the incandescent bulb is 100% efficient!

Of course, this also means the reverse is true. In the summer, if you are running the air conditioner to cool down your house, the incandescent bulb is worse than just "inefficient at making light" — it is actually counteracting the air conditioner!

Appendix A

Answers to Exercises

Answer to Exercise 1 (on page 10)

1. C:H = 1:4
2. Cu:S:O = 1:1:4
3. C:H:O = 6:12:6 = 1:2:1

Answer to Exercise 2 (on page 19)

1. balanced
2. unbalanced; oxygen
3. unbalanced; barium, sulfur, oxygen, and carbon
4. balanced

Answer to Exercise 3 (on page 22)

1. $v = \sqrt{2gh} = \sqrt{2\,(9.8\mathrm{m/s^2})\,(3.7\mathrm{m})} = 8.5\mathrm{m/s}$
2. $h = \frac{v^2}{2g} = \frac{(15\frac{m}{s})^2}{2\left(9.8\frac{m}{s^2}\right)} = 11.5m$. You can hit your little brother, since you could shoot the pebble as high as 11.5 m and he is only 5.0 m above you.
3. We know that $E_{p,initial} = E_{k,final} + E_{losttofriction}$. Therefore, $E_{losttofriction} = E_{p,initial} - E_{k,final} = mgh - \frac{1}{2}mv^2 = (63kg)\left(9.8\frac{m}{s^2}\right)(8m) - \frac{1}{2}(63kg)\left(11.2\frac{m}{s}\right)^2 = 4939.2J - 3951.4J = 988J$.

Answer to Exercise 4 (on page 25)

1. The human body is an open system because matter can enter (food, water, oxygen) and leave (waste, carbon dioxide, sweat) your body.
2. The Earth is an open system because matter can enter (asteroids falling, spaceships returning) and leave (space vehicles and astronauts). On the other hand, the Earth can be well-approximated as a closed system. Before the 20th Century, humans had no way to deliberately expel matter from the Earth, and the mass of asteroids that are pulled in by the Earth's gravity is negligible compared to the Earth. Therefore, in the right circumstances, it would be appropriate to model the Earth as a closed system. (It is closed because energy in the form of sunlight is constantly entering the system.)
3. A cell phone is a closed system - you don't put any matter in or take it out of your phone, but it constantly uses battery and then is recharged, showing that energy enters and leaves your phone.
4. Since the cooler is described as well-insulated and the lid is closed, it can be approximated as an isolated system. Scientific equipment, like bomb calorimeters, rely on this approximation.
5. With the lid open, matter can enter and leave and therefore the cooler is an open system (yes, even though it is well-insulated).
6. A sealed bottle of soda is a closed system - the soda and carbon dioxide can't escape, but energy in the form of heat can be transferred in and out of the system (the contents of the bottle will lose heat if you put it in the fridge and gain heat if you leave it in the sun).

Answer to Exercise 5 (on page 29)

$$F = G\frac{m_1 m_2}{r^2} = (6.674 \times 10^{-11})\frac{(6.8^3)(6 \times 10^{24})}{(10^5)^2} = 6.1 \times 10^6$$

About 6 million newtons.

Answer to Exercise 6 (on page 32)

1. acceleration: the satellite is moving in a circle, therefore changing direction and accelerating.
2. Not acceleration: the car isn't changing speed or direction.
3. Acceleration: the child is changing speed.

4. Acceleration: the roller coaster is changing direction.
5. Acceleration: the roller coaster is changing speed.
6. Not acceleration: the book isn't changing speed or direction.

Answer to Exercise 7 (on page 34)

If you accelerate to 20 m/s in 120 s, the acceleration is:

$$a = \frac{v_f - v_i}{t} = \frac{20 \text{ m/s} - 0 \text{ m/s}}{120 \text{ s}} = \frac{1}{6} \frac{\text{m}}{\text{s}^2}$$

To achieve this acceleration, you will need to apply a force of:

$$F = m \cdot a = 2400 \text{ kg} \cdot \frac{1}{6} \frac{\text{m}}{\text{s}^2} = 400 \text{ N}$$

Answer to Exercise 8 (on page 36)

C, A, E/F, B, D

$$a_A = \frac{F_{net,A}}{m_A} = \frac{4N - 3N}{1kg} = \frac{1N}{1kg} = 1\frac{m}{s^2}$$

$$a_B = \frac{F_{net,B}}{m_B} = \frac{3N - 4N}{1kg} = \frac{-1N}{1kg} = -1\frac{m}{s^2}$$

$$a_C = \frac{F_{net,C}}{m_C} = \frac{4N - 3N}{0.5kg} = \frac{1N}{0.5kg} = 2\frac{m}{s^2}$$

$$a_D = \frac{F_{net,D}}{m_D} = \frac{3N - 4N}{0.5kg} = \frac{-1N}{0.5kg} = -2\frac{m}{s^2}$$

$$a_E = \frac{F_{net,E}}{m_E} = \frac{5N - 5N}{0.5kg} = \frac{0N}{0.5kg} = 0\frac{m}{s^2}$$

$$a_F = \frac{F_{net,F}}{m_F} = \frac{5N - 5N}{1kg} = \frac{0N}{1kg} = 0\frac{m}{s^2}$$

Answer to Exercise 9 (on page 39)

3, 1/2, 6, 5, 4 The velocity does not affect the apparent weight, only the acceleration. If the elevator is accelerating upwards, the apparent weight increases. If the elevator is accelerating downwards, the apparent weight decreases. In fact, for scenario 4, the elevator is in free-fall and the person has no apparent weight. When the elevator is not accelerating, the person's apparent weight is their true weight.

Answer to Exercise 10 (on page 43)

$$4.5\ \cancel{\text{kWh}} \left(\frac{3.6 \times 10^6\ \cancel{\text{joules}}}{1\ \cancel{\text{kWh}}}\right) \left(\frac{1\ \text{calories}}{4.184\ \cancel{\text{joules}}}\right) = \frac{(4.5)(3.6 \times 10^6)}{4.184} = 1.08 \times 10^6 \text{calories}$$

Answer to Exercise 11 (on page 44)

$$\frac{0.1\ \cancel{\text{gallons}}}{2\ \cancel{\text{minutes}}} \left(\frac{3.7854\ \cancel{\text{liters}}}{1\ \cancel{\text{gallons}}}\right) \left(\frac{1000\ \text{milliliters}}{1\ \cancel{\text{liters}}}\right) \left(\frac{1\ \cancel{\text{minutes}}}{60\ \text{seconds}}\right) =$$
$$\frac{(0.1)(3.7854)(1000)}{(2)(60)}\ \text{ml/second} = 3.1545\ \text{ml/second}$$

Answer to Exercise 12 (on page 49)

1. 12.011 amu + 4(1.008 amu) = 16.043 amu
2. 63.546 amu + 32.06 amu + 4(15.999 amu) = 159.602 amu
3. 6(12.011 amu) + 12(1.008 amu) + 6(15.999) amu = 180.156 amu

Answer to Exercise 13 (on page 51)

Substance	num. of particles	num. of moles	grams
$NaHCO_3$	2.509×10^{23}	0.4166	35.00
HCl	7.53×10^{23}	1.25	45.58
KH_2PO_4	12.5×10^{24}	20.8	2820

$$\frac{35.00\ \text{g NaHCO}_3}{} \times \frac{1\ \text{mol NaHCO}_3}{84.007\ \text{g NaHCO}_3} = 0.4166\ \text{mol NaHCO}_3$$

$$\frac{0.4166 \text{ mol NaHCO}_3}{} \times \frac{6.02214076 \times 10^{23} \text{ molec NaHCO}_3}{1 \text{ mol NaHCO}_3} = 2.509 \times 10^{23} \text{ molec NaHCO}_3$$

$$\frac{1.25 \text{ mol HCl}}{} \times \frac{36.46 \text{ g HCl}}{1 \text{ mol HCl}} = 45.58 \text{ g HCl}$$

$$\frac{1.25 \text{ mol HCl}}{} \times \frac{6.02214076 \times 10^{23} \text{ molec HCl}}{1 \text{ mol HCl}} = 7.53 \times 10^{23} \text{ molec HCl}$$

$$\frac{12.5 \times 10^{24} \text{ molec KH}_2\text{PO}_4}{} \times \frac{1 \text{ mol KH}_2\text{PO}_4}{6.02214076 \times 10^{23} \text{ molec KH}_2\text{PO}_4} = 20.8 \text{ mol KH}_2\text{PO}_4$$

$$\frac{20.8 \text{ mol KH}_2\text{PO}_4}{} \times \frac{136.086 \text{ g KH}_2\text{PO}_4}{1 \text{ mol KH}_2\text{PO}_4} = 2820 \text{ g KH}_2\text{PO}_4$$

Answer to Exercise 14 (on page 52)

From the last exercise, you know that 1 mole of water weighs 18.01528 grams, meaning 200 grams of water is about 11.1 moles. So you need to burn 11.1 moles of methane.

What does one mole of methane weigh? Using the periodic table: $12.0107 + 4 \times 1.00794 = 16.04246$ grams.

$16.0424 \times 11.10 = 178.1$ grams of methane.

Answer to Exercise 15 (on page 56)

The average hydrogen atom has a mass of 1.00794 atomic mass units.

The average oxygen atom has a mass of 15.9994.

$2 \times 1.00794 + 15.9994 = 18.01528$ atomic mass units.

Answer to Exercise 16 (on page 60)

Element Name	Atomic Symbol	Protons	Most Common Number of Neutrons	Electrons
Francium	Fr	87	136	87
Arsenic	As	33	42	33
Erbium	Er	68	99	68
Cadmium	Cd	48	64	48

Answer to Exercise 17 (on page 65)

At the top of the ladder, the cannonball has $(9.8)(5)(3) = 147$ joules of potential energy.

At the bottom, the kinetic energy $\frac{1}{2}(5)v^2$ must be equal to 147 joules. So $v^2 = \frac{294}{5}$. This means it is going about 7.7 meters per second.

(You may be wondering about air resistance. Yes, a tiny amount of energy is lost to air resistance, but for a dense object moving at these relatively slow speeds, this energy is negligible.)

Answer to Exercise 18 (on page 66)

$$F_f \cdot d = \Delta KE = KE_f - KE_i$$

You'll reach the maximum distance you can slide when you stop moving, so we will use a final velocity of zero, which means a final kinetic energy of zero:

$$F_f \cdot d = -KE_i = \frac{1}{2}mv^2$$

Since the force of friction is 0.3 times your weight, we know that:

$$F_f = 0.3F_w = 0.3mg$$

Substituting and canceling the mass:

$$(0.3mg) \cdot d = \frac{1}{2}mv^2$$

$$0.3g \cdot d = \frac{1}{2}v^2$$

Since we know g and v, we can solve for d:

$$d = \frac{v^2}{0.6g} = \frac{\left(10\frac{m}{s}\right)^2}{0.6\left(9.8\frac{m}{s^2}\right)} \approx 1.7m$$

So, if you want to reach home base, you should start your slide no more than 1.7 m from home.

Answer to Exercise 19 (on page 67)

Since all the cars need to stop in the same distance, the cars with the greatest kinetic energy will take the most force to stop. Calculating the kinetic energies (we won't change the units from kilometers per hour to meters per second, since we're just comparing the values):

Car	Mass [kg]	Max speed [kph]	KE [kg $(kph)^2$]
A	2000	100	1×10^7
B	2400	120	1.728×10^7
C	3200	240	9.216×10^7
D	2400	140	2.352×10^7
E	3200	120	2.304×10^7
F	2000	240	5.67×10^7

The correct ranking is A, B, E, D, F, C.

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